A Class of Tractable Incomplete-Market Models to Study Asset Returns and Risk Exposure *

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Abstract

We present a class of tractable incomplete-market models, where agents face both aggregate risk and limited participation in financial markets. Tractability relies on the assumption of small asset volumes and of a periodic utility function which is linear beyond a threshold, following a contribution of Fishburn (1977) in decision theory. We prove equilibrium existence and derive theoretical results about asset prices and consumption choices. Despite its parsimony, this small-trade model is able to reproduce a low safe return and a high equity premium, together with a realistic exposure of households to both idiosyncratic and aggregate risks.

Keywords: Incomplete markets, risk sharing, consumption inequalities.

JEL codes: E21, E44, D91, D31.

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1 Introduction

Infinite-horizon incomplete-insurance market models with credit constraints are known to be difficult to solve in presence of aggregate shocks. These models generate a large amount of heterogeneity, reflected by a time-varying distribution of agents' wealth with large support, such that numerical methods are needed to approximate the equilibrium (Krusell and Smith, 1998). In this paper, we present a class of incomplete-market models allowing for theoretical investigations of equilibrium allocations and asset prices. We prove the existence of an equilibrium with aggregate shocks, heterogeneous levels of idiosyncratic risk and stock-market participation costs, for which we can analytically analyze the main determinants of the risk allocation and of asset prices.[1] The modeling strategy is based on two assumptions. First, we assume that the periodic utility function is linear beyond a certain threshold, while strictly concave before, though being globally concave. This utility function was first introduced in decision theory by Fishburn (1977) to analyze risk for "below-target returns".[2] This utility function provides tractability in incomplete-market models in an interesting way when compared to alternatives. In particular, incomplete-market models have often relied on quasi-linearity in the labor supply to reduce the dimension of the state space (Scheinkman and Weiss, 1986; Lagos and Wright, 2005; Challe, LeGrand and Ragot, 2013, Le Grand and Ragot 2015 among others). Assuming linearity in the labor supply has the drawback that the consumption of agents with infinite labor elasticity is constant and independent of wealth and income. This infinite elasticity is also far too high at the household level (see Hall, 2010, for a recent survey). The Fishburn utility function, linear beyond a threshold, may thus be an attractive alternative to study consumption dynamics together with realistic labor income processes, what we do in this paper. Our second assumption is that the supply of securities is not too large. This implies that credit constraints bind for agents having experienced only a small number of consecutive bad idiosyncratic shock realizations. This generates an equilibrium with

[1] Note that the existence of simple recursive equilibria in such environments is still an open question (Miao, 2006).

[2] This captures the idea that investors are averse to risk for low returns (below a given target), while they care much less about risk for high returns.

a small number of heterogeneous agents. Our economy therefore features a *small-trade* equilibrium, where prices can be analytically studied, as in no-trade equilibria (see below for references), but where we can also investigate consumption allocations as well as the role of security volumes.

The goal of the paper is to show the usefulness of this setup by theoretically investigating the properties of an environment where two groups of agents face two different labor income processes together with limited participation in financial markets. The motivation for such an environment is based on previous results in the literature. First, it is known that models with only incomplete insurance-markets contribute to solve some asset pricing puzzles but mostly fail to reproduce a high equity premium for a realistic calibration (Krusell and Smith, 1998; Krusell, Mukoyama and Smith 2011, among many others). Second, adding limited participation in financial markets can help to reproduce relevant aspects of asset prices (Guvenen, 2009) and is consistent with empirically relevance (Bricker et al., 2014). Third, empirical investigations of income risks in the US show that high-income households face a lower risk than low-income households, who mostly do not participate in financial markets.[3]

In this framework, we derive two main sets of results. First, we prove equilibrium existence and exhibit the simple equilibrium structure. We then theoretically show how the model can generate a low return for the safe asset and a high equity premium. In addition, we characterize the effect of risks and volumes on asset prices. We derive explicit formula identifying all effects at stake. In particular, a higher volume of securities decreases asset prices and improves consumption smoothing, whereas a higher level of idiosyncratic risk generates both a decrease in the bond interest rate and an increase in stock prices.

Second, the calibrated model can reproduce surprisingly well household risk exposures together with asset price properties. Realistic asset price dynamics are consistent with a higher volatility of the consumption growth rate for low-income households than for high-

[3]See De Giorgi and Gambetti (2012), Gârleanu, Kogan and Panageas (2012), Meyer and Sullivan (2013), Bricker et al. (2014) among others for empirical evidence.

income households, as in the data. High-income households are also found to bear a larger fraction of the aggregate risk than low income households (Parker and Vissing-Jorgensen, 2009), while the latter face a larger total risk than the former.

The paper contributes to the theoretical literature on incomplete-market models. In this literature, analytical tractability can be obtained in a no-trade equilibrium, as in Constantinides and Duffie (1996) or Krusell, Mukoyama and Smith (2011). In these economies, assets can be priced, even in absence of trade. In our model, trades do additionally occur at the equilibrium. We show that limited asset market participation is sufficient to explain both a high equity premium and the volatility of consumption, with a risk aversion as low as 1 in the concave part of our benchmark calibration. Our assumption of linearity in the utility function is reminiscent of several papers that consider linearity in consumption or leisure utility or in the production function, so as to reduce ex-post heterogeneity, such as Scheinkman and Weiss (1986), Lagos and Wright (2005), Kiyotaki and Moore (2005, 2008), Miao and Wang (2015) or Dang, Holmstrom, Gorton, and Ordoñez (2014) among others. Finally, this paper generalizes some previous works on small-trade (Challe, LeGrand, and Ragot 2013; Challe and Ragot 2014 or LeGrand and Ragot 2015). We here extend this work by connecting small trade to Fishburn (1977)'s utility function, which allows to investigating consumption allocations and household risk exposure. The paper is also related to the vast literature on asset prices with heterogeneous agents. Our contribution to this literature is to analyze the interaction of two frictions: limited participation and incomplete insurance-markets with heterogeneous income-risk exposure.[4]

The remainder of the paper is organized as follows. In Section 2, we present the model and derive our existence result. In Section 3, we present the intuition underlying our

[4] Among the recent quantitative papers, Guvenen (2009) studies a model with limited participation and household heterogeneity in intertemporal elasticities of substitution. Constantinides and Ghosh (2014) build on Constantinides and Duffie (1996) to construct a no-trade equilibrium with Epstein-Zin preferences. Chien, Cole, and Lustig (2011, 2012) consider an incomplete market model featuring exogenous trading restrictions, which can easily be simulated. Gomes and Michaelides (2008) and Favilukis (2013) consider models with preference heterogeneity (in intertemporal elasticity of substitution or in bequest motive) together with incomplete markets or limited participation.

model in simplified versions of our framework. In Section 4, we perform a quantitative exercise to show that the model can reproduce household risk exposures and asset returns. Section 5 discusses the key assumptions of the model. Section 6 concludes.

2 The model

The model relies on three assumptions: (i) incomplete insurance markets, (ii) limited stock market participation and (iii) concave-linear utility function.

2.1 Risks and securities

Time is discrete and indexed by $t = 0, 1, \ldots$ The economy is populated by two types of infinitely-lived and ex-ante different agents (the ex-ante heterogeneity will be made clearer later on). Each population of type $i = 1, 2$ is of size 1 and is distributed on a segment J_i according to a measure ℓ_i.[5] We call these two populations "type-1" and "type-2" agents, respectively, while "type-i" ($i = 1, 2$) agents will refer to one of the two types without further specification. The letter i consistently refers to the agent's type (1 or 2).

2.1.1 Aggregate risk

There is a single aggregate shock $(z_t)_{t\geq 0}$, which can take n different values in the set $Z = \{z_1, \ldots, z_n\}$. The aggregate risk process $(z_t)_{t\geq 0}$ is a time-homogeneous first-order Markov chain whose transition matrix is denoted $\Pi = (\pi_{kj})_{k,j=1,\ldots,n}$. The probability π_{kj} of moving from state k to state j is thus constant. For every date $t \geq 0$, $z^t \in Z^{t+1}$ denotes a possible history of aggregate shocks up to date t.

[5] Among others, Feldman and Gilles (1985) have identified issues when applying the law of large number to a continuum of random variables. Green (1994) describes a construction of the sets J_i and of the non-atomic measures ℓ_i to ensure that our statements hold. Feldman and Gilles (1985), Judd (1985), and Uhlig (1996) also propose other solutions to this issue. From now on, we assume that the law of large numbers applies.

2.1.2 Idiosyncratic risk

Agents face an idiosyncratic risk in addition to the aforementioned aggregate risk. This individual risk can neither be avoided nor insured. We call this a productivity risk even though it may cover many other individual risks (such as the risks of unemployment, income, health, etc.) that are likely to affect their productivity (see Chatterjee, Corbae, Nakajima and Rios-Rull, 2007 for a quantitative discussion). At any point in time, type-i agents can either be *productive* (denoted herein by p) earning income $\omega^i(z_t)$ or *unproductive* (denoted by u), earning income δ^i.[6] Both incomes may depend on the agent type i. To simplify the exposition, we assume that δ^i does not depend on the aggregate risk z_t, but all our results can be easily extended to stochastic incomes δ^i. We assume that, regardless of the aggregate state, $\omega^i(z_t)$ is greater than δ^i for both agent types. Moreover, type-1 agents when productive have a higher income than type-2 agents. These assumptions are summarized in Assumption C below.

