Why HANK matters for policy

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Abstract

When do optimal inflation and quantities differ significantly between Heterogeneous-Agent (HA) and Representative-Agent (RA) models, and what are the underlying mechanisms? To answer this question, we derive the jointly optimal fiscal-monetary Ramsey policy in HA and RA models that incorporate both price and wage stickiness. We examine different sets of fiscal tools and analyze both supply and demand shocks. Our findings show that HA economies diverge significantly from RA economies when the severity of credit constraints varies over time, which is the case for demand shocks but less so for supply shocks. Furthermore, inflation dynamics differ between HA and RA economies in response to demand shocks, particularly when fiscal policy is not employed as a stabilization tool over the business cycle. We identify the relevant fiscal tools to reduce inflation volatility over the business cycle.

Keywords: Heterogeneous agents, wage-price spiral, inflation, monetary policy, fiscal policy.

JEL codes: D31, E52, D52, E21.

1 Introduction

Heterogeneous-Agent (HA) models differ significantly from Representative-Agent (RA) models because some agents in HA models face credit constraints and therefore have a high Marginal Propensity to Consume (MPC), while unconstrained agents engage in time-varying precautionary savings. The primary reason why monetary and fiscal policy implications (if any) differ between these two types of models remains unclear. For instance, a high MPC can also be obtained in a Two-Agent (TA) model, where one agent is always credit-constrained. In such models, the volatility of certain instruments, such as the nominal interest rate, can differ from their

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counterparts in RA models, yet output and inflation dynamics may be very similar between TA and HA models. In this paper, we show that HA and RA models generate different implications for both monetary and fiscal policy due to time-varying precautionary savings, which arise from unconstrained agents' *expectations* of future credit constraints. In this case, the aggregate demand for self-insurance varies over time. Optimal monetary policy must therefore account for its effect on the opportunity cost of self-insurance through its impact on the real interest rate. Optimal fiscal policy must consider its effects on insurance through redistribution and public debt issuance. In both cases, HA and RA models differ. However, the extent to which monetary policy influences self-insurance—beyond its role in ensuring price stability—depends on the availability of fiscal tools. We precautionary saving is not time-varying, as in the TA model, we find that the dynamics of aggregate quantities are very similar in HA and RA model.

We prove this set of results by deriving optimal fiscal and monetary policy in an heterogeneous-agent (HA) model, after both demand and supply shocks, with both price and wage nominal rigidities. In this environment, we study optimal monetary and fiscal policies, with various sets of fiscal instruments, considering optimal Ramsey policy with commitment. Theoretically, it is known that if enough fiscal instruments are available, optimal monetary policy is able to implement price stability, and fiscal policy can use the fiscal instruments to implement the optimal allocation and redistribution (see Correia et al. 2008 or LeGrand et al., 2022). We first confirm this general result in this environment with both incomplete markets, sticky prices and sticky wages. We determine a set of fiscal instruments, which ensure price stability as the optimal outcome after both supply and demand shocks. We then use this benchmark environment to answer two main questions. First, what are the conditions on fiscal policy that optimally generate substantial deviations from price stability? Conversely, what are the fiscal instruments, whose absence that imply the largest optimal price deviations? Second, we investigate the role of heterogeneous agents in the design of optimal policies: When do representative-agent (RA) and heterogeneous-agent (HA) economies have different implications for aggregate variables? We provides three main contributions with respect to these questions.

First, considering demand shocks, time-varying labor tax and public debt are almost sufficient instruments to generate price stability as an optimal outcome. Indeed, demand shocks do not imply any change in the marginal product of labor. Therefore, they do not require a substantial adjustment in the real wage. Although the labor tax involves distortions, there is no need to deviate from price and wage stability, implying zero price and wage inflation close to zero (and thus constant real wage).

Second, considering supply shocks, the story is different: labor tax and public debt do not ensure price stability. These supply shocks can be interpreted as TFP shocks, or as shocks to the price of inputs, such as energy price shocks for instance. Supply shocks change the marginal productivity of labor and thus require an adjustment of the real wage. We show that this adjustment can be obtained with both price and wage stability, when an specific fiscal instrument is optimally designed. This instrument is a wage -subsidy, which is a subvention (or a tax credit) paid by firms to hire workers. It affects the real cost of labor, without having a direct impact on the wage bargained by workers. This tool can be implemented via either a time-varying employer social contribution, or a tax credit conditional on the total wage bill. It is noteworthy that such instruments were massively used in Europe during the Covid-19 crisis to stabilize employment. In Germany, it was called *Kurzarbeit*, while in France, this was called *activité partielle* policy. The time-varying wage subsidy stabilizes the business cycle, without targeting specifically the aggregate demand. For instance, the wage subsidy after a negative supply shocks generates an increase in public debt, but contributes to decrease inflation, by decreasing the cost of inputs. For this reason, we call these fiscal tools *non-Keynesian* stabilizers.

Finally, we find significant differences in the dynamics of aggregate variables between RA and HA economies, mostly because of the dynamic of public debt. In RA economies, changing public debt is a non-distorting tool to generate transfers, through interest payments, between the (representative) agent and the government. In HA economies, such movements of public debt are limited, because public debt is used by agents for self-insurance motives. As a consequence, time-varying liquidity requirements by private agents is a driving force of public debt that is specific to HA economies. Public debt dynamics is thus different in HA and RA economies and the allocations are different at the first-order. In other words, HA and RA economies differ in non-Ricardian environments due to self-insurance and not only due to redistribution. We find that time-varying liquidity requirements are more important for demand (like discount factor shocks) than supply shocks (like TFP shocks). As a consequence, HA and RA economies differ more after demand shocks. This last result happens to be also valid in flexible price economies but is magnified with nominal frictions.

Related literature. This paper belongs to the literature on optimal policy in heterogeneous agent model on one side, and on wage-price spirals on the other side. Deriving optimal policy in heterogeneous-agent models with aggregate shocks is a difficult theoretical and computational task. Some papers consider numerical methods to solve for optimal path of the instruments (Dyrda and Pedroni, 2022). Other papers rely on continuous-time techniques for the theoretical derivation of the first-order conditions of the planner (Nuño and Thomas, 2022 among others). Acharya et al. (2022) solve for optimal monetary policy using the tractability of the CARA-normal environment without capital. Bhandari et al. (2021) quantitatively solve for optimal policies in a new-Keynesian model with aggregate shocks. Yang (2022) solves for the optimal monetary policy by optimizing on the coefficients of a Taylor rule. McKay and Wolf (2022) derive a general quadratic-linear formulation to solve for optimal policy rules. McKay and Reis (2021) study optimal automatic stabilizers in the context of the optimal replacement rate. Their main focus is on the trade-off between insurance and incentives in the presence of an aggregate demand effect. The mechanism we identify is different in that it directly affects the gap between the real wage and the marginal productivity of labor.In this paper, we use the tools of LeGrand and Ragot (2022a) and the improvements of LeGrand and Ragot (2022b) to solve for optimal fiscal and monetary policy with aggregate shocks. The gain of this approach is to allows to easily solve for optimal policy with many tools and with various nominal frictions. On the theoretical side, the Lagrangian approach pioneered in Marcet and Marimon (2019) enables us to derive the first-order conditions of the Ramsey planner in an environment with both wage and price rigidities.

Regarding the literature on wage-price spirals, models including both price and wage stickiness have been studied in RA economies (Blanchard, 1986, Galí, 2015, chapter 6, or Blanchard and Gali, 2007 among others). Erceg et al. (2000) study optimal monetary policy in this environment. Chugh (2006) study both optimal monetary policy and an optimal labor tax. Recently, Lorenzoni and Werning (2023) analyze more deeply optimal policy and the real wage dynamics in this environment.

2 The environment

We consider a discrete-time economy populated by a continuum of size one of ex-ante identical agents. These agents are assumed to be distributed along a set J, with the non-atomic measure ℓ: $\ell(J) = 1$.[1]

2.1 Risk

We assume that the agents face an idiosyncratic productivity risk. The productivity process, denoted y, is assumed to take value in a finite set $\mathcal{Y}$ and to follow a first-order Markov chain with transition matrix $\boldsymbol{\pi} = (\pi_{yy'})_{y,y'}$. With wage w and labor supply l, an agent with productivity y earns the labor income wyl. In each period, the fraction of agents with productivity y is constant and denoted by n_y. We normalize average productivity to 1, i.e., such that $\sum_y n_y y = 1$. The history of idiosyncratic productivity shocks up to date t for an agent i is denoted by $y_i^t = \{y_{i,0}, \ldots, y_{i,t}\} \in \mathcal{Y}^{t+1}$, where $y_{i,\tau}$ is the date-τ productivity. The measure of idiosyncratic histories up-to-date t, denoted by θ_t, can be computed using the initial distribution and the transition matrix.

Aggregate risks. In addition to the previous idiosyncratic risk, agents face an aggregate supply shock, affecting either the economic TFP, denoted by Z, or demand shock, affecting public spending G. We show in Section A that a shock on energy price is equivalent to a negative shock to TFP for the relevant calibration. These aggregate shocks are persistent but are known at period 0, and should thus be considered as MIT shocks.

[1]We follow Green (1994) and assume that the law of large numbers holds.

2.2 Preferences

Households are expected-utility maximizers endowed with time-separable preferences and a constant discount factor $\beta \in (0,1)$. In each period, households enjoy utility $U(c,l)$ from the consumption c of the unique consumption good of the economy and suffer from the disutility of providing the labor supply l. We further assume that in each period, the instantaneous utility is separable in consumption and labor: $U(c,l) = u(c) - v(l)$, where $u, v : \mathbb{R}_+ \to \mathbb{R}$ are twice continuously differentiable and increasing. Furthermore, u is concave, with $u'(0) = \infty$, and v is convex.

2.3 Labor taxes

For the sake of generality, and for theoretical reasons which we develop in Section 2.8 below, we introduce a rich set of linear labor taxes. First, we assume that unions bargain over the nominal wage rate, denoted by $\hat{W}_t$. Workers pay a linear labor tax τ_t^L on this income such that their post-tax nominal wage is $(1-\tau_t^L)\hat{W}_t$. Second, firms pay an additional labor tax, τ_t^S, which implies a wedge between the labor cost per efficient unit of labor, $\tilde{W}_t$, paid by firms and the wage $\hat{W}_t$ bargained by workers. This additional tax can be thought of as an employer social contribution that does not appear on the payroll of workers. Formally, the labor cost $\tilde{W}_t$, the bargained wage $\hat{W}_t$ and the tax τ^S verify the following relationship: $\hat{W}_t = (1-\tau_t^S)\tilde{W}_t$. The tax τ_t^S will have an effect on labor demand that will be internalized by unions in their bargaining strategy. Similarly, the tax τ_t^L will have an effect on labor income that will also be internalized. The difference between the two taxes is that τ_t^S has a direct effect on employment for a given bargained wage $\hat{W}_t$ but not on the wage W_t, whereas τ_t^L has a direct effect on the wage W_t for a given bargained wage $\hat{W}_t$, but no direct effect on employment.[2]

2.4 Production

The specification of the production sector follows the New-Keynesian literature on price stickiness, adapted to the previous tax structure. The consumption good Y_t is produced by a unique profit-maximizing representative firm that combines intermediate goods $(y_{j,t}^f)_j$ from different sectors indexed by $j \in [0,1]$ using a standard Dixit-Stiglitz aggregator with an elasticity of substitution, denoted ε_P:

$$Y_t = \left[\int_0^1 {y_{j,t}^f}^{\frac{\varepsilon_P-1}{\varepsilon_P}} dj\right]^{\frac{\varepsilon_P}{\varepsilon_P-1}}.$$

[2]We call direct effect the partial equilibrium effect of each variable. In general equilibrium (with endogenous income), these taxes obviously affect all variables through price variations.

For any intermediate good $j \in [0,1]$, the production $y^f_{j,t}$ is realized by a monopolistic firm and sold at price $p_{j,t}$. The profit maximization for the firm producing the final good implies:

$$y^f_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon_P} Y_t, \text{ where } P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon_P} dj\right)^{\frac{1}{1-\varepsilon_P}}.$$

The quantity P_t is the price index of the consumption good. Intermediary firms are endowed with a Cobb-Douglas production technology and use only labor. The production technology involves that $\tilde{l}_{j,t}$ units of labor are transformed into $y^f_{j,t} = Z_t \tilde{l}_{j,t}$ units of intermediate good. Z_t is aggregate labor productivity. It is affected at period 0 by a shock ϵ^Z_0 and it follows a AR(1) process. $Z_t = e^{z_t}$,with

$$z_0 = 1 + \epsilon^Z_0 \text{ and } z_t = \rho^Z z_{t-1} \text{ for } t \geq 1, \rho^Z < 1.$$

Since intermediate firms have market power, the firm's objective is to minimize production costs, subject to producing the demand $y^f_{j,t}$. The cost function $C_{j,t}$ of firm j is therefore $C_{j,t} = \min_{\tilde{l}_{j,t}}\{\tilde{w}_t \tilde{l}_{j,t}\}$, subject to $y^f_{j,t} = Z_t \tilde{l}_{j,t}$, where $\tilde{w}_t = \tilde{W}_t / P_t$ is the real overall wage rate. The maximization implies the following mark-up:

$$m_t = \frac{1}{Z_t}\tilde{w}_t. \tag{1}$$

In addition to the production cost, intermediate firms face a quadratic price adjustment cost à la Rotemberg when setting their price. Following the literature, the price adjustment cost is proportional to the magnitude of the firm's relative price change and equal to $\frac{\psi_P}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - 1\right)^2$. We can thus deduce the real profit, denoted Ω_t at date t of firm j:

$$\Omega_{j.t} = \Big(\frac{p_{j,t}}{P_t} - m_t(1-\tau^Y_t)\Big)\Big(\frac{p_{j,t}}{P_t}\Big)^{-\epsilon} Y_t - \frac{\psi_P}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - 1\right)^2 Y_t - t^Y_t,$$

where t^Y_t is a lump-sum tax financing the subsidy τ^Y. Computing the firm j's intertemporal profit requires to define the firm's pricing kernel. We follow Bhandari et al. (2021) and assume a constant pricing kernel.[3] The firm j's thus sets its price schedule $(p_{j,t})_{t\geq 0}$ to maximize its intertemporal profit at date 0: $\max_{(p_{j,t})_{t\geq 0}} \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \Omega_{j,t}]$. The solution is independent of the firm type j and all firms in the symmetric equilibrium charge the same price: $p_{j,t} = P_t$. Denoting the price inflation rate as $\pi^P_t = \frac{P_t}{P_{t-1}} - 1$ and setting $\tau^Y = \frac{1}{\varepsilon_P}$ to obtain an efficient steady state, we obtain the standard equation characterizing the Phillips curve in our environment:

$$\pi^P_t(1+\pi^P_t) = \frac{\varepsilon_P - 1}{\psi_P}(m_t - 1) + \beta \mathbb{E}_t\left[\pi^P_{t+1}(1+\pi^P_{t+1})\frac{Y_{t+1}}{Y_t}\right], \tag{2}$$

[3]Our own computations also show us that the quantitative impact of the pricing kernel is limited.

where:

$$Y_t = Z_t L_t \tag{3}$$

The real profit is independent of the firm's type and can be expressed as follows:

$$\Omega_t = \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Y_t - \tilde{w}_t L_t. \tag{4}$$

2.5 Labor market: Labor supply and Union wage decision

Following the New Keynesian sticky-wage literature, labor hours are supplied monopolistically by unions (Erceg et al. (2000)Chugh (2006) Hagedorn et al. (2019) or Auclert et al., 2022 among others). There is a continuum of unions of size 1 indexed by k and each union k supplies L_{kt} hours of labor at date t with nominal wage $\hat{W}_{kt}$. Union-specific labor supplies are then aggregated into aggregate labor supply by a competitive technology featuring a constant elasticity of substitution ε_W:

$$L_t = \left(\int_k L_{kt}^{\frac{\varepsilon_W - 1}{\varepsilon_W}} dk\right)^{\frac{\varepsilon_W}{\varepsilon_W - 1}}. \tag{5}$$

The competitive aggregator demands the union labor supplies $(L_{kt})_k$ that minimize the total labor cost $\int_k \hat{W}_{kt} L_{k,t} dk$ subject to the aggregation constraint (5), where $\hat{W}_{kt}$ is the bargained nominal wage of the members of union k. The demand for labor of union k depends on the total labor cost paid by the firm $\tilde{W}_{kt}$: $L_{kt} = \left(\frac{\tilde{W}_{kt}}{\tilde{W}_t}\right)^{-\varepsilon_W}$, where $\tilde{W}_t = \left(\int_k \tilde{W}_{kt}^{1-\varepsilon_W} dk\right)^{\frac{1}{1-\varepsilon_W}}$ is the total nominal wage index. As the labor demand depends on relative wages, and $\frac{\tilde{W}_{kt}}{\tilde{W}_t} = \frac{\hat{W}_{kt}}{\hat{W}_t}\frac{1-\tau_t^S}{1-\tau_t^S} = \frac{\hat{W}_{kt}}{\hat{W}_t}$, total labor demand can be written as:

$$L_{kt} = \left(\frac{\hat{W}_{kt}}{\hat{W}_t}\right)^{-\varepsilon_W} L_t, \tag{6}$$

where $\hat{W}_t = \left(\int_k \hat{W}_{kt}^{1-\varepsilon_W} dk\right)^{\frac{1}{1-\varepsilon_W}}$ is the bargained nominal wage index. Each union k sets its wage $\hat{W}_{kt}$ so as to maximize the intertemporal welfare of its members subject to fulfilling the demand of equation (6). We assume the presence of quadratic utility costs related to the adjustment of the nominal wage and equal to $\frac{\psi_W}{2}(\hat{W}_{kt}/\hat{W}_{kt-1} - 1)^2 dk$. The objective of union k is thus:

$$\max_{(\hat{W}_{ks})_s} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \int_i \left(u(c_{i,s}) - v(l_{i,s}) - \frac{\psi_W}{2}\left(\frac{\hat{W}_{ks}}{\hat{W}_{ks-1}} - 1\right)^2\right)\ell(di),$$

subject to (6) and where $c_{i,t}$ and $l_{i,t}$ are the consumption and labor supply of agent i. The first-order condition with respect to W_{kt} thus writes as:

$$\pi_t^W(\pi_t^W + 1) = \frac{\hat{W}_{kt}}{\psi_W} \int_i \left(u'(c_{i,t})\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} - v'(l_{i,t})\frac{\partial l_{i,t}}{\partial \hat{W}_{kt}}\right)\ell(di) + \beta \mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W + 1)\Big], \tag{7}$$

where the wage inflation rate is denoted by:

$$\pi_t^W = \frac{\hat{W}_{k,t}}{\hat{W}_{k,t-1}} - 1.$$

The labor supply l_{it} of agent i is the sum of her hours l_{ikt} supplied to union k, summed over all unions: $l_{it} = \int_k l_{ikt} dk$. Each union is assumed to request its members to supply an uniform number of hours, such that: $l_{ikt} = L_{kt}$. We thus deduce from (6):

$$\hat{W}_{kt} \frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} = \hat{W}_{kt} \frac{\partial (\int_k (\frac{\hat{W}_{kt}}{\hat{W}_t})^{-\varepsilon_W} L_t dk)}{\partial \hat{W}_{kt}} = -\varepsilon_W L_{kt}. \tag{8}$$

To compute the derivative of consumption $\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}}$, it should observed that it is equal to the derivative of its net total income. The net total income of agent i writes as $(1-\tau_t^L)\hat{W}_{kt} y_{i,t} l_{i,t}/P_t$, where τ_t^L is the labor tax. Formally:

$$\begin{aligned}
\frac{1}{c_{i,t}} \frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} &= \frac{1}{\hat{W}_{kt}} + \frac{1}{l_{i,t}} \frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} \\
&= \frac{1}{\hat{W}_{kt}} - \frac{\varepsilon_W}{\hat{W}_{kt}} \frac{L_{kt}}{l_{i,t}} \\
\hat{W}_{kt} \frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} &= (1-\varepsilon_W)(1-\tau_t^L)\hat{W}_{kt} y_{i,t} l_{i,t}/P_t
\end{aligned} \tag{9}$$

We focus on the symmetric equilibrium where all unions choose to set the same wage $\hat{W}_{kt} = \hat{W}_t$, hence all households work the same number of hours, equal to $l_{it} = L_t$. Combining (7) with the partial derivatives (8) and (9), we deduce the following Phillips curve for wage inflation:

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W} \underbrace{\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} (1-\tau_t^L) \hat{w}_t \int_i y_{i,t} u'(c_{i,t}) \ell(di) \right)}_{\text{labor gap}} L_t + \beta \mathbb{E}_t \left[\pi_{t+1}^W (\pi_{t+1}^W + 1) \right], \tag{10}$$

where $\hat{w}_t = \hat{W}_t / P_t$ is the real pre-tax wage.

2.6 Assets

The only asset is nominal public debt, whose supply size is denoted by B_t at date t, and which pays off the pre-determined before-tax nominal interest rate i_{t-1}. Public debt is issued by the government and assumed to be default free. The financial market clearing implies that the net total savings of households, denoted A_t, must equal public debt:

$$A_t = B_t. \tag{11}$$

The corresponding real before-tax (net) interest rate for public debt, denoted by $\tilde{r}_t$, is defined by:

$$\tilde{r}_t = \frac{1 + i_{t-1}}{1 + \pi_t^P} - 1. \tag{12}$$

2.7 Agents' program

Each agent enters the economy with an initial endowment of public debt $a_{i,-1}$ and an initial productivity level $y_{i,0}$. The joint initial distribution over public debt and productivity levels is denoted Λ_0. In later periods, each agent learns her productivity level $y_{i,t}$, supplies labor, and earns her savings payoffs. Since the labor supply L_t is chosen by unions, the labor income is $(1-\tau_t^L)\hat{w}_t y_{i,t} L_t$. The before-tax real financial payoff amounts to $\tilde{r}_t a_{i,t-1}$.

We assume that agents pay two other taxes. First, a capital tax $\hat{\tau}_t^K$ is levied on interest payment and implies a net asset payoff $(1-\hat{\tau}_t^K)\tilde{r}_t a_{i,t-1}$. Second, an income tax τ_t^E is levied on total income, which implies a post-tax total income equal to $(1-\tau_t^E)\big((1-\tau_t^L)\hat{w}_t y_{i,t} L_t + (1-\hat{\tau}_t^K)\tilde{r}_t a_{i,t-1}\big)$. We assume that the latter income tax τ_t^E is not internalized by the unions, as the latter cannot observe total income.[4]

Agents earn this net total income and use it together with their past savings to consume $c_{i,t}$ and save $a_{i,t}$. Their budget constraint can be expressed as follows:

$$c_{i,t} + a_{i,t} = a_{i,t-1} + (1-\tau_t^E)\big((1-\hat{\tau}_t^K)\tilde{r}_t a_{i,t-1} + (1-\tau_t^L)\hat{w}_t y_{i,t} L_t\big). \tag{13}$$

To simplify the previous notation, we define the post-tax real interest and wage rates as:

$$r_t = (1-\tau_t^E)(1-\hat{\tau}_t^K)\tilde{r}_t, \tag{14}$$

$$w_t = (1-\tau_t^E)(1-\tau_t^L)\hat{w}_t = (1-\tau_t^E)(1-\tau_t^L)(1-\tau_t^S)\tilde{w}_t. \tag{15}$$

The agent's program can be finally be written as:

$$\max_{\{c_{i,t},a_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{i,t}) - v(L_t)\right), \tag{16}$$

$$c_{i,t} + a_{i,t} = (1+r_t)a_{i,t-1} + w_t y_{i,t} L_t, a_{i,t}, \tag{17}$$

and subject to the credit constraint $a_{i,t} \geq -\underline{a}$, and the consumption positivity constraint $c_{i,t} > 0$. The notation $\mathbb{E}_0$ is an expectation operator over both idiosyncratic and aggregate risks. The solution of the agent's program is a sequence of functions, defined over $([-\bar{a}; +\infty) \times \mathcal{Y}) \times \mathcal{Y}^t \times \mathbb{R}^t$

[4] The justification of this tax is presented in the next section. Although playing a major theoretical role, it has a modest quantitative impact, as we illustrate below.

and denoted by $(c_t, a_t)_{t\geq 0}$, such that:[5]

$$c_{i,t} = c_t((a_{i,-1}, y_{i,0}), y_i^t, z^t),\ a_{i,t} = a_t((a_{i,-1}, y_{i,0}), y_i^t, z^t). \tag{18}$$

For the sake of simplicity, we will keep using the notation with the i-index. Denoting by $\nu_{i,t}$ the discounted Lagrange multipliers of the credit constraint, the Euler equation corresponding to the agent's program (16) is:

$$u'(c_{i,t}) = \beta \mathbb{E}_t\Big[(1 + r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t}. \tag{19}$$

2.8 Government and market clearing conditions

The government has to finance an exogenous public good expenditure G_t, by raising a quite large number of taxes and by issuing one-period riskless public debt. G_t is affected at period 0 by a shock ϵ_0^G and it follows a AR(1) process. $G_t = e^{g_t}$,with

$$g_0 = 1 + \epsilon_0^G \text{ and } g_t = \rho^G g_{t-1} \text{ for } t \geq 1, \rho^G < 1.$$

The government raises three kinds of labor taxes: (i) a tax τ_t^S based on labor cost $\tilde{w}_t$ and paid by employers, (ii) a tax τ_t^L based on bargained wage $\hat{w}_t$ and paid by workers, and finally (iii) a tax τ_t^E based on total income and paid by workers. Importantly, the three labor instruments are independent and not redundant. Indeed, on the one hand, τ_t^S creates a wedge between the labor cost and the bargained wage, while τ_t^L and τ_t^E create wedges between the bargained wedge and the net wage. On the other hand, τ_t^L is internalized by unions, while τ_t^E is not. These three taxes will play on different margins and will allow us derive our equivalence result below. Hence, they should be understood as theoretical tools needed to generate price and wage stability. Each tax will be removed in turn to consider more realistic fiscal settings and to assess how each fiscal instrument contributes to inflation volatility.

In addition to capital and labor taxes and to public debt, the government also fully taxes the firms' profits, Ω_t, which limits the distortions implied by profit distribution. We can now express the government budget constraint. The government has to finance public spending and the repayment of past public debt. Its resources consist of all labor taxes, capital taxes, corporate profits, and newly issued public debt. We obtain:

$$\begin{aligned} G_t + \frac{1+i_{t-1}}{1+\pi_t^P} B_{t-1} \leq \Omega_t + B_t + \tau_t^E\big((1-\hat{\tau}_t^K)\tilde{r}_t \int_i a_{i,t-1}\ell(di) + (1-\tau_t^L)\hat{w}_t L_t\big) \\ + \hat{\tau}_t^K \tilde{r}_t \int_i a_{i,t-1}\ell(di) + \tau_t^L \hat{w}_t L_t + \tau_t^S \tilde{w}_t L_t. \end{aligned}$$

We can simplify the previous government budget constraint using the financial market clearing

[5] See e.g. Miao (2006), Cheridito and Sagredo (2016), and Açikgöz (2018) for a proof of the existence of such functions.

(11), the post-tax interest rate $\tilde{r}_t$ (12), and the profit definition (4):

$$G_t + (1 + (1 - \tau_t^E)(1 - \hat{\tau}_t^K))B_{t-1} + (1 - \tau_t^L)(1 - \tau_t^E)\hat{w}_t L_t \leq \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Y_t + B_t,$$

which using post-tax rate definitions (14) implies:

$$G_t + r_t B_{t-1} + w_t L_t \leq \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Y_t + B_t - B_{t-1}, \tag{20}$$

We finally express the financial market clearing condition and the economy resource constraints:

$$\int_i a_{i,t}\ell(di) = A_t = B_t, \tag{21}$$

$$\int_i c_{i,t}\ell(di) + G_t = \left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right) Z_t L_t. \tag{22}$$

Equilibrium definition. We can finally formulate our definition of competitive equilibrium.

Definition 1 (Sequential equilibrium) *For any exogenous paths of TFP $(Z_t)_t$ and of public spending $(G_t)_t$, a sequential competitive equilibrium is a collection of individual allocations $(c_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$, of aggregate quantities $(L_t, A_t, Y_t, \Omega_t, m_t)_{t\geq 0}$, of price processes $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$, of monetary policy $(i_t)_{t\geq 0}$, fiscal policies $(\tau_t^L, \tau_t^S, \tau_t^E, \hat{\tau}_t^K, B_t)_{t\geq 0}$, and inflation dynamics $(\pi_t^W, \pi_t^P)_{t\geq 0}$ such that, for an initial wealth and productivity distribution $(a_{i,-1}, y_{i,0})_{i\in\mathcal{I}}$, and for an initial value of public debt verifying $B_{-1} = \int_i a_{i,-1}\ell(di)$, we have:*

1. *given prices, the allocations $(c_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$ solve the agent's optimization program (16)–(17);*
2. *financial, and goods markets clear at all dates: for any $t \geq 0$, equations (21) and (22) hold;*
3. *the government budget is balanced at all dates: equation (20) holds for all $t \geq 0$;*
4. *firms' profits Ω_t and the mark-up m_t are consistent with equations (1) and (4);*
5. *the price inflation path $(\pi_t^P)_{t\geq 0}$ is consistent with the price Phillips curve (2), while the wage inflation path $(\pi_t^W)_{t\geq 0}$ is consistent with the wage Phillips curve (10);*
6. *the real and nominal rates $(\tilde{r}_t, i_t)_{t\geq 0}$ verify (12);*
7. *post tax rates $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$ are defined in equations (14)–(15).*

Social Welfare Function. Following LeGrand et al. (2022), we assume that the planner maximizes a generalized social welfare function, where the weights on each period utility can depend on the current productivity of the agent. The objective of the planner is thus:

$$W_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(l_t^i)\right) \ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right]. \tag{23}$$

This expression embeds the utilitarian case, where $\omega(y) = 1$ for all y. This generalization of the Standard Social Welfare Function is now used either in quantitative work, such as (LeGrand et al., 2022, McKay and Wolf, 2022), or in more theoretical investigations, as a deviation from the utilitarian case (Dávila and Schaab, 2022). It will be used to ease the simulations and comparisons of economies in Section 5. We can define the notion of Ramsey equilibrium using this notion of social welfare function.

Definition 2 (Ramsey equilibrium) *A Ramsey equilibrium is the path of of monetary policy $(i_t)_{t\geq 0}$, fiscal policies $(\tau_t^L, \tau_t^S, \tau_t^E, \hat{\tau}_t^K, B_t)_{t\geq 0}$, which selects a competitive equilibrium, which is maximizing the social welfare function (23).*

A steady-state Ramsey equilibrium is a Ramsey equilibrium where aggregate real variables $(L_t, A_t, Y_t, \Omega_t, m_t)_{t\geq 0}$, prices $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$, monetary policy $(i_t)_{t\geq 0}$, fiscal policies $(\tau_t^L, \tau_t^S, \tau_t^E, \hat{\tau}_t^K, B_t)_{t\geq 0}$, and inflation dynamics $(\pi_t^W, \pi_t^P)_{t\geq 0}$ are constant.

2.9 Considering different fiscal systems: The economic experiment

The previous model has introduced five fiscal instruments and monetary policy $(\tau_t^L, \tau_t^S, \tau_t^E, \hat{\tau}_t^K, B_t)_{t\geq 0}$. In what follows, we will determine the optimal path of each of them after both supply and demand shocks. This rich fiscal system is also a theoretical device to understand distortions in the HA economy with both price and wage stickiness. As will be clear below, this fiscal system is the minimal one such that there is no deviation from price stability in all cases.

We assume that the economy starts from the steady-state situation where the fiscal system is optimally determined. Then in period 0, the economy is hit either by a demand shock or a supply shock. The whole paths of these shocks is known at period 0, and the planner sets optimally its available instruments under commitment.

In what follows, we consider different fiscal systems, where only some fiscal instruments can be optimally time-varying to smooth the effect of the shock. These experiments will help understand the key margin for which HA and RA economies differ and for which the planner optimally implements deviation from price stability. More precisely, we solve for optimal monetary policy considering five economies:

1. Economy 1: The tools $(\tau_t^E, \tau_t^L, \tau_t^S)_{t\geq 0}$ are optimally time varying.
2. Economy 2: τ^E is constant for $t \geq 0$, and $(\tau_t^E, \tau_t^L, \tau_t^S)_{t\geq 0}$ are optimally chosen.
3. Economy 3: τ^E and τ^S is constant for $t \geq 0$, and $(\tau_t^L)_{t\geq 0}$ are optimally chosen.
4. Economy 4: τ^E and τ^L is constant for $t \geq 0$, and $(\tau_t^S)_{t\geq 0}$ are optimally chosen.
5. Economy 5: Fiscal instruments and the interest rate follow rules.

In the first four cases, we assume that $(B_t, \hat{\tau}_t^K)_{t\geq 0}$ can be time-varying to focus on fiscal policy affecting the labor market. The theoretical and quantitative analysis below will provide the rationale for the first four economies. Economy 5 is studied in Section 5.5 to compare the optimal outcomes with the ones implied by simple standard fiscal and monetary rules.

3 Optimal policies with a Representative Agent

This section first presents optimal policy in the Representative Agent (RA) economy, which will be a useful benchmark.

The first best allocation

The first-best allocation is straightforward to determine. The program is actually static and the planner solves in each period t, $\max_{L_t} u(C_t) - v(L_t)$, subject to the resource constraint $C_t + G_t = Z_t L_t$. For any G_t, Z_t, the labor supply that solves this program satisfies:

$$Z_t u'(Z_t L_t - G_t) = v'(L_t).$$

The left-hand side is decreasing in L_t, while the right-hand side is increasing in L_t. As a consequence, the previous equality uniquely determines the level L_t^{FB}. This optimal labor supply is increasing in G_t and which is either decreasing in Z_t (if the period utility function u is very concave) or increasing in Z_t if the utility function is not very concave.[6]

Optimal monetary and fiscal policy

We now consider the RA economy with both sticky prices and sticky wages and the instruments $(\tau_t^L, \tau_t^S, \tau_t^E, \hat{\tau}_t^K, B_t)_{t\geq 0}$. The folllwing proposition summarizes the main results. We provide the proof in Appendix.

Proposition 1 (Representative agent) *In the RA economy*

1. *If the economy is hit by demand shock only (G_t), the first-best allocation can be implemented in all four economies (Economy 1, 2, 3 and 4). Price and wage inflations are 0.*

2. *If the economy is hit by supply shocks (Z_t), the first-best allocation can be implemented in Economy 1 (where all fiscal tools are available) and Economy 2 (where τ^E is constant). Price and wage inflation rates are then 0.*

3. *If the economy is hit by supply shock, price and wage inflation rates are not zeros in economy 3 and 4, and the first-best allocation cannot be implemented.*

[6]This is a standard result. To see this, assume $G_t = 0$ and a constant IES $\frac{1}{\sigma}$ for u and a constant Frisch elasticity φ for v then $L_t = Z_t^{\frac{1-\sigma}{\sigma+1/\varphi}}$.

4. The path of public debt is undetermined in Economy 1.

Several comments are in order. First, there is clear difference between demand shocks and supply shocks in the RA economies. For demand shocks, the first-best best can be implemented without time-varying changes in taxes (Item 1 of the Proposition). The reason for this is that public debt movements and capital taxes can move optimally to generate resources for the government, who can then finance any stream of public spending. This is a standard outcome of the front-loading of capital tax. The representative agent provides resources to the government by paying interests to the government on the debt it is induced. As a consequence, public debt is negative in these economies. For demand shocks, in all economies the optimal price and wage inflation rates are 0.

Second, for supply shocks, the first-best allocation can be implemented in Economies 1 and 2 (Item 2 of the Proposition). The proof is simple to summarize. Optimality imposes that the real post-tax real wage w_t is equal to the marginal product of labor, such that the labor supply is optimal. In addition, the wage bargained by the union $\hat{w}_t$ must be constant to avoid utility cost. The set of fiscal instruments is sufficient to implement such an set of wages. Indeed, fiscal policy allows the planner to decouple the post-tax and bargained wages, since $w_t = (1-\tau_t^L)(1-\tau_t^E)\hat{w}_t$ and $\hat{w}_t = (1-\tau_t^S)\tilde{w}_t$. Setting $(1-\tau_t^L)(1-\tau_t^E) = Z_t$ and $1-\tau_t^S = 1/Z_t$ implies a constant bargained wage $\hat{w}_t = 1$ and efficient wages $w_t = \tilde{w}_t = Z_t$. Observe that in this situation, $(1-\tau_t^L)(1-\tau_t^E)$ and $1-\tau_t^S$ move in opposite direction.

In Economies 3 and 4, the first-best allocation cannot be implemented for supply shocks, as tools are missing to implement the strategy of the previous paragraph (Item 3 of the Proposition). Movements of price and wage inflation will be used to improve the allocation. The rational of the deviation from price stability for supply shocks is to affect the real post-tax wage rate $w_t = W_t/P_t$ to bring it closer to the marginal productivity of labor Z_t. We develop further these explanations in Section 5, where we simulate these economies.