For each type-i agent at any date t, the function $\xi_t^i(z^t)$ characterizes the current status of the agent's productivity, taking the value 1 when the agent is productive and 0 when unproductive. For both agents, the productivity risk process $(\xi_t^i(z^t))_{t\geq 0}$ is a two-state Markov-chain with transition matrix $T_t^i = \begin{pmatrix} \alpha_t^i(z^t) & 1-\alpha_t^i(z^t) \\ 1-\rho_t^i(z^t) & \rho_t^i(z^t) \end{pmatrix}$.

We call $\eta_t^i \in (0,1)$ the share of productive agents among type-i population. Initial values η_0^1 and η_0^2 being given, the laws of motion of productive shares are

$$\eta_t^i(z^t) = \alpha_t^i(z^t)\eta_{t-1}^i(z^{t-1}) + (1-\rho_t^i(z^t))(1-\eta_{t-1}^i(z^{t-1})), \text{ for } i=1,2 \text{ and } t\geq 1. \tag{1}$$

To obtain a tractable framework, we impose the following constraint:

Assumption A (Population shares) *The probabilities α_t^i and the shares η_t^i depend only on the current aggregate state and not on the whole history. Formally, we have: $\alpha_t^i(z^t) = \alpha^i(z_{t-1})$ and $\eta_t^i(z^t) = \eta^i(z_t)$ for $t \geq 0$ and $i = 1, 2$.*

[6] Our idiosyncratic productivity risk is reminiscent of Kiyotaki and Moore (2005, 2008), Kocherlakota (2009) and Miao and Wang (2015), although the model and the paper scope are very different.

This assumption simplifies the dynamics of the population structure, but does not guarantee analytical tractability, since in general, it does not prevent the wealth distribution from having an infinite support. Assumption A includes the standard case where the transition probability α_t^i and the share η_t^i $(i = 1, 2)$ are constant and equal to α^i and η^i. Furthermore, Assumption A implies that the primitives of our model are the probabilities α_t^i and the shares η_t^i, while the transition rates $(\rho_t^i)_{t\geq 0}$ adjust for the law of motion in equation (1) to hold.

2.1.3 Asset markets

There are two types of assets in the economy –a risky stock and a riskless bond, issued by the government. This is the simplest environment that enables us to study the price of the safe asset and the market price of risk.

The risky asset. There is a constant mass V_X of a Lucas tree. The tree dividend is stochastic and the payoff in state k be y_k $(k = 1, \ldots, n)$. At any date t, we denote by P_t the (endogenous) price of one "stock" or "risky asset" (i.e., a share of the tree).

The bond. There is also a riskless bond of maturity one. Purchased at date t at price Q_t, these bonds pay off one unit of the consumption good at the next date in all states of the world. The total supply of bonds is constant and equal to V_B. These bonds –or safe assets– are issued by the government and funded by taxes on productive agents.

Participation structure. We state an assumption regarding stock market participation costs so as to pin down asset market structure. Consistently with empirical facts presented in Section 4 below, we set participation costs, such that type-1 agents trade stocks, while type-2 agents do not. Participation costs are consistent with many empirical studies, such as Mankiw and Zeldes (1991) or Vissing-Jorgensen (2002), and they are a frequent device to generate limited participation.[7] The goal of this participation cost is not to provide a full-fledge theory of limited participation, but instead to quantify below the opportunity cost of not-participating in financial markets.

[7]The impact of participation costs on asset prices has for instance been studied in Basak and Cuoco (1998), Heaton and Lucas (1999), Polkovnichenko (2004), Gomes and Michaelides (2008), Guvenen (2009), Walentin (2010) or Favilukis (2013).

More precisely, participation in the stock market is costly for agents, and trading riskless bonds is free. Trading stocks requires type-i agents to payper period lump-sum participation cost χ_i. This cost is a shortcut for both monetary and non-monetary hurdles to stock market participation. In particular, non-monetary costs may cover informational aspects, such as acquiring financial literacy and building understanding of stock markets. There is a strong evidence of heterogeneity in financial literacy among the population and that people with a low financial literacy participate less in stock markets (see van Rooij, Lusardi, and Alessie (2011) among others). Consistently with this heterogeneity, we make the following assumption:

Assumption B (Participation costs) *We assume that χ_2 is large enough for type-2 agents not to trade stocks, while type-1 agents do not pay participation costs: $\chi_1 = 0$.*

This assumption implies that agents of type 2, that will be at the bottom of the income distribution, will not trade stocks. Equation (26) in Appendix A provides an explicit formula for an upper bound on χ_2, based on the observation that type-2 agents will never trade stocks if the stock return, net of participation cost, is lower than the bond return, in all states of the world.

2.2 Agents' preferences

Agents' preferences are a crucial feature of the model tractability, enabling us to derive our small-trade equilibrium with a finite number of states.

Description of preferences. The periodic utility function $\tilde{u}$ is continuous, strictly increasing and globally concave. It is strictly concave for low values of consumption and has possibly 2 linear parts. This assumption can formally be written through conditions imposed on marginal utility $\tilde{u}'$:

$$\tilde{u}'(c) = \begin{cases} u'(c) & \text{if } c \leq c_1^*, \\ \lambda^2 & \text{if } c_2^* \leq c \leq c_3^*, \\ \lambda^1 < \lambda^2 & \text{if } c_4^* \leq c \leq c_5^*, \end{cases} \tag{2}$$

where $0 < c_1^* \le c_2^* < c_3^* \le c_4^* < c_5^*$. When agents consume a low amount, they value their consumption with the marginal utility $u'(\cdot)$, which is the derivative of a function $u : \mathbb{R}^+ \to \mathbb{R}$ assumed to be twice derivable, strictly increasing, and strictly concave. When agents consume a higher amount, their marginal utility is constant for two consumption intervals, on which it is equal to λ^1 or λ^2.[8] Figure 1 plots the shape of such a periodic utility function. We show below that the two slopes of linear parts can be chosen such that this quasi-linear utility function are consistent with a standard utility function, such as a log utility function.

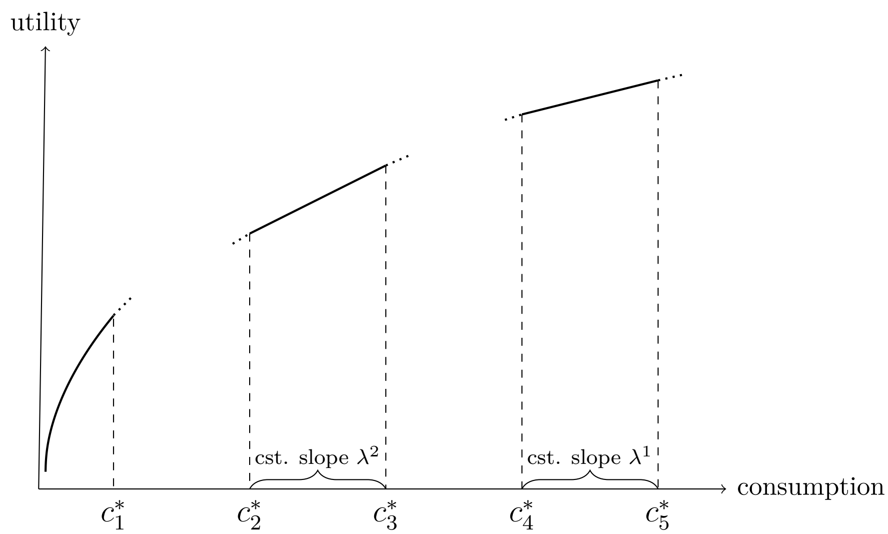

Figure 1: Shape of the periodic utility function

We now formulate our next assumption.

Assumption C (Income processes) *We assume that in any state $k = 1, \ldots, n$, we have $c_2^* < \omega^2(z_k) < c_3^*$, $c_4^* < \omega^1(z_k) < c_5^*$ and $\delta^i < c_1^*$ for $i = 1, 2$. This notably implies that $\delta^i < \omega^i(z_k)$ for both types $i = 1, 2$ and $\omega^1(z_k) > \omega^2(z_k)$ ($\forall k = 1, \ldots, n$).*

Assumption C states that the income of productive agents of types 1 and 2 lies in the set where the utility function is linear, and that the income of unproductive agents of types 1 and 2 lies in the set where the utility is strictly concave. A straightforward

[8] The two intervals can have connected support. There is no further restriction on $\tilde{u}$, except that it has to be continuous, strictly increasing and globally concave.

corollary is that in absence of trade, unproductive type-i agents are endowed with marginal utility $u'(\delta^i)$, while productive type-i agents have marginal utility $\lambda^i < u'(\delta^i)$, where the last inequality reflects that for any type i, being productive makes you better off than unproductive.

Consequence on the equilibrium. The assumption of constant marginal utility for productive agents helps to generate a limited heterogeneity equilibrium. Indeed, individual histories of productive agents do not matter for the pricing of securities since their marginal utility depends only on their type and not on their past saving choices.

Interpretation. The utility function of equation (2) is a generalization of the concave-linear utility function of Fishburn (1977), which is linear above a given threshold and strictly concave below it. In a portfolio choice problem, the agent endowed with such a utility, is risk-neutral for large payoffs and risk-averse for low ones. Loosely speaking, this functional form reflects the asymmetry in risk perception. Payoff realizations that are lower than a given threshold are perceived as actual risks, while payoffs greater than the threshold are perceived as being "nice surprises". As explained by Fishburn (1977, p. 123), this concave-linear functional form is "motivated by the observation that decision makers in investment contexts frequently associate risk with failure to attain a target return". In our paper, the concave-linear utility function can be understood as an approximation, according to which present wealth of productive agents is barely affected by saving choices, which implies a constant marginal utility. On the opposite, unproductive agents value their (lower) consumption with a strictly concave function. Importantly, this assumption does not imply that neither productive nor unproductive agents are risk-neutral: the concave part directly affects asset pricing and the behavior of productive agents (see Section 2.6). Furthermore, we generalize Fishburn's utility function and consider two thresholds and two linear parts.[9] We discuss this assumption further in Section 5.

[9] We could be equivalently assume that each agent type is endowed with a proper Fishburn concave-linear utility, with a common concave part but different linear parts.

2.3 Agent's program

Timing. At the beginning of every period, the agent observes her current productivity status and the dividend payoff.

Allocations. Due to the timing of the agent's program, agents' choices –consumption levels $(c_t^i)_{t\geq 0}$ and demands for stocks and bonds respectively denoted $(x_t^i)_{t\geq 0}$ and $(b_t^i)_{t\geq 0}$– at date t are mappings defined over the state space of possible shock histories $Z^t \times E^t$.[10] The consumption of any type-i agent is assumed to be positive:

$$\forall t \geq 0,\ c_t^i \geq 0. \tag{3}$$

Budget and borrowing constraints. The choices of a type-i agent are limited by a budget constraint in which total resources made up of income, stock dividends, and security-sale values are used to consume, pay taxes, and purchase securities. Formally, for all $t \geq 0$:

$$c_t^i + P_t\, x_t^i + Q_t b_t^i + \chi_i 1_{\zeta_t^i > 0} = (1 - \xi_t^i)\delta^i + \xi_t^i(\omega_t^i - \tau_t^i) + (P_t + y_t)x_{t-1}^i + b_{t-1}^i, \tag{4}$$

where $1_{\zeta_t^i > 0}$ is the indicator function equal to 1 if the agent trades bonds for the first time (i.e., if $x_t^i > 0$ and $1_{\zeta_{t-1}^i} = 0$), and equal to 0 otherwise. .

In addition, agents face borrowing constraints. They can neither produce any share of stocks nor short-sell the bond. This implies that:[11]

$$\forall t \geq 0,\ x_t^i, b_t^i \geq 0. \tag{5}$$

A *feasible allocation* for a type-i agent is a collection of plans $(c_t^i, x_t^i, b_t^i)_{t\geq 0}$ such that equations (4) and (5) hold at any date t. The *set of feasible allocations* $\mathcal{A}_i$ is

$$\mathcal{A}_i = \left\{(c_t^i, x_t^i, b_t^i)_{t\geq 0} :\ \text{equations (3), (4) and (5) hold}\right\} \tag{6}$$

[10] For the sake of clarity, we drop the dependence in shock histories.

[11] It would be possible to have strictly negative (but not too loose) borrowing constraints on bonds and stocks, while preserving the equilibrium existence. However, the set $\mathcal{V}$ of admissible security volumes compatible with equilibrium existence and defined below in Proposition 1 would be different.

Agent's program. The agent's program consists in finding a feasible allocation in the set $\mathcal{A}_i$ that maximizes her intertemporal utility subject to a transversality condition. Instantaneous utilities are discounted by a time preference parameter $\beta \in (0,1)$. The operator $E_0[\cdot]$ is the unconditional expectation over aggregate and idiosyncratic shocks. The initial financial asset endowments are denoted by x^i_{-1} and b^i_{-1}. Formally:

$$\max_{(c^i_t, x^i_t, b^i_t)_{t\geq 0} \in \mathcal{A}_i} E_0 \left[\sum_{t=0}^{\infty} \beta^t \tilde{u}(c^i_t) \right] \tag{7}$$
$$\text{s.t. } \lim_{t\to\infty} \beta^t E_0 \left[\tilde{u}'(c^i_t) x^i_t \right] = \lim_{t\to\infty} \beta^t E_0 \left[\tilde{u}'(c^i_t) b^i_t \right] = 0,$$
$$\{x^i_{-1}, b^i_{-1}, \xi^i_0, z_0\} \text{ are given.}$$

Agents' risk-sharing is limited along three dimensions. First, as in the Bewley-Huggett-Aiyagari literature, individual risk is uninsurable because no asset is contingent on the productivity status. Second, agents face participation and borrowing constraints. Finally, insurance market against the aggregate shock is possibly incomplete.

Budget of the State. In absence of government consumption, taxes and new bond issuances exactly cover the payoffs of maturing bonds. Moreover, the tax τ_t on productive agents of both types is assumed to be proportional to the productive agent's income. We recall that bond supply V_B is constant.[12] Hence, a balanced government budget constraint at any date t implies that the tax rate is

$$\tau_t = \frac{(1 - Q_t) V_B}{\omega^1_t \eta^1_t + \omega^2_t \eta^2_t}. \tag{8}$$

2.4 Equilibrium definition

We start with security market clearing conditions, stating that aggregate demand should be equal to total supply, which amounts to V_X for stocks and V_B for bonds. We define the probability measure Λ^i_t describing the distribution of type-i agents as a function of

[12] A time-varying bond supply would not change the conclusions of the paper. Moreover, a lump-sum tax would not quantitatively change the results, as will be clear after the presentation of the equilibrium structure.

their security holdings and the history of their individual status.[13] The market-clearing conditions can therefore be written as

$$\sum_{i=1,2} \int_{\mathbb{R}^2 \times E^t} x \Lambda_t^i(dx, db, d\xi) = V_X, \tag{9}$$

$$\sum_{i=1,2} \int_{\mathbb{R}^2 \times E^t} b \Lambda_t^i(dx, db, d\xi) = V_B. \tag{10}$$

Finally, by Walras' law, the good market clears when asset markets clear.

We can now define a sequential competitive equilibrium.

Definition 1 (Sequential competitive equilibrium) *A sequential competitive equilibrium is a collection of allocations* $(c_t^i, x_t^i, b_t^i)_{t \geq 0}$ *for* $i = 1, 2$ *and of price processes* $(P_t, Q_t)_{t \geq 0}$ *such that, for an initial distribution of stock and bond holdings and of idiosyncratic and aggregate shocks* $\{(x_{-1}^i, b_{-1}^i, \xi_0^i)_{i=1,2}, z_0\}$*, we have:*

1. *given prices, individual strategies solve the agents' optimization program in equation (7);*
2. *the security markets clear at all dates: for any* $t \geq 0$*, equations (9) and (10) hold;*
3. *the probability measures* Λ_t^i *evolve consistently with individual strategies in each period.*

2.5 Equilibrium existence

In standard economies featuring uninsurable idiosyncratic shocks, credit constraints and aggregate shocks, the equilibrium cannot be explicitly derived since it involves a large distribution of agents wealth. The usual strategy follows Krusell and Smith (1998) by computing approximate equilibria assuming a recursive structure. But, as pointed out by Heathcote, Storesletten, and Violante (2009), the existence of such an equilibrium is still an open question.

[13] More precisely, $\Lambda_t^i : \mathcal{B}(\mathbb{R})^2 \times \mathcal{B}(E^t) \to [0, 1]$, where for any metric space X, $\mathcal{B}(X)$ denotes the borel sets of X. As an example, $\Lambda_t^i(X, B^S, I)$ (with $(X, B^S, I) \in \mathcal{B}(\mathbb{R})^2 \times \mathcal{B}(E^t)$) is the measure of agents of type i, with holdings in risky assets $x \in X$, in bonds $b \in B^S$, and with an individual history $\xi \in I$.

In this paper, we prove the existence of an equilibrium and derive its theoretical properties under the assumption that the supply of both risky and riskless assets is not too large. In this case, unproductive agents of both types remain credit-constrained even after selling off their entire portfolio. They will not participate in the financial markets while productive agents are trading securities.

More precisely, we construct an equilibrium where the portfolio chosen by each agent depends only on her type, her current productive status, and the aggregate state. In other words, at each date, all type-1 productive agents have the same (time-varying) portfolio and all type-2 productive agents have the same portfolio. In this economy, there are thus only four different portfolios at each point in time. This stems from the fact that productive type-1 agents will have the marginal utility λ^1 and productive type-2 agents the marginal utility λ^2. Assumption C guarantees that this is the case when securities are in zero supply. Proposition 1 below extends it to positive supplies and proves the existence of a small-trade equilibrium.