Finally, the planner has "too many" tools in Economy 1in the RA case (Item 4 of the Proposition): any path of public debt is consistent with the first-best allocation, when taxes adequately moves, for both demand and supply shocks. In Economy 2 (where income tax τ^E is constant), the path of public debt is uniquely determined.

4 Optimal policies with Heterogeneous Agents

We now consider heterogeneous-agent economies and derive optimal policies for both supply and demand shocks, for the four fiscal systems discussed above (Economies 1 to 4). The Ramsey planner's program is

$$\max_{\left(\tau_t^L,\tau_t^S,\tau_t^E,\pi_t^P,\pi_t^W,w_t,r_t,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\int_i \omega(y_t^i)\left(u(c_t^i) - v(L)\right)\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right], \tag{24}$$

$$G_t + (1+r_t)\int_i a_{i,t-1}\ell(di) + w_t L_t \leq \left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right)Z_t L_t + \int_i a_{i,t}\ell(di), \tag{25}$$

$$\text{for all } i \in \mathcal{I}\text{: } c_{i,t} + a_{i,t} = (1+r_t)a_{i,t-1} + w_t y_{i,t} L_t, \tag{26}$$

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0,\ \nu_{i,t} \geq 0, \tag{27}$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t\left[(1+r_{t+1})u'(c_{i,t+1})\right] + \nu_{i,t}, \tag{28}$$

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}\frac{w_t}{1-\tau_t^E}\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)L_t + \beta\mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W + 1)\right], \tag{29}$$

$$\pi_t^P(1+\pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P}\left(\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)(1-\tau_t^S)(1-\tau_t^E)} - 1\right) + \beta\mathbb{E}_t\left[\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\right], \tag{30}$$

$$(1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L} = \frac{w_t}{1-\tau_t^L}(1+\pi_t^P), \tag{31}$$

and subject to the positivity of consumption choices, and initial conditions.

The constraints in the Ramsey program include: the governmental and individual budget constraints (25) and (26), the individual credit constraint (and related constraints on $\nu_{i,t}$) (27), the individual Euler equations (28), the Phillips curves (29) and (30), the relationship (31) between price and wage inflation rates. taxes and the nominal interest can be recovered from the allocation, using the relationships (12) and (14).

This economy faces different frictions, which are worth summarizing. The monetary economy features two sets of market imperfections. The first set is related to the goods market. Intermediary firms enjoy a monopoly power, which implies a price markup m_t that can differ from one. There is also a Rotemberg cost for price adjustment, which prevents firms from freely setting their price. Note that the good market imperfections are complementary: one vanishes when the other is absent, as can be seen from the price Phillips curve (2). The second set of imperfections is related to the labor market. The union implies that the labor supply of agents is not set optimally, while the Rotemberg cost for wages prevents unions from freely setting wages. Note that in the absence of Rotemberg cost, the labor supply still remains sub-optimal, as it remains set at the union level. Without Rotemberg cost, the equation characterizing the choice of the labor supply (common to all agents) would be $v'(L_t) = w_t \int_i y_{i,t}u'(c_{i,t})\ell(di)$, while it would be $v'(l_{i,t}) = w_t y_{i,t} u'(c_{i,t})$, if agents were able to choose their individual labor supply $l_{i,t}$. This sub-optimal common labor choice will play a major role in our equivalence results below.

To better understand the dynamics of the economy, it is worth formulating the Ramsey problem recursively. [7].Some notations are in order. First, we denote as λ the current lagrange multiplier on Euler equations and as λ_{-1} the Lagrange multiplier of the previous peroid.

Denote as $\Lambda\left(a,\lambda_{-1},y\right)$ the joint distribution measure over wealth, Lagrange multipliers and productivity level. In more technical terms, it is a probability measure over the Borel $\sigma-$algebra $\mathbb{R}^{+}\times\mathbb{R}\times Y$. For any function f, (and to simplify notations we note as) $\int f\left(a,\lambda,y\right)d\Lambda\left(a,\lambda,y\right)$ for $\sum_{y\in Y}\int_{\mathbb{R}^{+}\times\mathbb{R}}f\left(a,\lambda,y\right)\Lambda\left(da,d\lambda,y\right)$.

Then:

$$
\begin{aligned}
&V\left(\Lambda\left(a,\lambda_{-1},y\right),\tau_{-1}^{L},w_{-1},G,Z,\gamma_{P,-1},\gamma_{W,-1},B\right)\\
=&\min_{\mu,\mu^{B},\gamma_{P},\gamma_{W},\lambda,\Lambda}\ \max_{\left(\tau^{L},\tau^{S},\tau^{E},\pi^{P},\pi^{W},w,r,L,(c,a',\nu)_{i}\right)}\int_{(a,\lambda,y)}\omega(y)\left(u(c)-v(L)-\frac{\psi_{W}}{2}(\pi^{W})^{2}\right)d\Lambda\left(a,\lambda_{-1},y\right)\\
&-\int_{(a,\lambda_{-1},y)}\left(\lambda-(1+r)\lambda_{-1}\right)u'(c)d\Lambda\left(a,\lambda_{-1},y\right)\\
&-(\gamma_{W}-\gamma_{W-1})\pi^{W}(1+\pi^{W})+\frac{\varepsilon_{W}}{\psi_{W}}\gamma_{W}\left(v'(L)-\frac{\varepsilon_{W}-1}{\varepsilon_{W}}\frac{w}{1-\tau^{E}}\int_{(a,\lambda_{-1},y)}yu'(c)d\Lambda\left(a,\lambda_{-1},y\right)\right)L\\
&-(\gamma_{P}-\gamma_{P,-1})\pi^{P}(1+\pi^{P})ZL+\frac{\varepsilon_{P}-1}{\psi_{P}}\gamma_{P}\left(\frac{w}{\left(1-\tau^{L}\right)\left(1-\tau^{E}\right)\left(1-\tau^{S}\right)}-Z\right)L\\
&+\Lambda\left((1+\pi^{W})\frac{w_{-1}}{1-\tau_{-1}^{L}}-\frac{w}{1-\tau^{L}}(1+\pi^{P})\right)\\
&-\mu\left((1-\frac{\psi_{P}}{2}(\pi^{P})^{2})ZL+B'-G-(1+r)B-wL\right)\\
&+\mu^{B}\left(B'-\int_{(a,\lambda,y)}a'd\Lambda\left(a,\lambda,y\right)\right)\\
&+V\left(\Lambda'\left(a',\lambda,y'\right),\tau^{L},w,G',Z',\gamma_{P},\gamma_{W},B'\right)
\end{aligned}
$$

subject to the individual budget constraint and the complementarity slackness conditions:

$$
\begin{aligned}
c&=(1+r)a-a'+wyL\\
a'&\geq-\overline{a},\nu(a'+\overline{a})=0,\\
\nu&=u'(c)-\beta\mathbb{E}\Big[(1+r')u'(c')\Big],\\
\nu&\geq0
\end{aligned}
$$

To simplify notations, denote as $X^{agg}:=\left(B,\Lambda\left(a,\lambda_{-1},y\right),\tau_{-1}^{L},w_{-1},G,Z,\gamma_{P,-1},\gamma_{W,-1}\right)$. For individual agents, the solution of this probelm is the policy rules

$$
c\left(a,y,X^{agg}\right),a'\left(a,y,X^{agg}\right),\nu\left(a,y,X^{agg}\right),\lambda'\left(a,\lambda,y,X^{agg}\right).
$$

[7]This formulation is used to present some intuitions. We solve the first-order conditions of the planner in the sequential representation to avoid the difficult discussion of the existence of a recursive equlibrium.

An important restriction is that these policy rules don't depend on on λ, as ca be seen from sequential problem. The distribution of Lagrange multiplier $\Lambda(a, \lambda_{-1}, y)$ is important however for each households, as it helps forecasting the value of the instrument of the planner.

The law of motion of the distribution is the following. For any set $A' \times \Psi' \subset \mathbb{R}^+ \times \mathbb{R}$, the next period probability is

$$\Lambda'(A' \times \Psi', y') = \sum_y \Pi_{y,y'} \int_{\left\{(a,\lambda)\in\mathbb{R}^+\times\mathbb{R}|a'(a,\lambda,y,X^{agg})\in A' \text{ and } \lambda(a'(a,\lambda,y,X^{agg})\in\Psi')\right\}} \Lambda(da, d\lambda, y)$$

Roadmap. To decompose the different effects at play, we perform the experiment described in Section 2.9 in the context of the RA economy: the economy starts in period 0 from the Ramsey steady-state distribution of the HA economy (defined in 2) and is then hit once by a negative persistent productivity shock. We hence focus on so-called MIT shocks. We consider the same 5 economies as in Section 2.9.

To simplify the derivation of first-order conditions, we use some aspects of the methodology of Marcet and Marimon (2019) used in LeGrand et al. (2022), which is sometimes called the Lagrangian method (Golosov et al., 2016), applied to incomplete-market environments. This methodology connects to the public finance literature – that we further explain in the different environments listed above.

The summary is provided in Table 2 in Section 4.2. Section 5 provides a quantification of the different mechanisms.

4.1 The flexible-price economy

First, to derive the benchmark Ramsey allocation, we study the flexible-price economy featuring no price- and no wage-adjustment cost. In this economy, all workers are assumed to work the same number of hours and the planner is assumed to be able to directly choose this common labor supply.[8] The firms make no profit and we thus have $m_t = 1$. In addition, monetary policy has no role has price are fully flexible, and the real interest rate is determined in equilibrium.

To save some space, we provide the program in Appendix C.1, and focus here on the methodology and the main results. First, we denote by $\beta^t \lambda_{i,t}$ the Lagrange multipliers of the Euler equations (28) of agent i at date t. The Lagrange multiplier of the government budget constraint is $\beta^t \mu_t$. (25) with $\pi_t^P = 0$. We can then express the intertemporal Lagrangian of the program, denoted by $\mathcal{L}$. From this Lagrangian, we can define $\psi_{i,t}^{FP}$ as:

$$\psi_{i,t}^{FP} := \frac{\partial \mathcal{L}}{\partial c_{i,t}},$$

which is the value for the planner to transfer one extra unit of consumption good to agent i

[8]It is also possible to solve the model where the planner can differentiate hours across agents. The allocation is very different from the market one, and it it thus a useless benchmark.

in period t.[9] To some extent, this quantity can be understood as the planner's version of the agent's marginal utility of consumption. We call this amount, the *social valuation of liquidity for agent i*. The expression of $\psi_{i,t}^{FP}$ is:

$$\psi_{i,t}^{FP} := \underbrace{\omega_t^i u'(c_{i,t})}_{\text{direct effet}} - \underbrace{(\lambda_{i,t} - (1+r_t)\lambda_{i,t-1})\, u''(c_{i,t})}_{\text{effect on savings}}. \tag{32}$$

We add the upper-script FP to refer to flexible price, as the nature of the friction will change the expression of the valuation of liquidity for agents i. As can be seen in equation (32), this valuation consists of two terms. The first is the marginal utility of consumption $\omega_t^i u'(c_{i,t})$, which is the private valuation of liquidity for agent i multiplied by the current weight of agent i. The second term in (32) takes into consideration the impact of the extra consumption unit on saving incentives from periods $t-1$ to t and from periods t to $t+1$. An extra consumption unit makes the agent more willing to smooth out her consumption between periods t and $t+1$, and thus makes her Euler equation (either nominal or real) more "binding". This more "binding" constraint reduces the utility by the algebraic quantity $u''(c_{i,t})\lambda_{i,t}$. The extra consumption unit at t also makes the agent less willing to smooth her consumption between periods $t-1$ and t and therefore "relaxes" the constraint of date $t-1$. This is reflected in the quantity $\lambda_{i,t-1}$.

This marginal valuation $\psi_{i,t}^{FP}$ has the same economic meaning as the *Generalized Social Marginal Welfare Weights* (GSMWW) introduced by Saez and Stantcheva (2016), which they denote as g_i. It is the marginal valuation, which allows one to assess the welfare effect of a marginal change in tax systems.[10] This quantity appears in planner's first-order conditions. For instance, the FOC with respect to the labor supply L_t is:

$$\int_i \omega_{i,t}\ell(di)v'(L_t) = Z_t \int_i y_{i,t}\psi_{i,t}^{FP}\ell(di), \tag{33}$$

which has to be compared to $v'(l_{i,t}) = w_t y_{i,t} u'(c_{i,t})$ when agents individually decide of their labor supply. As in the individual FOC, the planner equalizes the marginal cost of one extra unit of labor to the marginal benefit, but there are three differences. First, since the labor supply is common to all agents, the planner has to take into account all individual situations, and hence needs to aggregate over the whole population. Second, the planner does not value marginal consumption through marginal utility as agents but through the marginal valuation of liquidity $\psi_{i,t}^{FP}$. Finally, the planner does not value the marginal benefit of labor supply with the net wage w_t but but the marginal productivity Z_t.

[9] To simplify the notation, we keep the index i, but the sequential representation (referring to histories and not the identity of agent i) can be derived along the lines of equation (18).

[10] The corresponding expression, following Saez and Stantcheva (2016) notation, in a static environment would thus be $\int_t g_i y_i \ell(di) = \mu$. In a dynamic setting, it appears that $\psi_{i,t}$ is not a sufficient statistics for agents i, and that the knowledge of the marginal utility of agent i is necessary to determine optimal policy (see equation (56) for instance). Note that compared to Saez and Stantcheva (2016), the elasticity of labor supply does not appear in the formula for taxation, because labour supply is determined by demand (as agents are not on their labor supply).

In addition to $\psi_{i,t}^{FP}$, another key quantity is the Lagrange multiplier, μ_t, on the governmental budget constraint. The quantity μ_t represents the marginal cost in period t of transferring one extra unit of consumption to households. Therefore, the quantity $\psi_{i,t} - \mu_t$ can be interpreted as the "net" valuation of liquidity. This is from the planner's perspective, the benefit of transferring one extra unit of consumption to agent i, net of the governmental cost. We thus define:

$$\hat{\psi}_{i,t}^{FP} := \psi_{i,t}^{FP} - \mu_t. \tag{34}$$

The interpretation of first-order conditions is greatly clarified by expressing them using $\hat{\psi}_{i,t}$ rather than the multiplier on Euler equations, $\lambda_{i,t}$. For instance, the first-order condition with respect to the post-tax wage rate w_t, is:

$$\int_i \hat{\psi}_{i,t}^{FP} y_{i,t} \ell(di) = 0. \tag{35}$$

The planner sets the labor tax (and thus the real wage) so as to tradeoff on the one hand the resources obtained from raising taxes (equal to the shadow price multiplied by labor supply $\mu_t L_t$) and on the other hand benefits of higher taxes, which depends on the productivity y_i for agent i, and on the marginal valuation $\hat{\psi}_{i,t}$.

The heterogeneous-agent model provides (with some obvious restrictions) some additional dynamic constraints on these valuation for the planner. For instance, we show that dynamics of this valuation for unconstrained agents is:

$$\hat{\psi}_{i,t}^{FP} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{i,t+1}^{FP} \right],$$

which can be seen as a generalized consumption Euler equation for the planner and not for agents. We derive all first-order conditions in Appendix C.1. We use this allocation to derive our equivalence results in the next section.

4.2 The sticky price economy

We now solve for optimal policy in an economy plagued with two nominal frictions, where the planner has use all fiscal and monetary instruments. The Ramsey planner can be written as:

$$\max_{(\tau_t^L, \tau_t^S, \tau_t^K, B_t, T_t, \pi_t^P, \pi_t^W, w_t, r_t, \Omega_t, i_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i)_{t \geq 0}} W_0, \tag{36}$$

subject to equations (23)–(31). We can state our main equivalence result.

Proposition 2 (An equivalence result) *In the HA economy,*

- *when all instruments* $(\tau_t^E, \tau_t^S, \tau_t^L, \tau_t^K, B_t, i_t)$ *are optimally chosen, the planner exactly reproduces the flexible-price allocation and the inflation on prices and wages is null in all periods, for both supply and demand shocks.*

– *When* $\tau_t^E = 0$*, and the other instruments* $(\tau_t^S, \tau_t^L, \tau_t^K, B_t, i_t)$ *are optimally chosen, the planner implements* $\pi_t^P = 0$ *but* $\pi_t^W \neq 0$*.*

– *When* $\tau_t^E = \tau_t^S = 0$ *and the other instruments* $(\tau_t^L, \tau_t^K, B_t, i_t)$ *are optimally chosen, the planner implements* $\pi_t^P \neq 0$ *and* $\pi_t^W \neq 0$*.*

Proposition 2 generalizes the equivalence result of Correia et al. (2008) and Correia et al. (2013) for representative agent economies and LeGrand et al. (2022) for heterogeneous-agent economy to the case where there are both sticky prices and sticky wages. Interestingly, compared to LeGrand et al. (2022), we need two additional instruments (τ_t^E, τ_t^S), whereas we introduce one additional nominal constraint. Indeed, we need one instrument to prevent wage inflation (which destroys resources) and another one to reproduce the flexible price labor supply and neutralize the market power of unions. In the presence of a sufficiently large fiscal system, monetary policy has no role but price stability. Importantly, the result requires the presence of two labor taxes. The first labor tax τ^S (internalized by the planner) enables the planner to "isolate" the pre-tax rate $\tilde{w}_t$ that is determined by the allocation (with a zero price inflation) from the union wage $\hat{w}_t$ that is determined by the inflation path $(\pi_t^W)_t$. Removing τ^S as an independent instrument imposes a constraint between the factor price $\tilde{w}_t$ and the wage inflation path. In other words, the planner would have to balance the effects of price inflation (determining $\tilde{w}_t$) and of wage inflation (determining $\hat{w}_t$). The second labor tax τ_t^E enables the planner to simultaneously set the labor supply optimally (as in equation (33)) and close the wage gap in the wage Phillips curve. Removing τ_t^E would imply that the planner would need to tradeoff two inefficiencies: (i) the sub-optimal labor supply due the market power of unions and (ii) the cost of wage inflation. Should one of these two instruments be removed, Proposition 2 would not hold anymore and the economy would feature positive inflation on wages or on prices.

Overall, the first item of Proposition 2 rationalizes our tax system, which is the minimal tax system for which price stability is optimal.[11]

The second and third items of Proposition 2 characterizes the impact of removing τ^E and then τ^S as independent instruments for the planner. When we remove the income tax τ_t^E, the planner still implements price stability, but now wage inflation is not constant after a TFP shock. This comes from the fact that the planner cannot close the wage gap of the wage Phillips curve and optimally set the common labor supply. Due to union labor market power, closing the wage gap would imply an inefficient labor supply. The planner chooses to change the number of worked hours along the business cycle by allowing an non-zero wage inflation. The planner thus trades off a more efficient labor supply at the cost of a quadratic wage adjustment.