Proposition 1 (Equilibrium existence) *We assume that:*

$$\forall k \in \{1, \ldots, n\}, \;\; \beta \left(\alpha^1(z_k) + (1 - \alpha^1(z_k)) \frac{u'(\delta^1)}{\lambda^1} \right) < 1. \tag{11}$$

If security volumes (V_B, V_X) belong to a set $\mathcal{V} \subset \mathbb{R}_+ \times \mathbb{R}_+$ defined in (47) –and containing $(0,0)$–, then there exists an equilibrium with the following features:

1. *the end-of-period security holdings of unproductive type-1 and type-2 agents is 0 for both the risky and the riskless assets;*
2. *the end-of-period security holdings of productive agents depend only on their type (1 or 2) and the current aggregate state;*
3. *the end-of-period holdings in stocks of type-2 agents is always 0;*
4. *the security prices depend only on the current aggregate state.*

The proof can be found in the Appendix[14]. The equilibrium exists under two conditions. The first one, $\beta(\alpha^1(z_k) + (1 - \alpha^1(z_k))\frac{u'(\delta^1)}{\lambda^1}) < 1$, ensures that stock prices are well-defined. If the condition does not hold, the stock price can possibly be infinite because agents are too patient or their desire to self-insure is too high.[15] The second condition is that security volumes belong to a set $\mathcal{V}$, including the zero volume case $V_X = V_B = 0$. This assumption implies that security volumes should not be too high, which guarantees that agents, when becoming unproductive, are credit-constrained. A second consequence of this condition on $\mathcal{V}$ is that unproductive agents still value consumption with a strictly concave utility and productive agents with an affine one.

The key feature of our equilibrium is that the saving choices of productive agents only depend on the current aggregate state and on the agent's type. This property critically relies on the quasi-linearity of the utility function, which implies that all productive agents of each type have the same marginal utility and therefore the same demand for assets. The quasi-linearity of utility function also explains the feature of our equilibrium, according to which security prices depend only on the current aggregate state.

We simplify our notations using the results of Proposition 1. Since security prices depend only on the current aggregate state, we call P_k the price of the risky asset and Q_k the price of the bond in state z_k $(k = 1, \ldots, n)$. Bond holdings only depend on the aggregate states and unproductive agents do not hold any assets. We therefore call b_k^i the holdings in bonds of any productive type-i agent in state z_k $(k = 1, \ldots, n)$. Since type-2 agents do not trade stocks ($x^2 = 0$), productive agents hold all stocks and $x_k^1 = \frac{V_X}{\eta_k^1}$. The equilibrium is therefore characterized by a finite sequence of $4 \times n$ variables $(b_k^1, b_k^2, P_k, Q_k)_{k=1,\ldots,n}$

In the heterogeneous agent literature, several existence results can already be found.

[14] To prove existence, we start from first-order conditions, as in Coleman (1991), by following the steps of the proof of Theorem 4.15 in Stokey and Lucas (1989). Indeed, the Kuhn-Tucker theorem requires a Hermitian space of allocations, which is not the case for the set of bounded real sequences (which (c_t^i), (e_t^i), (x_t^i), and (b_t^i) belong to).

[15] Technically, this condition ensures that the mapping $P \mapsto \beta E_k\left[(\alpha_k^1 + (1 - \alpha_k^1)u'(\delta_{k'}^1))(P_{k'} + y_{k'})\right]$, derived from the Euler equation, is a contraction with modulus strictly smaller than 1, where $E_k\left[X_{k'}\right] = \sum_{k'=1}^{n} \pi_{k,k'} X_{k'}$. The Banach fixed-point theorem then allows us to deduce the properties of the price. This condition does not appear in economies with only short-lived assets, which are simpler in this respect.

In Huggett (1993), agents trade short-lived riskless bonds in absence of aggregate shocks. Kuhn (2013) extends Huggett's result to permanent idiosyncratic shocks.[16] To our knowledge, Miao (2006) proves the sole existence result in an economy featuring asset trades, credit constraints, and idiosyncratic and aggregate risks with a continuum of agents. He considers an economy with general preferences in which agents can trade one short-lived asset, which are claims on capital. Our existence result concerns a setup with limited participation and both a short- and long-lived asset.[17]

2.6 Equilibrium structure

Due to the finite characterization of our equilibrium, we have a deeper understanding of the structure of the model. In the risky asset market, productive type-1 agents will always be the sole participants. In the bond market, both productive type-1 and productive type-2 agents can participate. The key result is that the equilibrium is a system of $4 \times n$ equations involving $4 \times n$ variables. The next Proposition presents this result.

Proposition 2 (Equilibrium properties) *There exist two distinct subsets $I_i \subset \{1, \ldots, n\}$ ($i = 1, 2$), characterizing the states of the world in which only type-i agents trade bond, such that the $4 \times n$ variables $(b_k^1, b_k^2, P_k, Q_k)_{k=1,\ldots,n}$ defining the equilibrium are given by*

[16] In a seminal paper, Duffie et al. (1994) consider endowment economies in which a finite number of ex-ante heterogeneous agents face aggregate risks and trade long-lived assets with borrowing constraints. They then prove the existence of ergodic equilibria with a recursive characterization, whose state space includes all endogenous variables (such as prices). In a similar vein, Becker and Zilcha (1997) prove the existence of a stationary equilibrium in a production economy with ex-ante heterogeneous agents facing aggregate risk. Krebs (2006) proves the existence of a no-trade equilibrium in a Krusell-Smith economy. Kubler and Schmedders (2002) prove existence of recursive equilibrium with a finite number of agents. These papers consider a finite number of households. This assumption helps in proving existence but makes the analysis of the properties of the equilibrium more difficult as all shocks are "aggregate". It may explain the wide use of Bewley-type model with a continuum of agents.

[17] In our setup it would also be possible to prove that the sequential competitive equilibrium is also a recursive competitive equilibrium in which the state variables are: current aggregate and idiosyncratic shocks and beginning-of-period security holdings for both agent types.

the following $4 \times n$ *equations:*

$$P_k = \beta \sum_{j=1}^{n} \pi_{kj} (\alpha_k^1 + (1-\alpha_k^1) \frac{1}{\lambda^1} u'(\delta^1 + (P_j + y_j) \frac{V_X}{\eta_k^1} + b_k^1))(P_j + y_j), \; k \in \{1, \ldots, n\}, \tag{12}$$

$$Q_k = \beta \sum_{j=1}^{n} \pi_{kj} (\alpha_k^1 + (1-\alpha_k^1) \frac{1}{\lambda^1} u'(\delta^1 + (P_j + y_j) \frac{V_X}{\eta_k^1} + b_k^1)), \; k \in \{1, \ldots, n\} - I_2, \tag{13}$$

$$Q_k = \beta \sum_{j=1}^{n} \pi_{kj} (\alpha_k^2 + (1-\alpha_k^2) \frac{1}{\lambda^2} u'(\delta^2 + b_k^2)), \; k \in \{1, \ldots, n\} - I_1, \tag{14}$$

$$V_B = \eta_k^1 b_k^1 \; \textit{and} \; 0 = b_k^2, \; k \in I_1, \tag{15}$$

$$V_B = \eta_k^2 b_k^2 \; \textit{and} \; 0 = b_k^1, \; k \in I_2, \tag{16}$$

$$V_B = \eta_k^1 b_k^1 + \eta_k^2 b_k^2, \; k \in \{1, \ldots, n\} - I_1 - I_2. \tag{17}$$

This equilibrium structure can be thought in two ways. First, one can consider it as a generalization of no-trade equilibria studied in Krusell, Mukoyama and Smith, (2011), in which we allow for limited participation and for positive traded volume. Second, this equilibrium structure can be thought as a simplification of the general incomplete-market equilibrium, where credit constraints bind after one period of unemployment. In any case, this limited-heterogeneity equilibrium allows us to study the effect of volume on asset prices. We now discuss the effects.

Our equilibrium is characterized by equalities (12)–(17). The first three sets of Euler equations provide security prices. Due to limited stock market participation, the risky asset price is only defined by the Euler equation (12) of productive type-1 agents, who hold all the stocks. The equation can be interpreted as follows. As it is standard, the price P_k in state k is equal to the discounted value of asset payoffs discounted by intertemporal marginal rate of substitution (hencefort, MRS). With the probability $\pi_{kj}\alpha_k^1$, the agent remains in the next period productive in state j, with marginal utility λ^1. Her MRS in that case is thus simply equal to 1, while asset payoff is $P_j + y_j$. With the probability $\pi_{kj}(1-\alpha_k^1)$, the agent becomes unproductive in state j. When becoming unproductive, the agent will sell her whole portfolio (and remain credit-constrained) and be endowed with the marginal utility $u'(\delta^1 + (P_j + y_j)\frac{V_X}{\eta_k^1} + b_k^1)$, which explains the MRS equal to $\frac{1}{\lambda^1}u'(\delta^1 + (P_j + y_j)\frac{V_X}{\eta_k^1} + b_k^1)$. Note that in spite of having a constant marginal utility, productive agents do not behave as if they were risk-neutral. Indeed, incomplete markets and the possibility of being unproductive and credit-constrained in the next period affect their pricing.