Finally, when we remove both τ_t^E and τ_t^S, both price and wage inflation move along the business cycles. Indeed, in addition to the previous mechanism for τ_t^E, removing τ^S prevents the

[11]More precisely, other tax systems could correspond to price and wage stability. For instance, it could be possible to consider time-varying consumption tax as in Correia et al. (2008). However, the number of independent instruments would not be smaller. We consider our tax system to be not unrealistic, at least in some countries.

planner from closing the price gap and to set the labor cost to marginal productivity of labor. The planner chooses to use a non-zero price inflation to change the cost of labor.

The expression of the social value of liquidity actually depends on the instruments of the planner. For instance, in the case where $\tau_t^E = \tau_t^S = 0$, such that the only instruments are $(\tau_t^L, \tau_t^K, B_t, i_t)$, the expression of the social valuation of liquidity for agent i is:

$$\psi_{i,t}^{ES} := \underbrace{\omega_t^i u'(c_{i,t})}_{\text{direct effet}} - \underbrace{(\lambda_{i,t} - (1+r_t)\lambda_{i,t-1})\, u''(c_{i,t})}_{\text{effect on savings}} - \underbrace{\frac{\varepsilon_W - 1}{\psi_W}\gamma_{W,t} w_t L_t y_{i,t} u''(c_{i,t})}_{\text{effect on wage inflation}}. \tag{37}$$

Compared to the expression (32) of $\psi_{i,t}^{FP}$ in the flexible price economy, the expression of $\psi_{i,t}^{ES}$ features a third effect that comes from the fact that the wage Phillips curve is a constraint for the planner. Indeed, in this case, the planner does not close the gap and the wage Phillips curve is a constraint for the planner, which implies the presence of the corresponding Lagrange multiplier $\gamma_{W,t}$. If the planner increases the consumption of agents i in period t, this will change the incentives to work and thus the union incentives to affect the wage dynamics. This is captured in the third term of equation (32). Furthermore, this new expression of $\psi_{i,t}^{FP}$ still verifies Euler-like equation for unconstrained agents: $\hat{\psi}_{i,t}^{ES} = \beta \mathbb{E}_t \left[(1+r_{t+1})\hat{\psi}_{i,t+1}^{ES}\right]$, where $\hat{\psi}_{i,t}^{ES} = \psi_{i,t}^{ES} - \mu_t^{ES}$.

Finally, these deviations to price or wage stability still need to be quantified in the quantitative section, so as to assess the economic relevance of the various instruments at play.

4.3 Comparing HA to RA

Table 1 summarizes the effect of missing instruments for demand shocks in both RA and HA economies.

Time-varying labor taxes	RA	HA
$\tau^L + \tau^S + \tau^E$	$\pi^P = 0$ and $\pi^W = 0$ (first-best alloc.)	$\pi^P = 0$ and $\pi^W = 0$ (flexible-price alloc.)
$\tau^L + \tau^S$		$\pi^P = 0$ and $\pi^W \neq 0$
τ^S		$\pi^P \neq 0$ and $\pi^W \neq 0$
τ^L		$\pi^P \neq 0$ and $\pi^W \neq 0$

Table 1: Price and wage inflation for demand shocks and for different instruments, Representative Agent economy (RA) and Heterogeneous-agent economy (HA).

Table 2 summarizes the effect of missing instruments for supply shocks.

Time-varying labor taxes	RA	HA
$\tau^L + \tau^S + \tau^E$	$\pi^P = 0$ and $\pi^W = 0$ (first-best alloc.)	$\pi^P = 0$ and $\pi^W = 0$ (flexible-price alloc.)
$\tau^L + \tau^S$	$\pi^P = 0$ and $\pi^W = 0$ (first-best alloc.)	$\pi^P = 0$ and $\pi^W \neq 0$
τ^S	$\pi^P \neq 0$ and $\pi^W \neq 0$	$\pi^P \neq 0$ and $\pi^W \neq 0$
τ^L	$\pi^P \neq 0$ and $\pi^W \neq 0$	$\pi^P \neq 0$ and $\pi^W \neq 0$

Table 2: Price and wage inflation for supply shocks and for different instruments, Representative Agent economy (RA) and Heterogeneous-agent economy (HA).

5 Quantitative analysis of optimal policies

This section quantifies the inflation dynamics under the different assumptions concerning the set of instruments available to the planner. The objective is to identify the most relevant instruments to stabilize inflation in HA models, among the ones presented in Table 2 and to understand how supply shocks differ from demand shocks. The calibration is described in Section 5.1. We explain how to compute optimal policies in HA economies in Section 5.2. We investigate supply shocks in Section 5.4 and demand shocks in Section 5.3. Finally, Section 5.5 presents the inflation dynamics with exogenous fiscal and monetary rules.

5.1 The calibration and steady-state distribution

Preferences. The period is a quarter. The discount factor is $\beta = 0.99$, and the period utility function is: $\frac{c^{1-\sigma}-1}{1-\sigma} - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}$. The Frisch elasticity of labor supply is set to $\varphi = 0.5$, which is the value recommended by Chetty et al. (2011) for the intensive margin in HA models. The scaling parameter is $\chi = 0.01$, which implies an aggregate labor supply of roughly 1/3.

Technology and TFP shock. The production function is: $Y = ZL$. The TFP process is a standard AR(1) process, with $Z_t = \exp(z_t)$ and $z_t = \rho_z z_{t-1}$, for $t \geq 1$, and $z_0 < 0$ is the period 0 negative TFP shock. We set $\rho_z = 0.95$,which the standard quarterly persistence.

Idiosyncratic risk. We use a standard productivity process: $\log y_t = \rho_y \log y_{t-1} + \varepsilon_t^y$, with $\varepsilon_t^y \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_y^2)$. We calibrate a persistence of the productivity process $\rho_y = 0.994$ and a standard deviation of $\sigma_y = 0.06$. These values are consistent with empirical estimates (Krueger et al., 2018), and generates a steady-state Gini of wealth of 0.78, which is again in line with the data.[12] Finally, we use the Rouwenhorst (1995) procedure to discretize the productivity process into 10 idiosyncratic states with a constant transition matrix.

[12]The Gini of wealth is 0.78 using the SCF data in 2007, before the 2008 Great Recession.

Steady state taxes and public debt. We first solve the model with constant exogenous taxes and explain below the choice of the Social Welfare Function (SWF). We first assume that steady-state taxes are 0, except for the labor tax τ^L: $\tau^E = \tau^S = 0$ and $\tau^L = 16\%$. This last value (together with the value of public debt explained below) implies that public spending over GDP is 15, which is close to the US value in 2007. The amount of public debt (which is the only asset here) is set to the annual value of 1.28. As public debt is the only asset in our economy, we target this amount to obtain an average Marginal Propensity to Consume (MPC) of 0.3.[13]

Monetary parameters. Following the literature and in particular Schmitt-Grohé and Uribe (2005), we assume that the elasticity of substitution is $\varepsilon_P = 6$ across goods and $\varepsilon_W = 21$ across labor types. The price adjustment cost is set to $\psi_P = 100$, such that the slope of the price Phillips curve is $\frac{\varepsilon_P - 1}{\psi_P} = 5\%$ (see Bilbiie and Ragot, 2021, for a discussion and references). The wage adjustment cost is set to $\psi_W = 2100$, such that the slope of the wage Phillips curve is 1%, assuming wages to be stickier than prices.[14] Finally, as there is no inflation on prices or wages at the steady state: $\pi^P = \pi^W = 0$, these coefficients only matter in the dynamics.

Table 3 provides a summary of the model parameters.

Calibration of the representative agent economy. The calibration of the RA economy considers the same preference parameters as in the HA economy. We denote with upper-script RA (HA) the allocation in the RA (HA) economy. In the RA economy, the steady-state labor supply L^{RA} (with $\pi^W = 0$) is determined by $v'(L^{RA}) = \frac{\varepsilon_W - 1}{\varepsilon_W}(1 - \tau^L)u'(c^{RA})$. Due to consumption inequality and the convexity of marginal utility, the average marginal utility in the RA economy is lower than the one in the HA economy. As a consequence, for the same parameters $L^{HA} > L^{RA}$. To consider comparable economies, we set public debt (B^{RA}) and public spending (G^{RA}), in the RA economy such that public-debt-to GDP and public-spending-to-GDP are identical in the two economies: $B^{RA}/Y^{RA} = B^{HA}/Y^{HA}$ and $G^{RA}/Y^{RA} = G^{HA}/Y^{HA}$.

5.2 Simulating optimal policies in the HA economies

To investigate the optimal dynamics of the model, we recall that we perform the following experiment – which is standard in the New Keynesian RA literature, but which must be adapted to the HA case. We first solve for the optimal policy for a given set of instruments and consider the steady-state allocation – which is the long run allocation in the absence of any aggregate shock. We then assume that the economy starts from the Ramsey steady-state and we then implement a period-0 transitory shock, which is either supply or demand driven. This procedure

[13] We thus adopt a liquid one-asset liquid wealth calibration to match a realistic MPC (Kaplan and Violante, 2022).

[14] We have performed sensitivity analysis regarding these coefficients. Our qualitative results appear not to be sensitive to these values, even if inflation and wage volatility increases with the slopes of Phillips curves.

Parameter	Description	Value	Target
	Preference and technology		
β	Discount factor	0.99	Quarterly calibration
σ	Curvature utility	2	
$\bar{a}$	Credit limit	0	
χ	Scaling param. labor supply	0.01	$L = 1/3$
φ	Frisch elasticity labor supply	0.5	Chetty et al. (2011)
	Shock process		
ρ_y	Autocorrelation idio. income	0.993	Krueger et al., 2018
σ_y	Standard dev. idio. income	6%	$Gini = 0.78$
ρ_z	Autocorrelation TFP shock	0.95	
	Tax system		
τ^L	Labor tax	16%	$G/Y = 15$
τ^S,τ^E,τ^K	Other tax	0%	
B/Y	Public debt over yearly GDP	128%	$MPC = 0.3$
G/Y	Public spending over yearly GDP	15%	Targeted
	Monetary parameters		
ε_p	Elasticity of sub. between goods	6	Schmitt-Grohé and Uribe (2005)
ψ_p	Price adjustment cost	100	Price PC 5%
ε_w	Elasticity of sub. labor inputs	21	Schmitt-Grohé and Uribe (2005)
ψ_w	Wage adjustment cost	2100	Wage PC 1%

Table 3: Parameter values in the baseline calibration. See text for descriptions and targets.

allows us to quantify how the economy is perturbated from that the steady state before converging back to the latter.

The steady state crucially depends on the Social Welfare Function used in the Ramsey program, as well as on the tools that the planner has access to. To overcome this difficulty and to start from the same steady state in all cases, we use the inverse optimal taxation approach, as in Heathcote and Tsujiyama (2021) and LeGrand and Ragot (2025). More precisely, we consider the same steady-state fiscal instruments, defined by $\tau^S = \tau^K = 0$, and $\tau^L > 0$, and estimate the weights of the SWF for each set of fiscal tools to ensure that this steady state is optimal. Indeed, each instrument of the planner generates a first-order condition, which imposes one restriction on the SWF.[15]We then choose the SWF satisfying these restrictions, which is the closest one to the utilitarian SWF (where all weights are equal). We also verify that the SWF does not quantitatively affect the dynamics of the allocation at the first order.

[15]As in standard New Keynesian models, optimal steady-state price and wage inflation is 0, whatever the social welfare function. As a consequence, steady-state price stability does not impose any restriction on the SWF.

The Ramsey problem in HA models cannot be solved with simple simulation techniques. Indeed, the Ramsey equilibrium is now a joint distribution across wealth and Lagrange multipliers, which is a high-dimensional object. While the steady-state values of Lagrange multipliers is already difficult to compute, the Ramsey solution actually requires the dynamics of this joint distribution. For this reason, we use the truncation method of LeGrand and Ragot (2022a) to determine the joint distribution of individual wealth and Lagrange multipliers.[16] The accuracy of optimal policies has been analyzed in LeGrand and Ragot (2023) for both the steady state and the dynamics. In addition, an improvement to efficiently reduce the state space is provided in LeGrand and Ragot (2022b). We detail the calculations in Appendix, and refer to these papers for details about the method.

To find the steady-state values of the Lagrange multipliers and SWF for a given fiscal policy, we use the following algorithm:

1. Set a truncation structure (a maximum truncation length N) and set instrument values.
2. Solve the steady-state allocation of the full-fledged Bewley model with the given instrument values, using standard techniques.
3. Consider the truncated representation of the economy, i.e., aggregate over truncated histories.
4. Compute the steady-state Ramsey solution in truncated economy
   (a) Derive first-order conditions of the planner for each instrument in the truncated representation.
   (b) Compute the SWF weights that are the closest to 1, for which all the planner's FOCs hold.
   (c) Compute associated Lagrange multipliers.
   (d) The truncated representation, together with the fiscal instruments, the estimated SWF, and Lagrange multipliers is a steady-state optimal Ramsey allocation for the truncated representation.
5. Compute the optimal dynamics of instruments and allocation in the truncated economy using the first order conditions of the planner – as is standard in any finite state space model.

We use the refined truncation approach, with a number of length for the refinement equals to $N = 8$. We check that the results do not depend on the choice of the truncation length. As in LeGrand and Ragot (2022a), the truncation provides accurate results, thanks to the introduction of the ξs parameters. We check that the dynamics does not depend on the truncation length.

[16] Optimizing on simple rules in the spirit of Krusell and Smith (1998) is also hard to implement as their are many independent instruments.

5.3 Optimal inflation and fiscal policy: the case of demand shocks

We solve for the optimal fiscal and monetary policies in different economies. We perform an exhaustive investigation with two driving questions: (i) When does optimal policy in the RANK and the HANK economies differ? (ii) When does monetary policy optimally deviate from price stability? We report in Figure 1 the simulation outcomes for negative negative demand shocks and for three key variables. These three variables summarize the allocation, which is provided in Appendix. We represent the evolution of GDP, price inflation, and wage inflation for the RA agent (in blue) and the HA agent (in red). As productivity is assumed to be 1 at the steady state, GDP is also the labor supply in this economy.

We solve for optimal policies in the RANK and HANK economies and compare them. Finally, we also solve for monetary policy with various fiscal systems, which allows one to identify which missing fiscal tools are key for the deviation from price stability. More precisely, we consider four different fiscal systems: 1) when $(\tau_t^E, \tau_t^S, \tau_t^L)$ are optimally time-varying , 2) when (τ_t^S, τ_t^L) are optimally time-varying, 3) when (τ_t^L) is optimally time-varying and finally 4) when (τ_t^S) is optimally time-varying. In all cases, public debt B_t and capital taxes are optimally chosen, and they satisfy the governmental budget constraint.

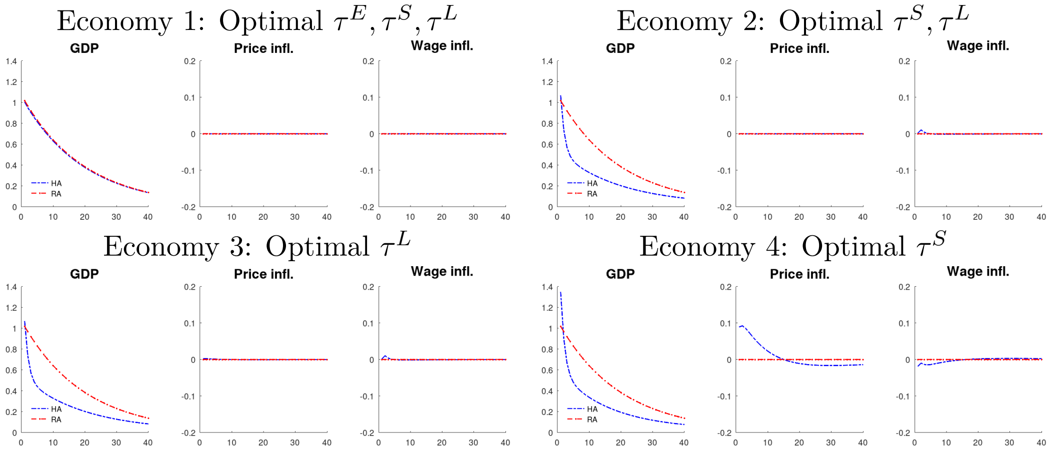

Figure 1: Summary results for a positive demand shock, for the 4 economies, comparing the Heterogeneous-Agent economy (HA) and the Representative-Agent economy (RA).

Panel 1 first plots the economy when all instruments $(\tau_t^E, \tau_t^S, \tau_t^L)$ are optimally time-varying. First, it confirms the theoretical results that price inflation and wage inflation are optimally 0 in this environment. In addition, it appears that there is no quantitative difference between the RA and the HA economy in this case. Panel 1 will thus be our benchmark to assess the deviations generating differences between HA and RA and form price stability.

Panel 2 reports the same variable for the same shock, but when τ^E is constant and equal to its steady-state value. One observes a difference between HA and RA economy. This difference also appears in the two other panels. Anticipating the analysis below, the difference between HA and RA economies comes from the path of public debt which is very different in HA economies, as public debt is used to for self-insurance motives.We observe only a tiny deviation from price-stability.

Panel 3 represents the economy with only time-varying (τ_t^L) and both $\tau^E$and τ^S kept at their steady-state values. The outcomes in Panel 3 are similar to those obtained in Panel 2, indicating that the variable τ_t^L is important for price stability, but τ^S less so. This is confirmed in Panel 4, where the outcome is plotted for optimal (τ_t^S). In this case, we observe a significant deviation from price stability. Price inflation decreases on impact, and wage inflation increases a little bit such that the real wage increases.

Understanding the difference in the allocation of HA and RA economies when τ^E is constant

Economy 2 with optimal time-varying (τ_t^S, τ_t^L) and a constant τ^E is the simplest deviation from the complete-fiscal system for which the allocation is differs between the HA and the RA economies. It is thus worth understanding deeper the economic mechanisms that are responsible for these differences. As we will see, some of these mechanisms are also relevant in the other fiscal systems.

Figure 2 plots the shock, aggregate consumption, the real wage, price and wage inflation (with a difference scale compared to Figure 1), together with the full set of fiscal tools: employer social contribution τ_t^S, employee Social Contribution τ_t^L, labor tax τ^L, income tax (τ^E) and public debt. By assumption τ^E is constant. First, considering the RA economy, one can observe that the first best is achieved with both price and wage stability. The path of public debt is well-defined, and public debt increases a little bit in the RA economy. Nominal wages $\hat{W}_t$ are constant, and the planner implements tax smoothing: taxes do not change, which is a standard outcome in the RA economy (Barro, 1979). On impact that government increases capital tax to decrease public debt (i.e. to hold more asset) and to finance public spending out of interest payment paid by the representative agent.

This allocation cannot be reproduced in the HA economy, because public debt is positive at the steady-state as a tool to smooth consumption. In the HA economy, nominal wage inflation is also almost 0, but we observe opposite movements in the path of employed social-contribution and employee social contribution. The employer social-contribution decreases a little bit on impact, whereas the employee social contribution increases sharply to finance public spending. Consumption and the real wage both fall on impact. Labor supply increases, as can be seen in Figure 1, because the average marginal utility of consumption increases after the shock. Public debt decreases after such a shock, because the average income of agents decreases as wage decreases. As a consequence, the saving of the economy, and thus public debt decreases.

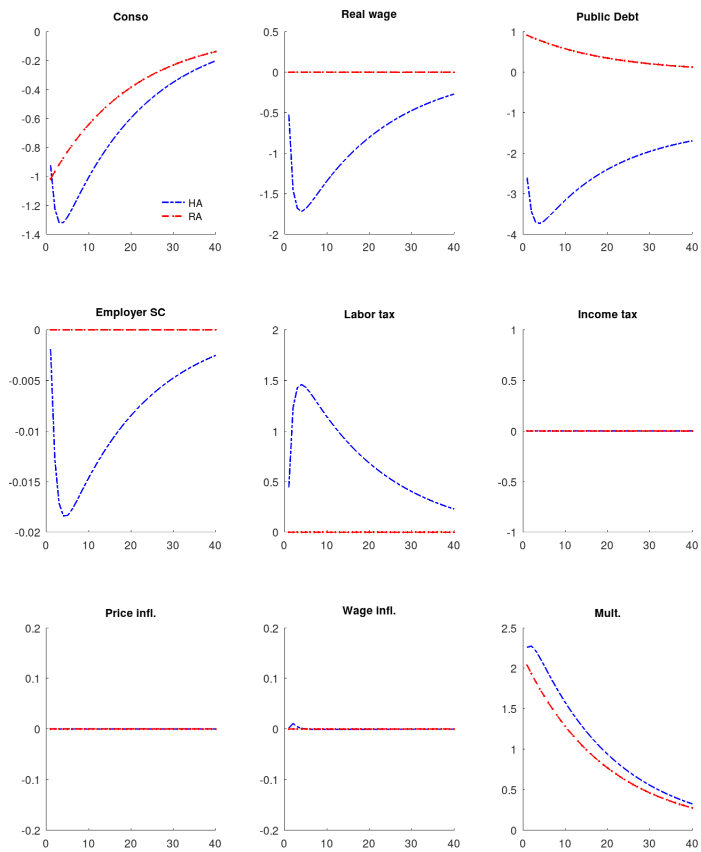

Figure 2: Dynamics of the economy for positive public spending shock for Economy 2 with optimal time-varying $\left\{\tau_t^S, \tau_t^L\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-Agent economy (RA) is in red. All variables are in percentage proportional change, except tax rates which are in percentage level change.

Understanding the difference in the allocation of HA and RA economies in other fiscal systems

The analysis of the Economy 2 with constant τ_t^E shows that optimal time-varying employer Social Contribution barely moves in the HA case. As a consequence, when we impose that this instrument is constant in Economy 3 (where only τ_t^L and B_t are time-varying), the allocation is not very different in the HA economy, as can be seen in Figure 1, comparing Panels 2 and 3. In addition, taxes were already constant for the RA economy in Economy 2. As a consequence, imposing that these taxes remain constant in Economies 3 and 4 is not a constraint and the allocation is the same. As a consequence, we do not observe deviation from price stability in Economy 3 and 4.

When only employer Social contributions are time-varying (Panel 4 in Figure 1), the allocation is different and we observe deviation from price stability in the HA economy (but not in the RA economy). To save some space, we represent the path of instruments in Appendix D.1, and summarize here the outcome. Price inflation is now used to decrease the real wage, as price inflation is less sticky than wage one. Employer social contribution has to increase to finance public spending.

Summary

From this analysis of demand shocks, we observe that HA and RA economies differ as soon as public debt becomes an independent instrument in the RA economy, because the use of public debt as liquidity create other mechanisms in the HA economies. We obtain deviation from price stability, when time-varying labor tax is not available.

5.4 Optimal inflation and fiscal policy: the case of supply shocks

We now present the analysis for negative supply shocks. As in the previous section, we first present a summary of the allocation in the same four economies as in Figure 1, and then focus on one economy to exhibit the key mechanisms at stake. Figure 3 presents model outcomes for the same negative supply shocks, which is temporary decreasing the marginal productivity of labor.

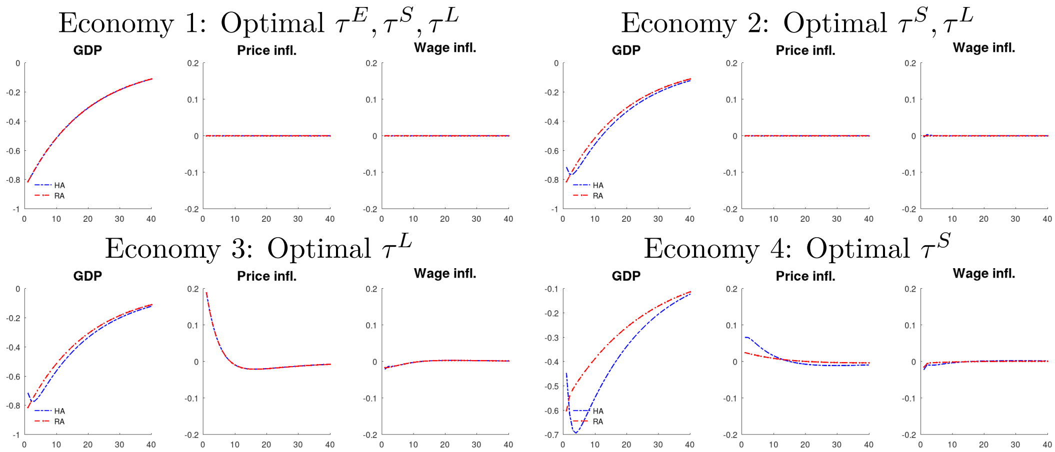

Figure 3: Summary results for a negative supply shock, for the 4 economies, comparing the Heterogeneous-Agent economy (HA) and the Representative-Agent economy (RA).

Economy 1 (as before) first plots the economy when all instruments $(\tau_t^E, \tau_t^S, \tau_t^L)$ are optimally time-varying. Economy 1 again confirms that price inflation and wage inflation are optimally 0 in this environment, and that there is no quantitative difference between the RA and the HA economy in this case.

Economy 2 reports the outcomes when τ^E is constant. In this case, there is no significant quantitative differences with Economy 1. HA and RA economies are similar and there little deviations from price stability. The intuitive reason for this new result is that the main issue in the RA and HA economy is to bring the real wage closer to the marginal product of labor closer, in an economy with both sticky price and sticky wages.

Economy 3 plots the economy with constant τ^S, τ^E and optimal τ_t^L. We find small differences between HA and RA economies, but significant deviation from price stability in this case. Optimal price inflation increases and wage inflation slightly decreases on impact, such that the real wages decrease to get closer to the marginal productivity of labor.

Economy 4 plots the economy with constant τ^L,τ^E and optimal τ^S. Again, we find that small differences between HA and RA economies, but the deviation from price and wage stability is reduced in this economy. Price inflation increases and wage inflation decreases as in Economy 3, but the increase in price inflation in 3 times smaller in this economy.

Understanding the difference in the allocation between HA and RA economies when τ^E is constant

We now present the dynamics of instruments for supply shocks for the Economy 2. Figure 4 presents the model outcomes for the same 9 variables as in Figure 2.

The shock in the first panel presents the evolution of the labor productivity Z_t. The RA economy implements the optimal allocation. In this case, the real wage follows labor productivity. To obtain this allocation with zero inflation, and thus a constant bargained prices $\hat{W}_t$,the planner increases labor tax such that the post tax real wage rate is decreasing and it decreases social contribution such that the pre-tax real wage rate paid by the firm $\tilde{W}_t = \hat{W}_t/(1-\tau_t^S)$ decreases as well. The intertemporal budget of the government requires a fall in public debt (so an increase in the asset held by the government issued by the representative agent). For the HA economy, the dynamics of the instruments is now roughly similar, because tax rates are now moving in the same directions in both RA and HA economies. This overall result in an aggregate consumption path which is roughly similar in the HA and the RA economies.

Understanding the difference in the allocation of HA and RA economies in other fiscal systems

For supply shock, the main objective of the planner is to close the gap between the real wage and the marginal productivity of labor in an economy with sticky prices and wages. When optimal Employer SC (τ_t^S) are not available (Economy 3), the planner uses price inflation to decrease the labor cost paid by the firm. The post tax real wage is decreasing due to an additional increase in the labor tax (see Figure 11 in Appendix E.2). When labor tax (τ_t^S) is not available (Economy 4), the planner uses a decrease in employer SC to decrease the real wage. As a consequence, the need of an increase in price inflation to further decrease the real wage is reduced.

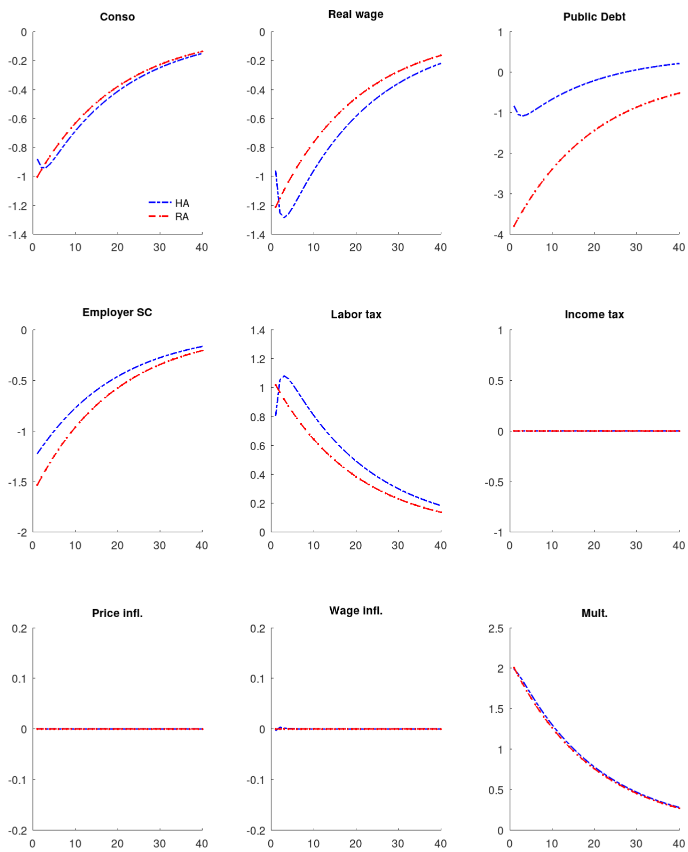

Figure 4: Dynamics of the economy for negative supply shock for Economy 2 with optimal time-varying $\left\{\tau_t^S, \tau_t^L\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-Agent economy (RA) is in red. All variables are in percentage proportional change, except tax rates which are in percentage level change.

Summary

The difference between allocations of HA and RA economies is smaller for supply shock compared to demand shocks. We obtain the biggest deviation from price stability, when only time-varying labor tax are available. In this case, inflation is a tool to reduce the real wage, in the HA economy, as in the RA economy.

5.5 Dynamics with fiscal and monetary rules

The previous optimal allocations can be compared to the one obtained with simple policy rules. We simulate the model with fiscal and monetary rules to assess the gain of implementing optimal policies in this environment. To save some space we only focus on negative supply shocks.

Concerning monetary policy, we introduce a standard Taylor rule, which depends on price inflation:

$$i_t = i_* + \phi_\pi \pi_t^P, \tag{38}$$

where i_t is the nominal interest rate between period t and period $t+1$. The constant $i_* = 1\%$ is the steady-state nominal rate, which is equal to the real interest rate, as steady-state inflation is 0. The parameter ϕ_π is the coefficient of the Taylor rule. As noted by Erceg et al. (2000) and Galí (2015), price determinacy generally requires the sum of the Taylor rule coefficients on both price and wage inflation to be larger than 1. In our case, $\phi_\pi > 1$ ensures price stability. We consider two values for this parameter: $\phi_\pi = 1.1$ and $\phi_\pi = 1.5$ to show the sensitivity of the dynamics to this coefficient.

Considering fiscal rules, we assume that tax rates are constant and set to their steady-state values. We only introduce an adjustment in the transfers related to the debt level à la Bohn (1998) to ensure debt sustainability. The fiscal rules thus involve $\tau_t^K = \tau_t^S = \tau_t^E = 0$ and:

$$\tau_t^L = \tau_*^L + \rho_B(B_t - B_*), \tag{39}$$

where we set $\rho_B = 0.08$, and then $\rho_B = 0.8$ to investigate the sensitivity of the economy dynamics to the fiscal rule. The value $B_*$is the steady-state level of public debt in either the RA or HA economy, while $\tau_*^L = 16\%$ is the steady-state value of this labor tax.

Figure 5 plots the Impulse Response Functions (IRFs) for the main variables in an economy with the previous rules, after a 1% negative TFP shock. Labor tax τ^L and wage and price inflation are reported in level deviations, while all other variables are reported in percent deviation from their steady-state values.

The shock is a negative supply shock, akin a energy price shock. Two results are worth mentioning.

1. HA and RA models generate qualitatively similar results, but the HA model exhibits a stronger fall in consumption. Indeed, as the MPC is higher in the HA model than in the RA model, the fall in consumption and output is higher in HA model, due to both direct and indirect effect (Kaplan et al., 2018).

2. Both HA and RA models generate a price-wage spiral that corresponds to an increase in both price and wage inflation, associated to a decrease in the real wage. This "spiral" is of a larger magnitude in the HA model than in the RA one. Indeed, both price and wage inflation responses are of larger magnitude: twice larger for the wage inflation and 50% larger for price inflation. Since wage inflation is much larger, this also translates to a smaller drop in the real wage in the HA economy than in the RA one. The reason is quite subtle and relates to the heterogeneity of the drop in consumption. Although the average consumption drop is higher in HA than in RA economy, the HA consumption drop is more severe (both in absolute and relative terms) for high-wealth agents than for low-wealth (credit-constrained) agents. In our simulation, credit-constrained agents suffer from a 0.1% consumption drop (which is roughly the real wage drop), whereas wealthy agents experience of 0.8% consumption drop. For the sake of comparison, note that the consumption drop in

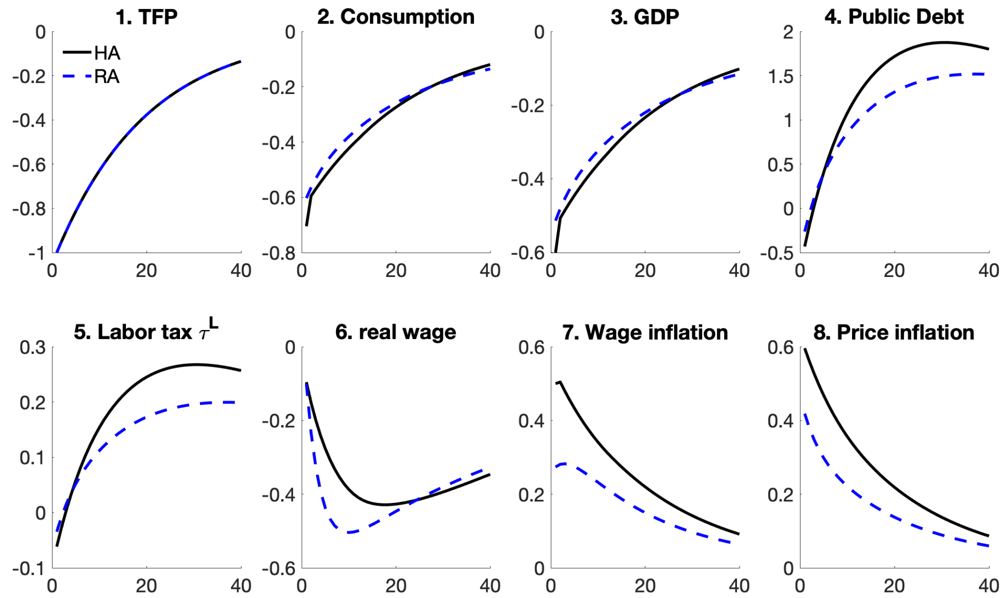

Figure 5: Impulse response functions of main variables after a negative TFP shock, for the model with a Taylor rule and simple fiscal rule. The labor tax and price and wage inflation are reported in level deviation (in percent), while all other ones are in proportional percentage deviations from steady state values. HA is the heterogeneous agent economy, RA is the representative agent economy.

the RA economy amounts to 0.6%. This stronger drop in consumption for wealthy (and hence high-consumption) agents than for poor (and hence low-consumption) agents is due to the drop in real interest rate that adds to the drop in real wage. As a consequence, the average marginal utility increases *less* in the HA economy than in the RA economy. Hence, the labor gap increases *more* in the HA economy and so does wage inflation.

We now investigate the sensitivity of these results, changing both the monetary and fiscal rules, plotting the same variables. In addition to the baseline environments ($\phi^\pi = 1.1$ and $\rho^B = 0.08$) in black solid line, we consider alternative economies. In the second economy (blue dashed line), the Taylor rule is more sensitive to inflation ($\phi^\pi = 1.5$ and $\rho^B = 0.08$). In the third economy (red dotted line), the fiscal rule is more sensitive to public debt ($\phi^\pi = 1.1$ and $\rho^B = 0.8$).