Regarding bonds, they may be traded by productive type-1 or type-2 agents possibly depending on the state of the world. Since our equilibrium features security prices that only depend on the current state of the world, there is one subset of states of the world, characterized by the index subset I_1, in which only type-1 agents trade bonds, while type-2 agents are excluded. In states of the world I_1: (i) the Euler equation (14) of type-2 agents does not hold, and (ii) the bond supply equals the demand of type-1 agents in equation (15). By the same token, there are states of the world, characterized by the subset index I_2, in which only type-2 agents hold bonds, while type-1 agents only hold stocks. This corresponds to the Euler equation (13) and the resource equality (16). Subsets I_1 and I_2 are possibly empty. Finally, in remaining states of world, i.e., those belonging to $\{1, \dots, n\} - I_1 - I_2$, type-1 and type-2 agents always trade bonds simultaneously. Note that the interpretation of equations (13)–(14) is the same as for (12)

Note that we need to explicitly check that the equilibrium exists, and that consumption levels implied by equations (12)–(17) are consistent with the equilibrium structure. For example, we need to verify that productive agents of type 2 actually do not wish to trade bonds in states I_1 and that unproductive agents are actually credit-constrained at the equilibrium. These conditions are provided in Appendix with equations (48)–(49) on the one hand and equations (50)–(52) in the other hand. These conditions implicitly define the set $\mathcal{V}$ of admissible security supplies V_X and V_B.

This system of equations can easily be simulated to match key moments of the data as we do in Section 4. More importantly, this equilibrium enables us to derive theoretical insights about the household risk exposure and the equity premium throughout the business cycle.

3 Intuitions through simpler setups

Our model features heterogeneous uninsurable individual risk, aggregate risk and positive security volumes. We now examine, in turn, the role of the different model features.

Throughout this section –and only in this section–, we further simplify our setup so as to make mechanisms as transparent as possible. In particular, we make the following two assumptions: (i) aggregate risk follows an IID process, and (ii) productivity transition probabilities α^i and ρ^i are constant.

No idiosyncratic risk. As a first benchmark, we study the case where agents do not face idiosyncratic risk (i.e., $\alpha^i = 1$ for $i = 1, 2$). Due to limited participation, only type-1 agents trade stocks, whose constant price P^{NIR} (NIR stands for "No Idiosyncratic Risk") verifies $P^{NIR} = \beta E[P^{NIR} + \tilde{y}]$, where $\tilde{y}$ is the next period uncertain dividend. The gross average stock return R_s^{NIR} is therefore constant and equal to β^{-1}. The riskless bond is traded by both agents and its price is $Q^{NIR} = \beta$. The riskless gross interest rate R_f^{NIR} is identical to the stock return. The equity premium in this environment is null: $R_s^{NIR} - R_f^{NIR} = 0$. Limited participation alone does not imply a non-zero risk premium.

Zero volumes. We now assume that agents face heterogeneous but constant transition probabilities across idiosyncratic states. Aggregate risk affects dividends while the income of productive (ω^i) and unproductive (δ^i) agents are constant. Moreover, both riskless and risky securities are in zero volume –henceforth, denoted ZV.

In this economy, the equilibrium features a complete asset market segmentation where productive type-1 agents trade stocks and productive type-2 agents trade bonds if

$$\kappa_2 > \kappa_1, \tag{18}$$

$$\text{where: } \kappa_i = (1 - \alpha^i)(\frac{u'(\delta^i)}{\lambda^i} - 1) \tag{19}$$

In other words, this segmentation relies on the heterogeneity in idiosyncratic risk. Indeed, for a type-i agent, the expression $\kappa_i = (1-\alpha^i)(\frac{u'(\delta^i)}{\lambda^i} - 1)$ can be interpreted as the expected magnitude of idiosyncratic (or productivity) risk since (i) $(1-\alpha^i)$ is the probability that a type-i agent faces a bad outcome due to the productivity risk, and (ii) $\frac{u'(\delta^i)}{\lambda^i}-1$ is the relative fall in marginal utility experienced by a type-i agent due to the productivity risk. As will be clear in the following propositions, this quantity drives the demand for self-insurance against the uninsurable individual risk.

Proposition 3 (Zero volumes) *In the zero-volume economy described above, the risk premium is an increasing function of heterogeneity in productivity risk:*

$$R_s^{ZV}-R_f^{ZV}=\frac{1}{\kappa_1}-\frac{1}{\kappa_2}. \tag{20}$$

The risk premium in equation (20) is strictly positive because of market segmentation, even in the absence of correlation between dividend payouts and marginal utility. The greater the heterogeneity in productivity risk between the both agents' types, the higher the equity premium. On the one hand, the stronger the self-insurance need of type-2 agents, the more they demand riskless bonds to hedge against the risk, causing the return on riskless bonds to decrease. On the other hand, the lower the self-insurance need of type-1 agents, the less they demand stocks and the greater the risky return they require to hold stocks. As a result, heterogeneous demands for self-insurance –in combination with limited stock market participation– are sufficient to generate a strictly positive risk premium, even though both asset payouts are uncorrelated with marginal utilities.

Positive volumes. We now relax the assumption of zero volumes. For the sake of simplicity, we assume that both bond and stock volumes are small, so as to allow us to derive closed-form expressions –as first-order expressions– for the equity premium, consumption levels and growth rates. In addition, and to simplify expressions, we assume that incomes are not time-varying: δ^i and ω^i are constant for $i=1,2$. Only stock dividends are time-varying. PV stands for positive volume.

Proposition 4 (Positive volumes) *If the condition (18) holds, the economy exhibits the following features:*

- *the equity premium, compared to the ZV case, is augmented by two terms: one reflecting the security supply and another reflecting the equity risk:*

$$R_s^{PV} - R_f^{PV} \approx R_s^{ZV} - R_f^{ZV} + \beta(1-\alpha^1)\frac{-\frac{u''(\delta^1)}{\lambda^1}}{\alpha^1 + (1-\alpha^1)\frac{u'(\delta^1)}{\lambda^1}} \tag{21}$$

$$\times\left(\frac{E\left[P^{ZV}+y(\tilde{z})\right]}{P^{ZV}}(E_t\left[P^{ZV}+y(\tilde{z})\right]\frac{V_X}{\eta^1}+b^1) + \frac{V\left[P^{ZV}+y(\tilde{z})\right]}{P^{ZV}}\frac{V_X}{\eta^1}\right)$$

$$\textit{with: } P^{ZV} = \frac{\beta(\alpha^1 + (1-\alpha^1)\frac{u'(\delta^1)}{\lambda^1})}{1-\beta(\alpha^1 + (1-\alpha^1)\frac{u'(\delta^1)}{\lambda^1})}E[y(\tilde{z})]. \tag{22}$$

- *the bond holdings of productive agents are such that*
  - *either* $b^1 = 0$ *and* $\eta^2 b^2 = V_B$ *in case of (endogenous) complete market separation;*
  - *or* $b_1 \geq 0$ *and* $b_2 \geq 0$*. Bond holdings are then determined by idiosyncratic risk heterogeneity and security volumes:*

$$\eta^2(\mu_1+\mu_2)b^2 \approx \kappa_2 - \kappa_1 + \mu_1(E^{\tilde{z}}[P^{ZV}+y(\tilde{z})]V_X + V_B), \tag{23}$$

$$\eta^1(\mu_1+\mu_2)b^1 \approx \kappa_2 - \kappa_1 - \mu_1 E^{\tilde{z}}[P^{ZV}+y(\tilde{z})]V_X + \mu_2 V_B, \tag{24}$$

$$\textit{with: } \mu_i = -(1-\alpha^i)\frac{u''(\delta^i)}{\eta^i\lambda^i} > 0.$$

This proposition illustrates the role of positive asset volumes along two dimensions: the equity premium and bond trading. Equation (21) describes the role of positive volumes on the equity premium. An increase in volumes raises the equity premium through two channels. First, positive volumes increase the ability to self-insure for productive type-1 agents who can trade both risky and riskless securities. Type-1 agents therefore require a greater return to hold the risky asset, increasing the equity premium. This corresponds to the expectation term in (21). Second, once they become unproductive, type-1 agents now sell a positive volume of risky assets whose liquidation value is uncertain. Type-1 agents want to be compensated for the uncertainty related to the liquidation value. This corresponds to the variance term in equation (21). Since bonds payoffs are riskless, there is no liquidation premium related to bonds.

Equations (23) and (24) determine the bond trading (in the absence of full market segmentation) and therefore risk hedging demand. From (23), we deduce that the bond demand b^2 of type-2 agents is mainly driven by two factors, heterogeneity in idiosyncratic risk and security volumes. The first determinant –in $\kappa_2 - \kappa_1$– is the heterogeneity in idiosyncratic risk. The greater this risk for type-2 agents –with respect to type-1 agents–, the more type-2 agents need to self-insure themselves and the more they demand bonds. The second determinant –proportional to μ_1– is the total quantity of securities available, including both bonds and stocks. The greater the security supply, the smaller the bond price and the more type-2 agents can purchase bonds. For type-1 agents, the intuition is similar except for the role of stock volumes. Indeed, type-1 agents can purchase either stocks or bonds, which are therefore partly substitutes. An increase in stock volumes makes stock cheaper and therefore crowds out bonds in favor of stocks for type-1 agents –this i sthe term proportional to μ_2. Type-1 agents purchase more stocks, but need fewer bonds to achieve the same degree of self-insurance.

4 Quantitative exercise

We now assess the ability of our simple model to match asset prices and the allocation of risk across households. The goal of this exercise is to show that the model can be made quantitatively relevant, though based on the simplifying assumptions introduced above. In the description of Section 2, Assumption B implies that only type-1 agents hold stocks, while type-2 do not, while the population of both types is identical. This is consistent with the empirical observation that only 50% of US households hold stocks either directly or indirectly and that stock-holders are mostly in the top 50% of the income distribution (Bricker et al., 2014). We identify type-1 participating households to the top 50% of US households in the income distribution (henceforth the *top 50%*) and type-2 nonparticipating agents to the bottom 50% (henceforth the *bottom 50%*).