We first observe that the monetary policy rule affects both the the allocation and the inflation dynamics. A more aggressive monetary policy (higher ϕ^π) generates a larger drop in consumption and a much smaller inflation reaction. The change in the fiscal rule translates to a more volatile response of the labor tax and a smoother public debt variation. The real wage drops much more but inflation and output reactions are barely affected.[17]

[17]The small effect of fiscal policy on output is due to the fact than we use the linear labor tax as a fiscal

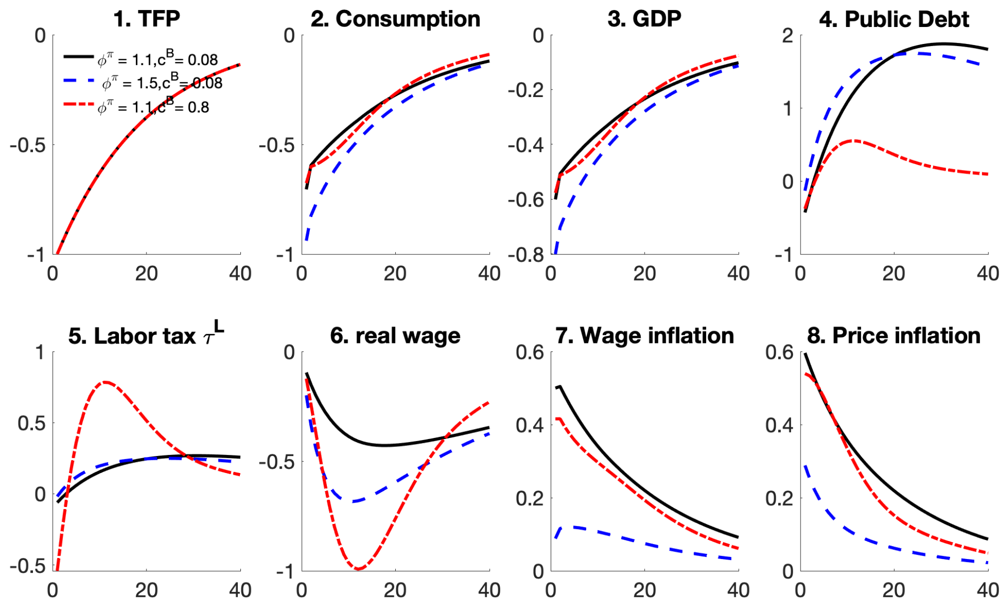

Figure 6: Impulse response functions of main variables after a negative TFP shock, for the model with a Taylor rule and simple fiscal rule. The labor tax and price and wage inflation are reported in level deviation (in percent), while all other ones are in proportional percentage deviations from steady state values. the HA economy for different monetary and fiscal rules.

6 Conclusion

We derive joint optimal monetary-and-fiscal policy in an HA model with both sticky prices and sticky wages, for both supply and demand shocks. Our first main finding is that a sufficiently rich fiscal policy can efficiently stabilize both inflation and activity. The key instrument for supply shocks appears to be a time-varying wage subsidy for supply shocks, and time-varying labor tax for demand shocks. Their primary goal is to reduce the gap between marginal labor productivity and the sticky labor cost, even though indirect effects on aggregate demand and inflation are also present. It is noteworthy that these tools have been recently been used in Europe to stabilize employment. In Germany, the so-called *kurzarbeit* device played this role, while in France, the *activité partielle* policy was a wage subsidy to reduce layoffs during the Covid-19 crisis. We have named these time-varying fiscal policy non-Keynesian stabilizers are their goal is not to simply manage aggregate demand and economic activity due to fiscal multipliers. The fiscal tools directly affect distortions on the labor market, generating as indirect effect, some changes in aggregate demand.

Our second main finding is that HA and RA economies significantly differ, when the set set of

instrument to be consistent with the analysis. A fiscal rule based on a lump-sum transfer would generate a higher variation in output due to the higher MPC in the HA economy.

fiscal instruments imply that movements in public debt are important to implement the desired allocation (in other words when we deviate from Ricardian equivalence for both the RA and the HA economies).

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Appendix

A Interpretation the TFP shock as an energy price shock

We explain how the TFP shock can be interpreted as an energy price shock. We do so in a general case featuring capital. We consider a CRS production function $\tilde{F}$ using capital, labor, and energy. Energy is denoted E and its price is denoted by $\tilde{q}$. We thus have:

$$\tilde{F}(K, L, E) = \tilde{Z} K^{\alpha_K} L^{\alpha_L} E^{1-\alpha_K-\alpha_L},$$

where α_K and α_L are capital and labor shares respectively. We can easily generalize the construction of Section 2.4. The markup of equation (1) is denoted with a tilde and becomes: $\tilde{m}_t = \frac{1}{\tilde{Z}_t}\left(\frac{\tilde{r}_t^K+\delta}{\alpha_K}\right)^{\alpha_K}\left(\frac{\tilde{w}_t}{\alpha_L}\right)^{\alpha_L}\left(\frac{\tilde{q}_t}{1-\alpha_K-\alpha_L}\right)^{1-\alpha_K-\alpha_L}$, while factor prices are defined as follows:

$$\tilde{r}_t^K + \delta = \tilde{m}_t \alpha_K \tilde{Z}_t K_{t-1}^{\alpha_K-1} L_t^{\alpha_L} E_t^{1-\alpha_K-\alpha_L}, \tag{40}$$

$$\tilde{w}_t = \tilde{m}_t \alpha_L \tilde{Z}_t K_{t-1}^{\alpha_K-1} L^{\alpha_L-1} E_t^{1-\alpha_K-\alpha_L}, \tag{41}$$

$$\tilde{q}_t = \tilde{m}_t (1-\alpha_K-\alpha_L) \tilde{Z}_t K_{t-1}^{\alpha_K} L_t^{\alpha_L} E_t^{-\alpha_K-\alpha_L} \tag{42}$$

Using the expression (42) of $\tilde{q}_t$, we obtain:

$$E_t = \left(\frac{\tilde{m}_t(1-\alpha_K-\alpha_L)\tilde{Z}_t}{\tilde{q}_t}\right)^{\frac{1}{\alpha_K+\alpha_L}} K_t^{\frac{\alpha_K}{\alpha_K+\alpha_L}} L_t^{\frac{\alpha_L}{\alpha_K+\alpha_L}}. \tag{43}$$

We introduce the following notation:

$$Z_t = \tilde{Z}_t^{\frac{1}{\alpha_K+\alpha_L}} \left(\frac{\tilde{q}_t}{1-\alpha_K-\alpha_L}\right)^{1-\frac{1}{\alpha_K+\alpha_L}}, \tag{44}$$

$$\alpha = \frac{\alpha_K}{\alpha_K+\alpha_L}, \tag{45}$$

$$m_t = (\alpha_K+\alpha_L)\tilde{m}_t^{\frac{1}{\alpha_K+\alpha_L}}. \tag{46}$$

Substituting for the expression (43) of E_t into factor prices (40), we obtain:

$$\tilde{r}_t^K + \delta = m_t \alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha}, \tag{47}$$

where the second equality comes from rearrangement and the last from the definitions (44)–(46). Similarly for (41):

$$\tilde{w}_t = m_t(1-\alpha) Z_t K_{t-1}^{\alpha} L_t^{-\alpha}. \tag{48}$$

We have been able to rewrite factor prices $\tilde{r}_t$ and $\tilde{w}_t$ consistently with factor price definition. We now have to find a consistent definition of the production function. Adapting (3), we have:

$$\tilde{F}(K_{t-1}, L_t, E_t) = \frac{(\tilde{r}_t^K + \delta)K_{t-1} + \tilde{w}_t L_t + \tilde{q}_t E_t}{\tilde{m}_t},$$

or after substituting the expressions of $\tilde{F}$ and E_t and

$$\frac{(\tilde{r}_t^K + \delta)K_{t-1} + \tilde{w}_t L_t}{\tilde{m}_t} = Z_t(\alpha_K + \alpha_L)\tilde{m}_t^{\frac{1}{\alpha_K+\alpha_L}-1} K_{t-1}^{\alpha} L_t^{1-\alpha},$$

where we have used the definitions (44) of $Z_t$and (45) of α. Using the definition (46) of $\tilde{m}_t$, we finally obtain:

$$Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} = \frac{(\tilde{r}_t^K + \delta)K_{t-1} + \tilde{w}_t L_t}{m_t},$$

which is thus similar to (3). The function $F(K, L) = ZK^{\alpha}L^{1-\alpha}$ with Z and α defined in (44) and (45) is thus consistent with the new definitions of factor prices (47) and (48), the markup (46), as well as with the equation (3) connection output, factor prices and markups.

Interestingly, the TFP expression is $Z_t = \tilde{Z}_t^{\frac{1}{\alpha_K+\alpha_L}} \left(\frac{\tilde{q}_t}{1-\alpha_K-\alpha_L}\right)^{1-\frac{1}{\alpha_K+\alpha_L}}$ with $0 < \alpha_K + \alpha_L < 1$: an increase in energy prices (a higher $\tilde{q}_t$) can thus be interpreted as a drop in TFP Z_t. We will use this analogy in our quantitative exercise of Section 5.

Alternatively to a Cobb Douglas production function, one could consider a production function with Constant Elasticity of Substitution (CES) of the following form:

$$F(K_{t-1}, L_t, E_t) = Z_t\Big[(1-\epsilon)^{\frac{1}{\eta}}\left(K_{t-1}^{\alpha}L_t^{1-\alpha}\right)^{\frac{\eta-1}{\eta}} + \epsilon^{\frac{1}{\eta}}\left(E_t\right)^{\frac{\eta-1}{\eta}}\Big]^{\frac{\eta}{\eta-1}}$$

where ϵ is the energy share and η and the elasticity of substitution between energy inputs and value-added $V_t = K_{t-1}^{\alpha}L_t^{1-\alpha}$. When the elasticity of substitution is close to zero, small fluctuation in quantity of energy supplied can cause large spike in energy prices and marginal costs for firms. This ordering of the nests between energy, labor and capital is suggested by empirical evidence that the energy share in the economy follows closely the fluctuation in energy prices. Such CES expression is well suited for matching such patterns, as suggested in Hassler et al. (2021).

B Proof of proposition 1

We solve for the Ramsey allocation in the RA case, for both demand and supply shocks.

B.1 First-best allocation

In the first-best allocation, the resource constraint imposes that total consumption is financed out of production: $G_t + C_t = Z_t L_t$. The labor supply is thus determined by the solution to the following program: $\max_{L_t} u(Z_t L_t - G_t) - v(L_t)$. The first-order condition defines the first-best labor supply L_t^{FB} as the solution of:

$$Z_t u'(Z_t L_t^{FB} - G_t) \quad = \quad v'(L_t^{FB}), \tag{49}$$

which can be shown to admit a unique solution under standard assumption (u increasing concave with $u'(0) = \infty$ and $u'(\infty) = 0$ and v increasing convex).

Consider the following particular case. We set $G_t = 0$, $u'(c) = c^{-\gamma}$, and $v'(L) = \chi^{-1} L^{1/\phi}$ such that $\gamma > 0$ is the inverse of the IES and $\phi > 0$ is the Frisch elasticity of labor supply. We obtain: $L_t^{FB} = \chi^{\frac{1}{\frac{1}{\phi}+\gamma}} Z_t^{\frac{1-\gamma}{\frac{1}{\phi}+\gamma}}$.

B.2 Representative-agent model with a full set of instruments

We show that when the planner has access to the full set of instrument, the first-best allocation can be implemented for both demand and supply shocks. This requires $\pi^W = \pi^P = 0$, to avoid price or wage adjustment costs. The equations defining the equilibrium allocation are

$$w_t = \left(1 - \tau_t^E\right)\left(1 - \tau_t^L\right)\left(1 - \tau_t^S\right) Z_t,$$

$$v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} \frac{w_t}{1 - \tau_t^E} u'(C_t) = 0 \tag{50}$$

$$\frac{1}{Z_t} \frac{w_t}{(1 - \tau_t^L)(1 - \tau_t^S)\left(1 - \tau_t^E\right)} = 1 \tag{51}$$

$$1 = \frac{w_t}{w_{t-1}} \frac{(1 - \tau_{t-1}^L)}{(1 - \tau_t^L)} \tag{52}$$

$$C_t = Z_t L_t - G_t$$

$$G_t + (1 + r_t) B_{t-1} + (w_t - Z_t) L_t = B_t \tag{53}$$

$$u'(C_t) = \beta (1 + r_{t+1}) u'(C_{t+1}) \tag{54}$$

Note that the Euler equation (54) determines the real interest r_t^{FB} $t \geq 1$ from the first-best path of consumption C_t^{FB}. Importantly, this equations don't determine the period-0 interest rate r_0.

Equation (53à is the budget constraint of the government.

Equation (52) implies that there is a α such that $1 - \tau_t^L := \alpha w_t$. For the allocation to be the first best, equations (50) and (49) implies that

$$1 - \tau_t^E \quad := \quad \frac{\varepsilon_W - 1}{\varepsilon_W} \frac{w_t}{Z_t}$$

Then equation (51) implies

$$1-\tau_t^S := \frac{\varepsilon_W}{\varepsilon_W-1}\frac{1}{\alpha w_t}$$

Then the budget of the government implies

$$w_t = Z_t - \frac{G_t + (1+r_t) B_{t-1} - B_t}{L_t} \tag{55}$$

Implementation results: For any path of G_t, Z_t and path of public debt B_t, for $t \geq 0$, the first best can be implemented.

The proof is direct. Consider a path G_t, Z_t, B_t and the first-best labor supply L_t^{FB}. It gives a path of consumption determining the real interest rate r_t, $t \geq 1$. For any r_0 (which is an additional free variable), the equation (55) determines a path for the real wage rate. Then for any α, $1-\tau_t^L = \alpha w_t$, $1-\tau_t^E = \frac{\varepsilon_W-1}{\varepsilon_W}\frac{w_t}{Z_t}$, $1-\tau_t^S = \frac{\varepsilon_W}{\varepsilon_W-1}\frac{1}{\alpha w_t}$ is a market equilibrium.

Note thus that public debt is not determined in this implementation.

Note that in a steady-state equilibrium (where $Z=1$ and B, G, w are constant) , we have

$$B_{SS} = \frac{\beta}{1-\beta}\left((1-w) L_{SS}^{FB} - G_{SS}\right)$$

B.3 RA model : Representative agents without time-varying τ_t^E

We assume that the economy is in steady state, where public debt is B_{SS}. and hit by the shock at period 0. In the previous analysis, we can impose $\tau_t^E = \tau_{SS}^E$.

$$w_t = \frac{\varepsilon_W}{\varepsilon_W-1}\left(1-\tau_{ss}^E\right) Z_t$$

Then, $1-\tau_t^L = \alpha\frac{\varepsilon_W}{\varepsilon_W-1}\left(1-\tau_{ss}^E\right) Z_t$, , $1-\tau_t^S = \frac{1}{\alpha(1-\tau_{ss}^E)Z_t}$.

The budget of the state implies (for $t \geq 0$, with the notation $B_{-1} = B_{SS}$)

$$(1+r_t) B_{t-1} - B_t = \Theta_t$$

with

$$\Theta_t := \left(1 - \frac{\varepsilon_W}{\varepsilon_W-1}\left(1-\tau_{ss}^E\right)\right) Z_t L_t^{FB} - G_t$$

The variable Θ_t is uniquely determined. This uniquely determines the path of public converging back to the steady state. To see that, first observe that the period-0 interest rate 0 r_0 is a free parameter determined by period-0 capital tax $\hat{\tau}_0^K$.

$$B_{-1}(1+r_0) = \sum_{t=0} \frac{G_t}{R_{0,t}} + \lim_{T\to\infty}\frac{B_T}{R_{t,T}}$$

To have $\lim_{T\to\infty}\frac{B_T}{R_{t,T}} = 0$, we must choose the initial capital tax such that

$$(1+r_0)\,B_{-1}=\sum_{k=t}^{\infty}\frac{\Theta_k}{\prod_{j=t+1}^{k}\left(1+r_j^{FB}\right)}+\lim_{T\to\infty}\frac{B_T}{\prod_{j=t+1}^{T}\left(1+r_j^{FB}\right)}$$

(with the notation $\prod_{j=t+1}^{t}=1$). The term $\lim_{T\to\infty}\frac{B_T}{\prod_{j=t+1}^{T}(1+r_j)}=0$ if the economy converges back to the steady state. The unique period 0 allowing the public debt to converge back to the steady state is

$$1+r_0=\frac{\sum_{k=t}^{\infty}\frac{\Theta_k}{\prod_{j=t+1}^{k}\left(1+r_j^{FB}\right)}}{B_{SS}},$$

which is uniquely determined.

B.4 RA with Demand shock

We now show that whatever the fiscal system (economy 3 and 4), the first-best allocation can be implemented with demand shocks. The proof follows the consideration of the previous Section.

We now assume that $\tau^E=\tau^E_{SS}$ and $\tau^E=\tau^E_{SS}$. We now focus on the case where $1-\tau^E_{ss}=\frac{\varepsilon_W-1}{\varepsilon_W}$, to determine uniquely the path of the instruments. Any other value would not quantitatively change the allocation, and qualitatively the path of the instruments.

In this case, we have $w=Z=1$. Then $1-\tau_t^L=\alpha$, and $1-\tau^S=\frac{\varepsilon_W}{\varepsilon_W-1}\frac{1}{\alpha}$ and

$$1+r_0=\frac{1-\beta}{\beta}\sum_{k=t}^{\infty}\frac{G_k/G_{SS}}{\prod_{j=t+1}^{k}\left(1+r_j^{FB}\right)},$$

implements the first-best allocation.

B.5 The RA economy without time-varying τ_t^E and τ_t^S, with optimal τ_t^L and supply shocks

The first-best cannot be implemented, and we must solve for the Ramsey allocation. We provide equations for both demand and supply shocks and then discuss each case in turn.

The program is:

$$\max_{\left(\tau_t^L,\tau_t^S,\tau_t^K,B_t,T_t,\pi_t^P,\pi_t^W,w_t,r_t,\Omega_t,\tilde{R}_t^N,L_t,c_t,a_t\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-v(L_t)\right)-\frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t+(1+r_t)B_{t-1}+w_tL_t\leq\Big(1-\frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_tL_t+B_t,$$

$$c_t+a_t=(1+r_t)B_{t-1}+w_tL_t,$$

$$u'(c_t)=\beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{t+1})\Big],$$

$$\pi_t^W(\pi_t^W+1)=\frac{\varepsilon_W}{\psi_W}\Big(v'(L_t)-\frac{\varepsilon_W-1}{\varepsilon_W}\frac{w_t}{1-\tau_{ss}^E}u'(c_t)\Big)L_t+\beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W+1)\Big],$$

$$\pi_t^P(1+\pi_t^P)=\frac{\varepsilon_P-1}{\psi_P}(\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)\,(1-\tau_{ss}^E)}-1)+\beta\mathbb{E}_t\Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_tL_t}\Big),$$

$$(1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L}=\frac{w_t}{1-\tau_t^L}(1+\pi_t^P),$$

Define

$$T_t=(1+r_t)B_{t-1}-B_t$$
$$x_t=\frac{w_t}{1-\tau_t^L}$$

Then the program is (using $\frac{\varepsilon_W-1}{\varepsilon_W}=1-\tau_{ss}^E$)

$$\max_{\left(x_tT_t,\pi_t^P,\pi_t^W,w_t,L_t,c_t\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-v(L_t)\right)-\frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t+T_t+\left(1-\tau_t^L\right)x_tL_t\leq\Big(1-\frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_tL_t,$$

$$c_t=T_t+\left(1-\tau_t^L\right)x_tL_t,$$

$$\pi_t^W(\pi_t^W+1)=\frac{\varepsilon_W}{\psi_W}\Big(v'(L_t)-\left(1-\tau_t^L\right)x_tu'(c_t)\Big)L_t+\beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W+1)\Big],$$

$$\pi_t^P(1+\pi_t^P)=\frac{\varepsilon_P-1}{\psi_P}(\frac{\varepsilon_W}{\varepsilon_W-1}\frac{1}{Z_t}x_t-1)+\beta\mathbb{E}_t\Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_tL_t}\Big),$$

$$(1+\pi_t^W)x_{t-1}=x_t(1+\pi_t^P),$$

while the corresponding Lagrangian becomes $c_t = T_t + \left(1 - \tau_t^L\right) x_t L_t$

$$\begin{aligned}
\mathcal{L} = &\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) - v(L_t) - \frac{\psi_W}{2} (\pi_t^W)^2) \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1}) \pi_t^W (1 + \pi_t^W) \\
&+ \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \left(1 - \tau_t^L\right) x_t u'(c_t) \right) L_t \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1}) \pi_t^P (1 + \pi_t^P) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{\varepsilon_W}{\varepsilon_W - 1} x_t - Z_t \right) L_t \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big((1 - \frac{\psi_P}{2} (\pi_t^P)^2) Z_t L_t - G_t - T_t - \left(1 - \tau_t^L\right) x_t L_t \Big) \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1 + \pi_t^W) x_{t-1} - x_t (1 + \pi_t^P) \right)
\end{aligned}$$

We now turn to the computation of the FOCs.

Consider

$$\psi_t := \frac{d\mathcal{L}}{dc} = u'(c_t) - \underbrace{\frac{\varepsilon_W}{\psi_W} \gamma_{W,t} \left(1 - \tau_t^L\right) x_t L_t u''(c_t)}_{\text{effect on wage inflation}}$$

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t x_{t-1} = 0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) - \mu_t \psi_P \pi_t^P - \frac{\Lambda_t}{Z_t L_t} x_t = 0.$$

FOC wrt x_t.

$$0 = \left(1 - \tau_t^L\right) L_t \psi_t - \frac{\varepsilon_W}{\psi_W} \gamma_{W,t} \left(1 - \tau_t^L\right) u'(c_t) L_t + \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} \frac{\varepsilon_W}{\varepsilon_W - 1} L_t - \mu_t \left(1 - \tau_t^L\right) L_t - \Lambda_t (1 + \pi_t^P) + \beta \Lambda_{t+1} ($$

FOC wrt L_t.

$$\begin{aligned}
0 = &\left(1 - \tau_t^L\right) x_t \psi_t - v'(L_t) + \mu_t \left(\left(1 - \frac{\psi_P}{2} (\pi_t^P)^2\right) Z_t - \left(1 - \tau_t^L\right) x_t \right) + \frac{\varepsilon_W}{\psi_W} \gamma_{W,t} \left(v''(L_t) L_t + v'(L_t) - \left(1 - \tau_t^L\right) x \right. \\
&- (\gamma_{P,t} - \gamma_{P,t-1}) \pi_t^P (1 + \pi_t^P) Z_t + \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} \left(\frac{\varepsilon_W}{\varepsilon_W - 1} x_t - Z_t \right).
\end{aligned}$$

FOC wrt T_t.

$$\mu_t = u'(c_t).$$

FOC wrt $1-\tau_t^L$.

$$0 = L_t x_t \psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t} x_t u'(c_t) L_t - \mu_t x_t L_t.$$

Simplifying

FOC wrt T_t.

$$\mu_t = u'(c_t).$$

FOC wrt $1-\tau_t^L$.

$$0 = \psi_t - \frac{\varepsilon_W}{\psi_W}\gamma_{W,t} u'(c_t) - \mu_t.$$
$$0 = \frac{\varepsilon_W}{\psi_W}\gamma_{W,t} u'(c_t)\left(1 - \frac{-\left(c_t - T_t\right)u''(c_t)}{u'(c_t)}\right)$$

In this case, one can check that one has $\gamma_{W,t} = 0$ (The wage Phillips curve is not a constraint)

FOC wrt π_t^W.

$$-\psi_W \pi_t^W + \Lambda_t x_{t-1} = 0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) - \mu_t \psi_P \pi_t^P - \frac{\Lambda_t}{Z_t L_t} x_t = 0.$$

FOC wrt x_t.

$$0 = \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\frac{\varepsilon_W}{\varepsilon_W - 1} L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

Using the FOC wrt to τ^L, we have:

FOC wrt L_t.

$$0 = -v'(L_t) + \mu_t\Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t$$
$$-(\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t + \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\left(\frac{\varepsilon_W}{\varepsilon_W - 1}x_t - Z_t\right).$$

Simplifying

$$\psi_W \pi_t^W = \Lambda_t x_{t-1}.$$

and

FOC wrt π_t^P.

$$-(\gamma_{P,t}-\gamma_{P,t-1})(2\pi_t^P+1)=\mu_t\psi_P\pi_t^P+\frac{\Lambda_t}{Z_tL_t}x_t.$$

FOC wrt x_t.

$$0=\frac{\varepsilon_P-1}{\psi_P}\gamma_{P,t}\frac{\varepsilon_W}{\varepsilon_W-1}L_t-\Lambda_t(1+\pi_t^P)+\beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

FOC wrt L_t.

$$\begin{aligned}v'(L_t)&=\mu_t\Big(1-\frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t\\&-(\gamma_{P,t}-\gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_t+\frac{\varepsilon_P-1}{\psi_P}\gamma_{P,t}\left(\frac{\varepsilon_W}{\varepsilon_W-1}x_t-Z_t\right).\end{aligned}$$

Determining the path of public debt from the path of T_t

The dynamics of public debt is

$$B_t=(1+r_t)B_{t-1}+T_t$$

At the moment of the shock, at period 0, the planner can change capital tax.

$$1+r_0=\frac{-\sum_{k=t}^{\infty}\frac{T_k}{\prod_{j=t+1}^{k}(1+r_j^{FB})}}{B_{SS}},$$

B.6 RA analysis without (time-varying) τ_t^E, τ_t^L, with time-varying τ_t^S

With the same change of variable as in the previous case,

$$\max_{\left(\tau_t^L,\tau_t^S,\tau_t^E,\tau_t^K,\pi_t^P,\pi_t^W,w_t,r_t,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-v(L_t)\right)\ell(di)-\frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t+T+w_tL_t\leq\Big(1-\frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_tL_t,$$

$$c_t=T_t+w_tL_t,$$

$$\pi_t^W(\pi_t^W+1)=\frac{\varepsilon_W}{\psi_W}\Big(v'(L_t)-w_tu'(c_t)\ell(di)\Big)L_t+\beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W+1)\Big],$$

$$\pi_t^P(1+\pi_t^P)=\frac{\varepsilon_P-1}{\psi_P}\big(\frac{1}{Z_t}\frac{w_t}{(1-\tau_{ss}^L)(1-\tau_t^S)(1-\tau_{ss}^E)}-1\big)+\beta\mathbb{E}_t\Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_tL_t}\Big),$$

$$(1+\pi_t^W)w_{t-1}=w_t(1+\pi_t^P).$$

Define

$$z_t = \frac{1}{(1-\tau^L_{ss})(1-\tau^S_t)(1-\tau^E_{ss})} = \frac{1-\tau^S_{ss}}{1-\tau^S_t}$$

$$\begin{aligned}
\mathcal{L} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) - v(L_t) - \frac{\psi_W}{2}(\pi^W_t)^2) \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1}) \pi^W_t (1+\pi^W_t) \\
&+ \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - w_t u'(c_t)\right) L_t \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1}) \pi^P_t (1+\pi^P_t) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(w z_t - Z_t\right) L_t \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big((1 - \frac{\psi_P}{2}(\pi^P_t)^2) Z_t L_t - G_t - T_t - w_t L_t \Big) \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1+\pi^W_t) w_{t-1} - w_t (1+\pi^P_t) \right)
\end{aligned}$$

FOC wrt π^W_t.

$$-\psi_W \pi^W_t - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi^W_t + 1) + \Lambda_t w_{t-1} = 0.$$

FOC wrt π^P_t.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi^P_t + 1) - \mu_t \psi_P \pi^P_t - \frac{\Lambda_t}{Z_t L_t} w_t = 0.$$

FOC wrt w_t.

$$0 = L_t \psi_t - \frac{\varepsilon_W}{\psi_W} \gamma_{W,t} u'(c_t) L_t + \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} L_t - \mu_t L_t - \Lambda_t (1+\pi^P_t) + \beta \Lambda_{t+1} (1 + \pi^W_{t+1}).$$

FOC wrt L_t.

$$\begin{aligned}
0 &= w_t \psi_t - v'(L_t) + \mu_t \left(\Big(1 - \frac{\psi_P}{2}(\pi^P_t)^2\Big) Z_t - w_t \right) + \frac{\varepsilon_W}{\psi_W} \gamma_{W,t} \left(v''(L_t) L_t + v'(L_t) - w_t u'(c_t) \right) \\
&- (\gamma_{P,t} - \gamma_{P,t-1}) \pi^P_t (1+\pi^P_t) Z_t + \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} \left(w z_t - Z_t \right).
\end{aligned}$$

FOC wrt T_t.

$$\mu_t = u'(c_t).$$

FOC wrt z_t.

$$0 = \gamma_{P,t}.$$

Simplifying

FOC wrt π_t^W.

$$\psi_W \pi_t^W = -(\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t w_{t-1}.$$

FOC wrt π_t^P.

$$-\mu_t \psi_P \pi_t^P = \frac{\Lambda_t}{Z_t L_t} w_{t-1}.$$

FOC wrt w_t.

$$0 = -\frac{\varepsilon_W}{\psi_W}\gamma_{W,t} u'(c_t) L_t - \Lambda_t(1+\pi_t^P) + \beta\Lambda_{t+1}(1+\pi_{t+1}^W).$$

FOC wrt L_t.

$$v'(L_t) = +\mu_t\Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t) - w_t u'(c_t)\right)$$

$$\mu_t = u'(c_t).$$

C Ramsey program for HA models

C.1 Flexible-price equilibrium

We here assume here that the planner must choose a common labor supply for all agents, in a flexible price economy: $\pi_t^P = \pi_t^W = 0$. The program is:

$$\max_{\left(\tau_t^L, \tau_t^S, \tau_t^K, w_t, r_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i)\left(u(c_t^i) - v(L_t)\right)\ell(di)\right],$$

$$G_t + (1+r_t)\int_i a_{i,t-1}\ell(di) + w_t L_t + T_t \leq Z_t L_t + \int_i a_{i,t}\ell(di),$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1+r_t)a_{i,t-1} + w_t y_{i,t} L_t,$

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0,\ \nu_{i,t} \geq 0,$$

$$u'(c_{i,t}) = \beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t}.$$

The Lagrangian can be written as:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di)$$
$$+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t - T_t \right).$$

We recall that $\psi_{i,t} = \omega_t^i u'(c_{i,t}) - (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u''(c_{i,t})$. Compared to (32), we drop the FP subscript for the sake of simplicity. We compute the FOCs wrt four independent instruments: r_t, w_t, L_t and $(a_{i,t})_i$. The other instruments can be recovered from the constraints.

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t})\ell(di) = 0. \tag{56}$$

FOC wrt w_t.

$$\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) = 0.$$

FOC wrt L_t. Using the FOC on w_t:

$$\int_i \omega_{i,t}\ell(di) v'(L_t) = \mu_t Z_t = Z_t \int_i y_{i,t}\psi_{i,t}\ell(di).$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1+r_{t+1})\hat{\psi}_{i,t+1} \right].$$

C.2 The HA economy with all instruments

The program is:

$$\max_{\left(\tau_t^L,\tau_t^S,\tau_t^E,\tau_t^K,\pi_t^P,\pi_t^W,w_t,r_t,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega(y_t^i)\left(u(c_t^i)-v(L_t)\right)\ell(di)-\frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t+(1+r_t)\int_i a_{i,t-1}\ell(di)+w_tL_t\leq\left(1-\frac{\psi_P}{2}(\pi_t^P)^2\right)Z_tL_t+\int_i a_{i,t}\ell(di),$$

$$\text{for all } i\in\mathcal{I}\text{: } c_{i,t}+a_{i,t}=(1+r_t)a_{i,t-1}+w_ty_{i,t}L_t,$$

$$a_{i,t}\geq-\overline{a},\nu_{i,t}(a_{i,t}+\overline{a})=0,\ \nu_{i,t}\geq 0,$$

$$u'(c_{i,t})=\beta\mathbb{E}_t\left[(1+r_{t+1})u'(c_{i,t+1})\right]+\nu_{i,t},$$

$$\pi_t^W(\pi_t^W+1)=\frac{\varepsilon_W}{\psi_W}\left(v'(L_t)-\frac{\varepsilon_W-1}{\varepsilon_W}\frac{w_t}{1-\tau_t^E}\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)L_t+\beta\mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W+1)\right],$$

$$\pi_t^P(1+\pi_t^P)=\frac{\varepsilon_P-1}{\psi_P}\left(\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)(1-\tau_t^S)(1-\tau_t^E)}-1\right)+\beta\mathbb{E}_t\left(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_tL_t}\right),$$

$$(1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L}=\frac{w_t}{1-\tau_t^L}(1+\pi_t^P).$$

We can set:

- τ_t^S such that $1-\tau_t^S=\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)(1-\tau_t^E)}$, hence $\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)(1-\tau_t^S)(1-\tau_t^E)}-1$ and $\pi_t^P=0$.
- τ_t^E is a free parameter that can be deduced from π_t^W and the allocation. Hence, the wage Phillips curve is not a constraint.
- π_t^W only reduces utility and is an independent parameter that can be set through τ^L, hence $\pi_t^W=0$

The program then reduces to the same one as in the flexible-price economy without union:

Recovering taxes from the allocation We then have

$$1-\tau_t^E=\frac{\varepsilon_W-1}{\varepsilon_W}w_t\frac{\int_i y_{i,t}u'(c_{i,t})\ell(di)}{v'(L_t)}$$

$$1-\tau_t^L=\alpha w_t$$

$$1-\tau_t^S=\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)(1-\tau_t^E)}$$

C.3 The HA economy without τ_t^E

We impose $\tau_t^E=0$. The program is otherwise the same as in Section C.2. In particular, τ_t^S only appears in the price Phillips curve. As consequence, this equation is not a constraint and

τ_t^S is set, such that $\pi_t^P = 0$. Inflation indeed only destroys resources here. We then obtain the following program:

$$\max_{\left(\tau_t^L, B_t, T_t, \pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(L_t)\right) \ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t + (1 + r_t) \int_i a_{i,t-1}\ell(di) + w_t L_t + T_t \leq Z_t L_t + \int_i a_{i,t}\ell(di),$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1 + r_t)a_{i,t-1} + y_{i,t}w_t L_t + T_t$,

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1 + r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t},$$

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t}u'(c_{i,t})\ell(di)\right) L_t + \beta \mathbb{E}_t \Big[\pi_{t+1}^W(\pi_{t+1}^W + 1)\Big],$$

Because of $\tau_t^E = 0$, we cannot have simultaneously optimal labor supply and $\pi_t^W = 0$: the planner has to balance the relative costs of wage inflation with the suboptimal provision of labor supply. The Lagrangian is:

$$\begin{aligned}
\mathcal{L} = {} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_{i,c,t} - (1 + r_t)\lambda_{i,c,t-1}\right) u'(c_{i,t})\ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1 + \pi_t^W) \\
& + \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) L_t \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1 + r_t) \int_i a_{i,t-1}\ell(di) - w_t L_t - T_t\right).
\end{aligned}$$

We recall that in this economy, we have $\psi_{i,t} = \omega_t^i u'(c_{i,t}) - \left(\lambda_{i,c,t} - (1 + r_t)\lambda_{i,c,t-1}\right) u''(c_{i,t}) - \frac{\varepsilon_W - 1}{\psi_W}\gamma_{W,t} w_t y_{i,t} u''(c_{i,t}) L_t$, where compared to (37), we also drop the superscript.

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1}u'(c_{i,t})\ell(di) = 0.$$

FOC wrt w_t.

$$\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) = \gamma_{W,t}\frac{\varepsilon_W - 1}{\psi_W} \int_i y_{i,t}u'(c_{i,t})\ell(di).$$

FOC wrt L_t. Using the FOC wrt w_t:

$$-\int_i \omega_{i,t}\ell(di)v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right) = 0.$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta\mathbb{E}_t\left[(1+r_{t+1})\hat{\psi}_{i,t+1}\right].$$

C.4 The HA economy without τ_t^E and τ_t^S with τ_t^L

In this case, there is no obvious simplification and the program is:

$$\max_{\left(\tau_t^L,\tau_t^S,\tau_t^K,B_t,T_t,\pi_t^P,\pi_t^W,w_t,r_t,\Omega_t,\tilde{R}_t^N,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega(y_t^i)\left(u(c_t^i)-v(L_t)\right)\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t + (1+r_t)\int_i a_{i,t-1}\ell(di) + w_t L_t + T_t \leq \left(1-\frac{\psi_P}{2}(\pi_t^P)^2\right)Z_t L_t + \int_i a_{i,t}\ell(di),$$

for all $i\in\mathcal{I}$: $c_{i,t}+a_{i,t} = (1+r_t)a_{i,t-1} + w_t y_{i,t}L_t,$

$$a_{i,t}\geq -\overline{a}, \nu_{i,t}(a_{i,t}+\overline{a}) = 0,\ \nu_{i,t}\geq 0,$$

$$u'(c_{i,t}) = \beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t},$$

$$\pi_t^W(\pi_t^W+1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W-1}{\varepsilon_W}w_t\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)L_t + \beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W+1)\Big],$$

$$\pi_t^P(1+\pi_t^P) = \frac{\varepsilon_P-1}{\psi_P}\left(\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)} - 1\right) + \beta\mathbb{E}_t\Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\Big),$$

$$(1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L} = \frac{w_t}{1-\tau_t^L}(1+\pi_t^P),$$

while the corresponding Lagrangian becomes:

$$\begin{aligned}
\mathcal{L} = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1+\pi_t^W) \\
& + \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P (1+\pi_t^P) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{w_t}{(1-\tau_t^L)} - Z_t \right) L_t \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big((1 - \frac{\psi_P}{2}(\pi_t^P)^2) Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t) \int_i a_{i,t-1}\ell(di) - w_t L_t \Big) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1+\pi_t^W) \frac{w_{t-1}}{1-\tau_{t-1}^L} - \frac{w_t}{1-\tau_t^L}(1+\pi_t^P) \right)
\end{aligned}$$

We now turn to the computation of the FOCs.