The model parameters are divided into two groups. First, we calibrate some parameters to standard values (and to ensure that our equilibrium exists). Second, we use the tractability of our framework to simulate the model to match several targets (described below).

4.1 Parameter restrictions

4.1.1 Aggregate risk and asset volumes

The period is a quarter. There are two aggregate states ($n = 2$), which can be either G (for good) –that we identify to a boom– or B (for bad) –that we identify to a recession. For transition probabilities, we rely on the estimation of Hamilton (1994) for recession and booms and choose the values $\pi_{GG} = 0.75$ and $\pi_{BB} = 0.5$. The good state is thus more persistent than the bad one.

The volumes of assets are not chosen to match empirical targets but to ensure that our equilibrium exists. The stock volume is $V_X = 0.002$ and a bond volume is $V_B = 0.1$. We justify this choice by the following observations. First, the bottom 50% of US households (in the consumption distribution) holds a small amount of liquid wealth, which amounts to less than one thousand dollars according to the SCF. The assumption of small asset volumes is therefore not unrealistic for this fraction of the population. Second, the top 50% of US households obviously holds a much higher amount of assets and the model is not able to match this large amount. We use the flexibility given by time-varying idiosyncratic risks to reproduce the consumption volatility of this group of agents. The effectiveness of tools available for self-insurance is thus embedded in the income process, which should be considered as a "post-insurance" income process. This modeling strategy is the same as Krusell, Mukoyama, and Smith (2011), applied to a subgroup of agents. For these reasons, it is important in this class of model to reproduce key facts about consumption dynamics for both groups of agents as well as asset prices.

4.1.2 Preference parameters

The shape of the periodic utility function $\tilde{u}$ is defined by three parameters –see equation (2). First, we set $\sigma = 1$ and $\tilde{u}(c) = \log(c)$ in the non-linear part. Second, to avoid arbitrariness in the choice of the slopes $\lambda^1 < \lambda^2$ of the two linear parts, we require them to be equal to the derivatives –computed at the relevant point– of the utility function $\log(c)$:

$$\lambda^i = \frac{1}{c_{GG}^{i,pp}},\ i = 1, 2, \tag{25}$$

where $c_{GG}^{i,pp}$ is the consumption level of type-$i = 1, 2$ agents, who have been productive for at least two consecutive periods, while the aggregate state is G and was G in the previous period. Our choice for λ^1 and λ^2 is consistent with our interpretation of the linear parts in the utility function as an approximation of a more general utility function. Obviously, the values of consumption level $c_{GG}^{i,pp}$ in turn depend on λ^i, implying that equation (25) defining λ^i in fact involves solving a fixed-point problem.

4.1.3 Parameter restrictions

To bring discipline to the calibration strategy, we impose some constraints on the model parameters, which are consistent with the mechanisms identified in Section 3. We first set $\omega_B^1 = 1.0$ to scale the income process of type-1 agents. Second, we set $\eta_k^1 = \eta_k^2 = \eta$ ($k = G, B$) such that a constant fraction of the population is productive in every period. Third, we assume that the income risk faced by type-2 agents is not time-varying, such that $\omega^2 \equiv \omega_G^2 = \omega_B^2$. The model could easily accommodate these parameters being time-varying, but this is not necessary to match key moments of the data. Finally, we assume that the average value of dividends is $\overline{y} = 1$, such that only the ratio y_G/y_B is used in the calibration strategy.

4.2 Calibration

We are left with 11 parameters to calibrate: the discount factor β, the dividend process y_G/y_B and 9 parameters driving the income process: 4 probabilities α_k^i ($i = 1, 2$ and $k = G, B$), 4 income levels ω_G^1, ω^2, δ^1, δ^2 and the share η of productive agents. Using equation (1), the values of $\rho_{k_1 k_2}^i$ ($i = 1, 2$ and $k_1, k_2 = G, B$) are uniquely determined by the values of η and α_k^i for $i = 1, 2$ and $k = G, B$. To calibrate these 11 parameters, we match 12 empirical targets, that we denote $T = [T_1, \dots, T_{12}]$.

4.2.1 Consumption and household risk exposure

We first target parameters concerning the consumption risk. Following the literature (Parker and Vissing-Jorgensen, 2009, among others), the risk faced by each category of households is proxied by the volatility of the consumption growth rate for non-durable goods and services. Consumption is measured by quarterly expenditures on non-durable goods and on a subset of services deflated with the relevant price index. We use data of the Consumer Expenditure Survey (CEX) from 1980 to 2007.[18] A detailed discussion can be found in Appendix E. Our first two targets are the standard deviations of the quarterly consumption growth rate for the top 50%, equal to $T_1 \equiv \sigma(\Delta \log C_1) = 0.14$, and for the bottom 50%, equal to $T_2 \equiv \sigma(\Delta \log C_2) = 0.19$. The bottom 50% face higher total risk than the top 50%, as is standard in the literature. Our third target is the standard deviation of aggregate consumption which is $T_3 \equiv \sigma(\Delta \log C^{tot}) = 1\%$. This last value is not implied by the first two targets, because of agent heterogeneity.

We also target the exposure of both groups to aggregate shocks. Following Parker and Vissing-Jorgensen (2009), we compute, for each group, the coefficient equal to: (Change in real group consumption per household)∗(Group share of population)/(Lagged aggregate real consumption per household).[19] The coefficients, that sum to one for both groups,

[18] In what follows, we apply the methodology of Parker and Vissing-Jorgensen (2009) to a different subset of households, so as to be consistent with our model. Moreover, it is known that consumption data are not as accurate as data on household income (see Aguiar and Bils, 2011, among others, for a discussion). Nevertheless, as our results are consistent with those derived using different datasets, we are confident that the facts presented here are robust.

[19] This measure does not depend on the share of the type-1 agent consumption in total consumption,

can be interpreted as the fraction of aggregate risk born by each group. According to this metric, the top 50% bear $T_4 \equiv S_1 = 84\%$ of aggregate risk, whereas the bottom 50% bears the remaining 16%. Finally, the consumption share of the top 50% amounts to $T_5 \equiv C_1/C_{tot} = 72.1\%$, which drives consumption inequalities in our economy.

4.2.2 Asset returns and dividend process

We target 4 moments for stock and bond returns, and 3 moments for the dividend process. These seven moments are taken from Campbell (1999)'s dataset. First, the stock returns are computed from the S&P 500 index, while the bond returns are computed from the six-month commercial paper rate. To ease the comparison, these returns are annualized quarterly real returns and correspond to historical US data from 1890–1991. The average bond interest rate is $T_6 \equiv E(R_f) = 0.9\%$, while the standard deviation of the bond interest rate is $T_7 \equiv \sigma(R_f) = 1.7\%$. The average average stock return is equal $T_8 \equiv E(R_s) = 8.1\%$, whereas its standard deviation is $T_9 \equiv \sigma(R_s) = 15.6\%$. The last three targets are the average price dividend ratio $T_{10} \equiv E(P_s/D) = 21$, the standard deviation of the log of the price dividend ratio $T_{11} \equiv \sigma(\log(P_s/D)) = 30\%$, and the standard deviation of the log of dividend growth $T_{12} \equiv \sigma(\Delta\log(D)) = 28.3\%$.

4.2.3 Parameter values

Table 1 presents the model parameter values. First, we find that the income of type-1 agents barely moves, as the income in the good state $\omega_G^1 = 1.01$ is very close to the income in the bad state $\omega_B^1 = 1$. As a consequence, time variation in the income process is mostly driven by time-varying probabilities. The probabilities α_G^1, α_B^1, α_G^2 and α_B^2 oscillate around the value of 0.9, which is close but slightly lower than the value obtained when identifying the idiosyncratic risk with the employment risk. Indeed, on US data from 1948Q1-2007Q4, the quarterly probability of remaining employed equals 0.953, using Shimer (2005) methodology (see Challe and Ragot, 2014). The share of productive agents is $\eta = 0.89$. The discount factor β amounts to 0.86, which ensures equilibrium existence

as explained by Parker and Vissing-Jorgensen (2009).

Calibrated parameters								***"Equilibrium" parameters***		
π_{GG}	π_{BB}	σ	ω_B^1	V_X	V_B				λ^1	λ^2
0.75	0.50	1	1	0.002	0.1				0.99	2.25
Other parameters										
α_G^1	α_B^1	α_G^2	α_B^2	ω_G^1	$\omega_G^2 = \omega_B^2$	δ^1	δ^2	η	β	y_G/y_B
0.91	0.94	0.90	0.88	1.01	0.45	0.32	0.07	0.89	0.86	1.12

Table 1: Parameter values

(see condition (11)) and a realistic price dividend ratio. This discount factor is lower than the typical value used in complete-market models. Incomplete insurance-market models are indeed known to require a lower discount factor to reach the same average returns (for instance Krusell, Mukoyama, and Smith et al. 2011). The dividend process $y_G/y_B = 1.12$ enables to match the standard deviation of the dividend quarterly growth rate. Finally, the two preference parameters λ^1 and λ^2 are pinned down by equation (25).