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t \frac{w_{t-1}}{1-\tau_{t-1}^L} = 0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) - \mu_t \psi_P \pi_t^P - \frac{\Lambda_t}{Z_t L_t} \frac{w_t}{1-\tau_t^L} = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1} \hat{\psi}_{i,t} \ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t})\ell(di) = 0.$$

FOC wrt w_t. Using the FOC wrt to τ^L, we have:

$$0 = \int_i y_{i,t} \hat{\psi}_{i,t} \ell(di) - \gamma_{W,t} \frac{\varepsilon_W - 1}{\psi_W} \int_i y_{i,t} u'(c_{i,t})\ell(di).$$

FOC wrt L_t. Using the FOC wrt w_t:

$$\begin{aligned}
0 = & -\int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big) Z_t + \frac{\varepsilon_W}{\psi_W} \gamma_{W,t} \left(v''(L_t) L_t + v'(L_t) \right) \\
& - (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P (1+\pi_t^P) Z_t + \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} \left(\frac{w_t}{(1-\tau_t^L)} - Z_t \right).
\end{aligned}$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1+r_{t+1}) \hat{\psi}_{i,t+1} \right].$$

FOC wrt τ_t^L. We derive wrt $\frac{1}{1-\tau_t^L}$ and obtain:

$$0 = \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} L_t - \Lambda_t (1+\pi_t^P) + \beta \mathbb{E}_t \left[\Lambda_{t+1} (1+\pi_{t+1}^W) \right].$$

C.5 The HA economy without time-varying τ_t^E and τ_t^L with τ_t^S

I consider $\tau_{ss}^E = \left(\frac{\varepsilon_W - 1}{\varepsilon_W} \right)^{-1}$ and $\tau_t^L = \tau_{ss}^L$

In this case, there is no obvious simplification and the program is:

$$\max_{\left(\tau_t^S, \tau_t^K, B_t, T_t, \pi_t^P, \pi_t^W, w_t, r_t, \Omega_t, \tilde{R}_t^N, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t \geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(L_t) \right) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right],$$

$$G_t + (1+r_t) \int_i a_{i,t-1} \ell(di) + w_t L_t + T_t \leq \left(1 - \frac{\psi_P}{2} (\pi_t^P)^2 \right) Z_t L_t + \int_i a_{i,t} \ell(di),$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1+r_t) a_{i,t-1} + w_t y_{i,t} L_t,$

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0, \ \nu_{i,t} \geq 0,$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1+r_{t+1}) u'(c_{i,t+1}) \Big] + \nu_{i,t},$$

$$\pi_t^W (\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W} \left(v'(L_t) - w_t \int_i y_{i,t} u'(c_{i,t}) \ell(di) \right) L_t + \beta \mathbb{E}_t \left[\pi_{t+1}^W (\pi_{t+1}^W + 1) \right],$$

$$\pi_t^P (1+\pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P} \left(\frac{1}{Z_t} \frac{w_t}{(1-\tau^L)(1-\tau_t^S)(1-\tau_{ss}^E)} - 1 \right) + \beta \mathbb{E}_t \left(\pi_{t+1}^P (1+\pi_{t+1}^P) \frac{Z_{t+1} L_{t+1}}{Z_t L_t} \right),$$

$$(1+\pi_t^W) w_{t-1} = w_t (1+\pi_t^P),$$

while the corresponding Lagrangian becomes:

$$\begin{aligned}
\mathcal{L} = &\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W(1+\pi_t^W) \\
&+ \frac{\varepsilon_W}{\psi_W}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P(1+\pi_t^P) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{w_t}{(1-\tau_{SS}^L)(1-\tau_{SS}^E)(1-\tau_t^S)} - Z_t \right) L_t \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big((1 - \frac{\psi_P}{2}(\pi_t^P)^2) Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t \Big) \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1+\pi_t^W) w_{t-1} - w_t(1+\pi_t^P) \right)
\end{aligned}$$

We now turn to the computation of the FOCs.

FOC wrt τ_t^S. We derive wrt $\frac{1}{1-\tau_t^S}$ and obtain:

$$0 = \frac{\varepsilon_P - 1}{\psi_P}\gamma_{P,t}\frac{w_t}{(1-\tau_{SS}^L)(1-\tau_{SS}^E)} L_t$$

or

$$\gamma_{P,t} = 0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) - \mu_t \psi_P \pi_t^P - \frac{\Lambda_t}{Z_t L_t} w_t = 0,$$

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) + \Lambda_t \frac{w_{t-1}}{1-\tau_{t-1}^L} = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t})\ell(di) = 0.$$

FOC wrt w_t.

$$0 = \int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) - \gamma_{W,t}\frac{\varepsilon_W}{\psi_W}\int_i y_{i,t} u'(c_{i,t})\ell(di).$$

FOC wrt L_t.

$$\begin{aligned}\mathcal{L} = &\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\ &- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\ &- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W(1+\pi_t^W) \\ &+ \frac{\varepsilon_W}{\psi_W}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t}\left(v'(L_t) - \frac{\varepsilon_W}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) L_t \\ &+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Big(Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t \Big)\end{aligned}$$

$$\begin{aligned}0 = &- \int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right) \\ &+ w_t \left(\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) - \gamma_{W,t}\frac{\varepsilon_W}{\psi_W}\int_i y_{i,t} u'(c_{i,t})\ell(di) \right)\end{aligned}$$

Using the FOC wrt w_t:

$$0 = - \int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right)$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1 + r_{t+1})\hat{\psi}_{i,t+1} \right].$$

D Optimal policies for demand shocks

D.1 Economy 1, with all instruments

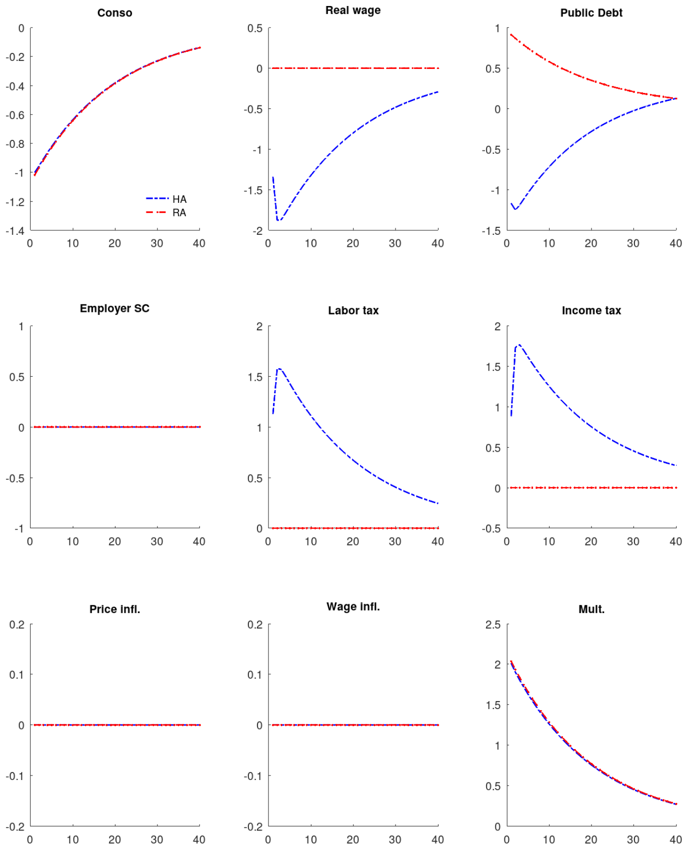


Figure 7: Dynamics of the economy for positive public spending shock for Economy 1 with optimal time-varying $\left\{\tau_t^E, \tau_t^S, \tau_t^L\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-Agent economy (RA) is in red. All variables are in percentage proportional change, except tax rates which are in percentage level change.

D.2 Economy 3, with time-varying τ_t^L

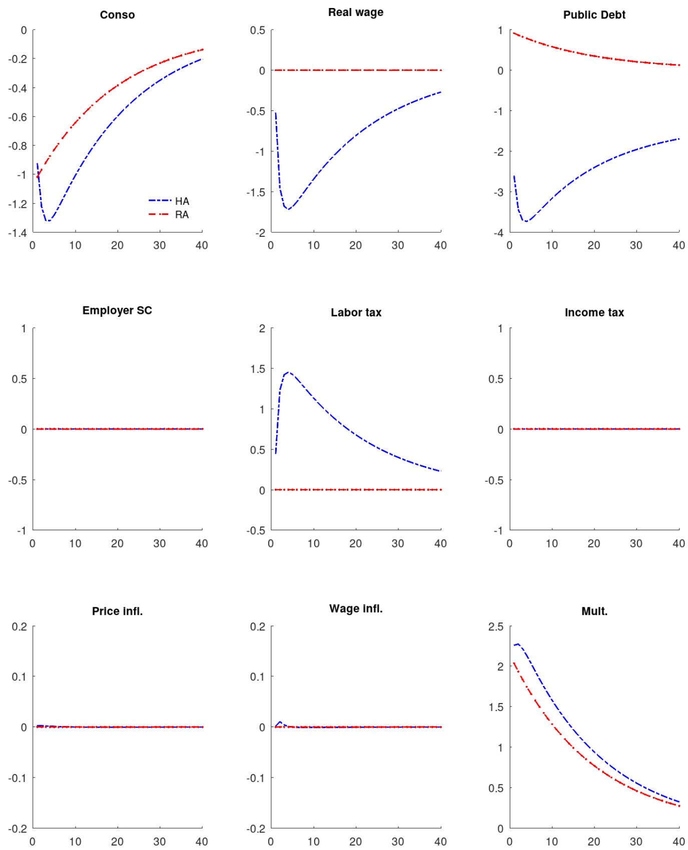


Figure 8: Dynamics of the economy for positive public spending shock for Economy 3 with optimal time-varying $\left\{\tau_t^L\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-Agent economy (RA) is in red. All variables are in percentage proportional change, except tax rates which are in percentage level change.

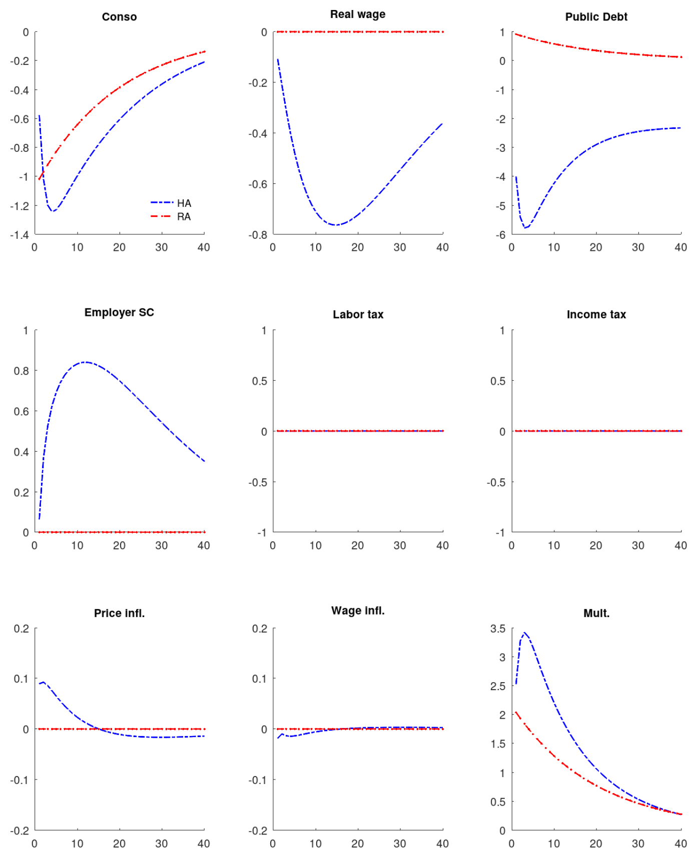
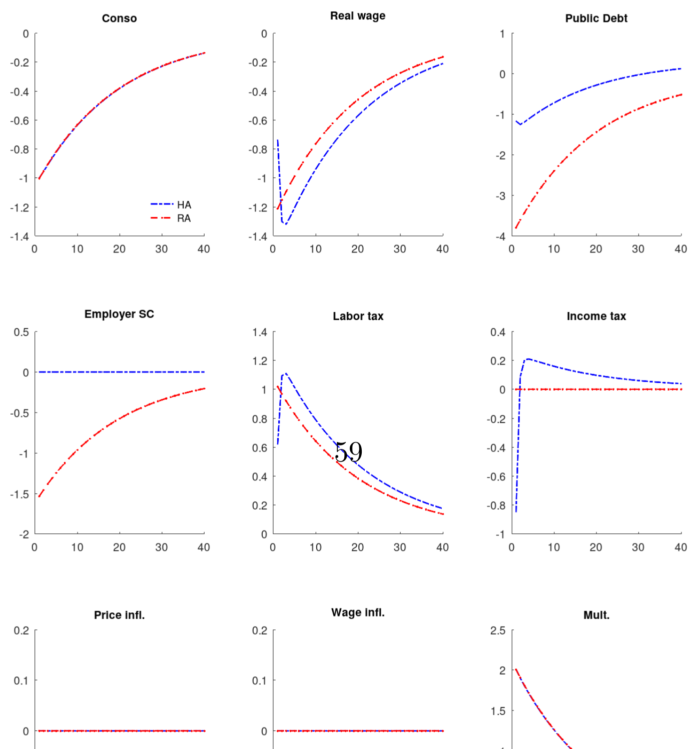


Figure 9: Dynamics of the economy for positive public spending shock for Economy 4 with optimal time-varying $\left\{\tau_t^S\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-Agent economy (RA) is in red. All variables are in percentage proportional change, except tax rates which are in percentage level change.

D.3 Economy 4, with time-varying τ_t^S

E Optimal policies for supply shocks

E.1 Economy 1, with all instruments



E.2 Economy 3, with time-varying τ_t^L

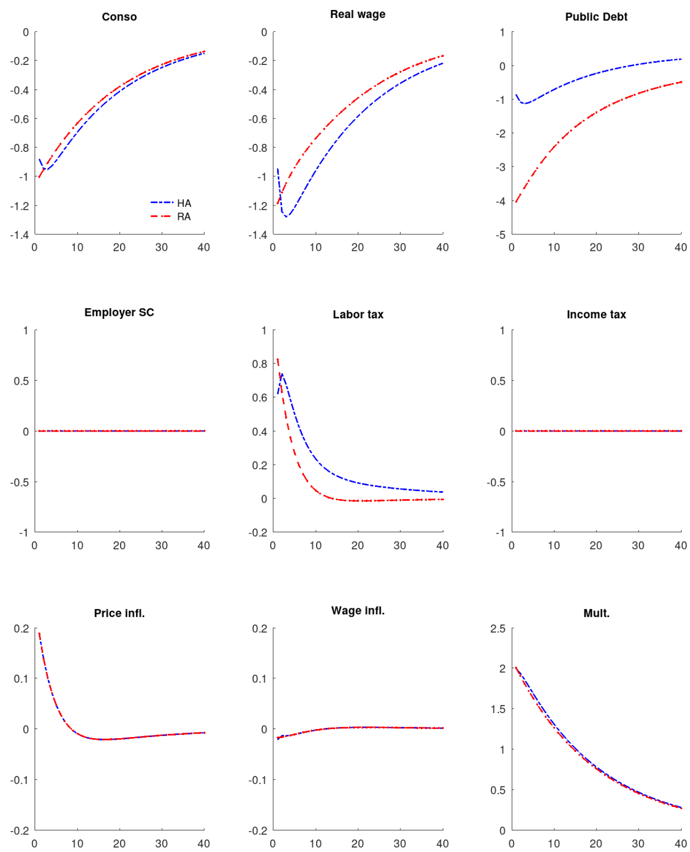


Figure 11: Dynamics of the economy for negative supply shock for Economy 3 with optimal time-varying $\left\{\tau_t^L\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-Agent economy (RA) is in red. All variables are in percentage proportional change, except tax rates which are in percentage level change.

E.3 Economy 4, with time-varying τ_t^S

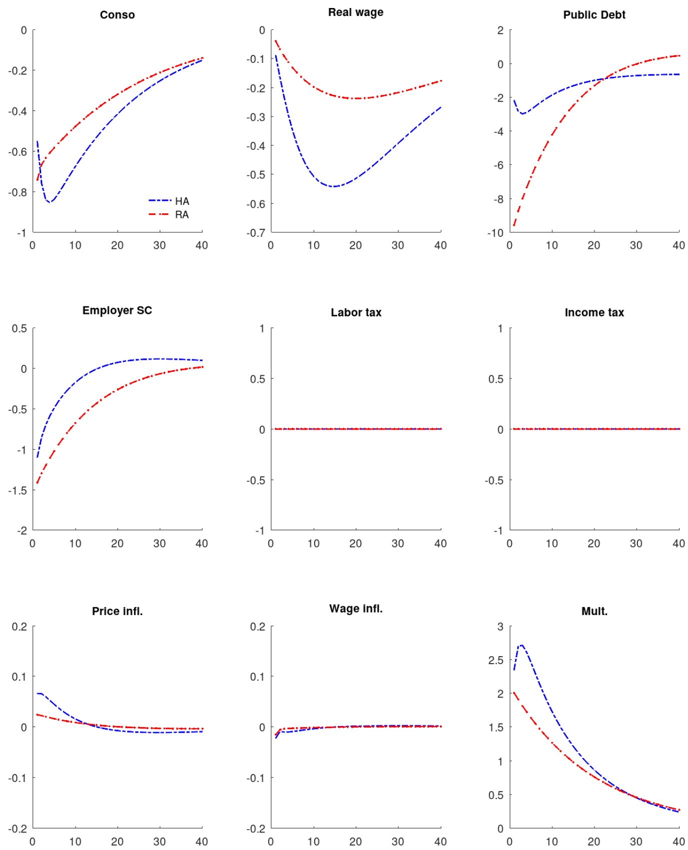


Figure 12: Dynamics of the economy for negative supply shock for Economy 4 with optimal time-varying $\left\{\tau_t^S\right\}$. The Heterogeneous-Agent economy (HA) is in blue and the Representative-Agent economy (RA) is in red. All variables are in percentage proportional change, except tax rates which are in percentage level change.