4.3 Results

Table 2 summarizes the targets and provides the model outcome. This simple model does a surprisingly good job in reproducing empirical moments. The volatility of consumption growth for each group is close to its empirical counterpart and the returns are in line with their empirical values. We find a low return for the safe asset and a high equity premium, which amounts to 7%.

The model closely reproduces average moments of asset returns, a low volatility of the safe return and a high volatility of the risky return although the latter is lower than the data, which may not be surprising for such a simple framework. We find that the average income of type-1 agents is roughly twice as high as the one for type-2 agents. These values are consistent with empirical estimates of the skill premium, which is between 1.4 and 1.7 (Murphy and Welch, 1992) and with the fact that populations of skilled and unskilled workers are roughly of the same size over the last 20 years (Mukoyama and Sahin, 2006).

Implicit valuation of the risky asset by type-2 agents. In Assumption B, we set a high participation cost of type-2 agent to ensure that these agents will not trade any stock. We

	US Data	Model	Description and Remarks
Consumption Growth			
$\sigma(\Delta\log C_1)$(in %)	14	15	std. dev. of agg. type 1 cons. growth
$\sigma(\Delta\log C_2)$(in %)	19	22	std. dev. of type 2 cons. growth
$\sigma(\Delta\log C^{tot})$(in %)	1.0	1.0	std. dev. of agg. cons. growth
S_1(in %)	84	85	share of agg. risk born by type 1
C_1/C_{tot}(in %)	72	70	cons. share of type 1 in agg. cons.
Asset Returns			
$E(R_f)$(in %)	0.9	0.9	average safe return
$\sigma(R_f)$(in %)	1.7	1.2	std. dev. of the safe return
$E(R_s)$(in %)	8.1	7.9	average risky return
$\sigma(R_s)$(in %)	15.6	6.8	std. dev. of the risky return
Price-Dividend (P/D) Ratio			
$E(P_s/D)$	21	13	average P/D ratio
$\sigma(\log(P_s/D))$(in %)	30	17	std. dev. of log of P/D ratio
$\sigma(\Delta\log(D))$(in %)	28.3	27.7	std. dev. of log dividend growth

Note: See the text for the description of the statistics.

Table 2: Targets and model outcomes

provide in equation (26) a value of $\overline{\chi}_2$ which ensures that holding stocks is a dominated choice. Using our calibration and as the median annual income of US households was \$52,250 in 2014, we find an annual participation cost of \$33. This is relatively small compared to other estimates of the participation cost (Vissing-Jorgensen, 2002, among others).[20]

Role of participation costs. The ability of the previous model to reproduce asset prices crucially relies on stock market limited participation. To see this, we perform the same quantitative analysis as in the previous section but we relax Assumption B and set all participation costs to zero: $\chi_1 = \chi_2 = 0$. As explained in Proposition 2, both agent types may trade stocks and bonds. We provide the full model in Appendix G. We re-calibrate the model without participation costs and we find that the safe and risky returns are almost identical and equal to 7.0%. The model without participation costs thus fails to

[20] As this calibration may depend on the small volume of debt, we also compute the implicit valuation of the risky asset by type-2 agents (i.e., their valuation with their own pricing kernel). We find that type-2 agents will never participate in the stock market, if they face a proportional participation cost as low as 1.1%.

reproduce realistic asset prices. The reason is the following. A higher idiosyncratic risk for type-2 agents is necessary to match the difference in the consumption growth rate volatilities. Such a difference generates a higher desire to self-insure for type-2 agents and thus a higher valuation of all assets. In consequence, only one type of agents prices all the assets at the equilibrium and type-2 agents hold all stocks and bonds. [21]

5 Discussion of our assumptions

As explained in the discussion of Proposition 1, our equilibrium relies on two assumptions: (i) the linear parts in the utility function and (ii) the upper bound on security volumes. The concave-linear utility function generalizes Fishburn (1977)'s contribution. The shape of the periodic utility function implies that agents with a low consumption level are sensitive to small variations in consumption levels, while agents consuming a higher amount (i.e., those in a linear part) have a marginal utility which is invariant to small variations in consumption. However, these agents can experience a sensible increase in marginal utility if they are hit by a negative idiosyncratic shock that would force them to consume a low amount (that would be valued by the strictly concave part of the utility function). The concave-linear utility function accounts for extensive variations in consumption due to individual shocks but neglects the impact of small intensive variations. We believe it to be a simplified but relevant representation of consumption smoothing and of the behavior with respect to idiosyncratic risk. Consumption variations matter much more when consumption levels are low than when they are high. The quantitative exercise has shown that the linear part can be chosen for the marginal utility to be consistent with a regular and globally concave utility function. Finally, productive agents are *not* risk neutral as they always have a positive probability of valuing next period consumption with a strictly concave utility function.

[21] The role for participation costs in asset prices confirms findings in Guvenen (2009) in a model with heterogeneous preferences. Krusell, Mukoyama and Smith (2011) have shown that in such an economy, it is not possible to reproduce empirical asset prices with realistic idiosyncratic risks and a low risk aversion. Participation cost is thus a key ingredient for the ability of our model to match empirical data.

The upper bound on security volumes is the second important assumption. Indeed, for our limited-heterogeneity equilibrium to exist, we have to limit the amount of self-insurance, such that unproductive agents remain credit-constrained. As explained above, the assumption of small asset volumes is not unrealistic for the bottom 50% of US households. For the top 50%, we justify our assumption by the fact that the assets that we consider here only represent of small share of the total amount of assets observed in the data. Not all assets are available for self-insurance, either because they are not liquid (such as stocks locked in retirement plans –see Kaplan and Violante 2014a and Kaplan, Violante, and Weidner 2014b) or because households do not wish to trade them (due to the so-called portfolio inertia reported in Brunnermeier and Nagel, 2008, and Bilias, Georgarakos, and Haliassos, 2010). The modeling strategy is thus relevant for agents who have a low amount of assets to self-insure, or if one assumes that the main tools available for self-insurance are captured in the income process (as a reduced form). Our small-trade equilibrium therefore enables us to analyse the effect of *additional* liquidity.

Other assumptions are much less critical with respect to our equilibrium existence. For instance, Assumption A could be replaced by a less strict assumption, at the cost of a greater number of agent classes in the equilibrium. As discussed in Footnote 9, we could also allow agents to hold a negative wealth, as long as the borrowing limit is not too loose. Our equilibrium would still exist, provided that the set $\mathcal{V}$ of admissible volumes in Proposition 1 is changed accordingly.[22]

6 Conclusion

We have constructed an analytically tractable incomplete insurance market model with limited participation in financial markets, heterogeneity in risk exposure, and aggregate shocks. Our small-trade equilibrium relies on not-too-large security volumes and a concave-linear utility function introduced by Fishburn (1977). Though simple, this

[22]There is a kind of substitution between the negative wealth constraint and the maximal bounds (in $\mathcal{V}$) allowing for equilibrium existence.

model reproduces asset price properties and household risk exposure quite well. This parsimonious setup could be used to study other forms of heterogeneity with aggregate shocks and for instance the heterogeneity of agents according to both sides of their balance sheet (i.e., asset and liability sides). This would allow us to study financial intermediation in an incomplete market setting with aggregate shocks. Such an environment could improve our understanding of the functioning of financial markets.

Appendix

A Participation costs

We now derive explicitly the condition on the participation cost $\overline{\chi}_2$ of Assumption B for type-2 agents to never participate in the stock market. This cost is such that participating in the stock market is a dominated strategy, no matter the state of the world.

If type-2 agents participate in stock markets, their portfolio choice is denoted $\{\tilde{x}^2_k, \tilde{b}^2_k\}_{k=1,\ldots,n}$ given equilibrium prices $(P_k, Q_k)_{k=1,\ldots,n}$. Purchasing the quantity of stock $\tilde{x}^2_k$ is a dominated strategy in any state of the world k, if investing the same amount in bonds offers in every state a greater payoff. Due to participation cost, purchasing $\tilde{x}^2_k$ costs $P_k\tilde{x}^2_k + \chi_2$ and pays off $\tilde{x}^2_k(P_j + y_j)$ in the next period when the state of the world is $j = 1, \ldots, n$. Investing the same amount $P_k\tilde{x}^2_k + \overline{\chi}_2$ in bonds pays off $\frac{P_k\tilde{x}^2_k+\overline{\chi}_2}{Q_k}$ units of consumption in all states of the next period. In consequence, if $\frac{P_k\tilde{x}^2_k+\chi_2}{Q_k} > \tilde{x}^2_k(P_j + y_j)$ for any k, j, type-2 agents never wish to trade stocks. We deduce the following expression for $\overline{\chi}_2$ that ensures Assumption B to hold:

$$\overline{\chi}_2 = \max_{k,j=1,\ldots,n} (Q_k(P_j + y_j) - P_k)\tilde{x}^2_k. \tag{26}$$

Following the same steps as in Proposition 2, we obtain for the portfolio $\{\tilde{x}^2_k, \tilde{b}^2_k\}_{k=1,\ldots,n}$:[23]

$$P_k \geq \beta \sum_{j=1}^{n} \pi_{kj}(\alpha^2_k + (1-\alpha^2_k)\frac{1}{\lambda^2}u'(\delta^2 + (P_j + y_j)\tilde{x}^2_k + \tilde{b}^2_k))(P_j + y_j),\ k \in \{1, \ldots, n\}, \tag{27}$$

$$Q_k \geq \beta \sum_{j=1}^{n} \pi_{kj}(\alpha^2_k + (1-\alpha^2_k)\frac{1}{\lambda^2}u'(\delta^2 + (P_j + y_j)\tilde{x}^2_k + \tilde{b}^2_k)),\ k \in \{1, \ldots, n\} - I_2, \tag{28}$$

where (27) and (28) hold with equality if $\tilde{x}^2_k > 0$ and $\tilde{b}^2_k > 0$ respectively.

B Proof of Proposition 1

We prove that the market arrangement implied by Assumption B, in which type-1 agents trade stocks, while type-2 do not, is an equilibrium. We proceed in two steps: (i) we prove that we can find prices and quantities such that equations (12)–(17) hold and

[23] We have used the fact that credit constraints bind for unproductive type-2 agents after this deviation, which is true in the equilibrium under consideration.

(ii) we check that unproductive agents do not participate in security markets and that equilibrium consumption levels lie in proper definition sets of the utility function.

First step: the existence of prices and quantities. We define a correspondence on a compact set to invoke the Kakutani's fixed-point theorem. First, we define the compact convex sets $D_b = \{b \in \mathbb{R} : \; V_B \geq b \geq 0\}$ and $D_p = \{(P, Q) \in \mathbb{R}^2 : \underline{P} \leq P \leq \overline{P}$ and $0 \leq Q \leq \overline{Q}\}$ with $\underline{P} = \frac{\beta \min_{z\in Z} \alpha^1(z) y(z)}{1-\beta \min_{z\in Z} \beta\alpha^1(z)} > 0$, $\overline{P} = \frac{\beta \max_{z\in Z}(\alpha^1(z)+(1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1))y(z)}{1-\beta\max_{z\in Z}\alpha^1(z)+(1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1)} \geq \underline{P}$ and $\overline{Q} = \max_{z\in Z} \beta(\alpha^2(z) + (1-\alpha^2(z))\frac{1}{\lambda^2}u'(\delta^2))$.[24]

We define the mapping T^x from $(b\mathcal{C}(Z \times D_b), \|\cdot\|_\infty)$ onto itself as follows:[25]

$$T^x : X \mapsto y(z) + \beta\left[(\alpha^1(z) + (1-\alpha^1(z))f(X, z, z', b))X\right],$$
$$f(X, b, z, z') = \frac{1}{\lambda^1}u'\left(\delta^1 + X\frac{V_X}{\eta^1(z)} + \frac{V_B - b}{\eta^1(z)}\right),$$

where f decreasing in its first argument and where we can find $\underline{\beta}$ and $\overline{\beta}$ such that for all $z \in Z$ and for all $X \geq 0$, $0 < \underline{\beta} \leq \beta(\alpha^1(z) + (1-\alpha^1(z))f(X, z, z', b)) \leq \overline{\beta} < 1$ (condition (11)). We wish to prove that T^x is a contraction. We define $R : (X, X') \mapsto \frac{T^x X - T^x X'}{X - X'}$ for $0 \leq X' < X$. First, we can notice that $R(X, X') \leq R(X, 0)$. Indeed, it is equivalent to: $\frac{T^x X - T^x 0}{X} \leq \frac{T^x X' - T^x 0}{X'}$, which holds since f is decreasing in the first argument. Second, we have for $X' < X$ (since f is decreasing in the first argument):

$$R(X, X') = \beta\alpha^1(z) + \beta(1-\alpha^1(z))\frac{f(X,\cdot) - f(X',\cdot)}{X - X'} \geq \beta\alpha^1(z).$$

We deduce that for all $X \geq 0$ and $X' \geq 0$, we have $|E^z[T^x X] - E^z[T^x X']| \leq \overline{\beta}|E^z[X] - E^z[X']|$, where $E^z[\zeta] = \sum_{z'\in Z}\pi_{zz'}\zeta_{z'}$ is the conditional expectation of ζ. The Banach fixed-point theorem implies that there exists a unique $X \in b\mathcal{C}(Z \times D_b)$ such that:

$$X(z, b) = y + \beta E^z\left[(\alpha^1(z) + (1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1 + X(z',\cdot)\frac{V_X}{\eta^1(z)} + \frac{V_B - b}{\eta^1(z)}))X(z',\cdot)\right].$$

X is by construction a continuous function in b. We have just proven that the stock price $P(\cdot) = X(\cdot) - y$ is well-defined and is a continuous function of bond demand.

[24]It will be straightforward to check that equilibrium prices and quantities respectively belong to D_p and D_b.

[25]$b\mathcal{C}(\star)$ is the set of continuous bounded functions over the metric space $\star$, endowed with the sup. norm.

We now define the following correspondence $\psi^P : \mathcal{F}(Z, D_b) \rightrightarrows \mathcal{P}\left(\mathcal{F}(Z, D_p)\right)$, as:[26]

$$\begin{aligned}
&\psi^P(b) = \{(P, Q) \in \mathcal{F}(Z, D_p) | \\
&\quad P = \beta E^z[(\alpha^1(z) + (1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1 + (P(z') + y(z'))\frac{V_X}{\eta^1(z)} + \frac{V_B - b}{\eta^1(z)}))(P(z') + y(z'))], \\
&\quad Q = \beta E^z[\alpha^2(z) + (1-\alpha^2(z))\frac{1}{\lambda^2}u'(\delta^2 + \frac{b}{\eta^2(z)})]\Big\}.
\end{aligned}$$

If security demands solely depend on the current aggregate and idiosyncratic states, we deduce from Assumption A that the bond market clearing implies that $\forall z \in Z,\ b^2(z) = \frac{b(z)}{\eta^2(z)}$ and $b^1(z) = \frac{V_B - b(z)}{\eta^1(z)}$ where b^i denotes the bond demand of a type-i agent ($i = 1, 2$).

We introduce the correspondence $\psi^x : \mathcal{F}(Z, D_p) \rightrightarrows \mathcal{P}\left(\mathcal{F}(Z, D_b)\right)$, as follows:

$$\psi^x(P, Q) = \left\{ b \in \mathcal{F}(Z, D_b) | T^p_{P,Q}(b) = 0, V_B \geq b(z) \geq 0 \right\} \tag{29}$$

where $\forall (P, Q) \in \mathcal{F}(Z, D_p),\ T^p_{P,Q} :\ b \in \mathcal{F}(Z, D_b) \mapsto \forall z \in Z,$

$$\begin{aligned}
&(V_B - b(z)) \times 1_{E^z[\alpha^2(z)+(1-\alpha^2(z))\frac{1}{\lambda^2}u'(\delta^2+\frac{V_B}{\eta^2(z)})] > E^z[\alpha^1(z)+(1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1+(P+y(z'))\frac{V_X}{\eta^1(z)})]} \\
&+ b(z) \times 1_{E^z[\alpha^2(z)+(1-\alpha^2(z))\frac{1}{\lambda^2}u'(\delta^2)] < E^z[\alpha^1(z)+(1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1+(P+y(z'))\frac{V_X}{\eta^1(z)}+\frac{V_B}{\eta^1(z)})]} \\
&+ (E^z[\alpha^2(z) + (1-\alpha^2(z))\frac{u'(\delta^2 + \frac{b(z)}{\eta^2(z)})}{\lambda^2}] \\
&\qquad\qquad - E^z[\alpha^1(z) + (1-\alpha^1(z))\frac{u'(\delta^1 + (P + y(z'))\frac{V_X}{\eta^1(z)} + \frac{V_B - b}{\eta^1(z)})}{\lambda^1}]) \\
&\quad \times 1_{E^z[\alpha^2(z)+(1-\alpha^2(z))\frac{1}{\lambda^2}u'(\delta^2+\frac{V_B}{\eta^2(z)})] \leq E^z[\alpha^1(z)+(1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1+(P+y(z'))\frac{V_X}{\eta^1(z)})]} \\
&\quad \times 1_{E^z[\alpha^2(z)+(1-\alpha^2(z))\frac{1}{\lambda^2}u'(\delta^2)] \geq E^z[\alpha^1(z)+(1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1+(P+y(z'))\frac{V_X}{\eta^1(z)}+\frac{V_B}{\eta^1(z)})]},
\end{aligned}$$

where $1_A = 1$ if A is true and 0 otherwise. The mapping $T^p_{P,Q}$ considers the three possible cases of bond market participation. Bonds are traded by: (i) only type-2 agents, (ii) only type-1 agents and (iii) both agents. These three cases correspond to three mutually exclusive conditions. We can therefore check that ψ^x is compact- and convex-valued and upper semi-continuous (since it is compact-valued and its graph is closed).[27] ψ^x is also non-empty: either there is complete market separation (with only type-1 or type-2 agents

[26] Correspondences are set-valued functions (see Mas-Collel, Whinston and Green(1995), Section M.H). $\mathcal{P}(\star)$ is the set of all subsets of $\star$. For any compact K, $\mathcal{F}(Z, K)$ is the set of functions from Z to K and is isomorphic to K^n (and thus compact) since Z is of a cardinal n.

[27] Considering $\phi : p \mapsto \left\{ x \in [\underline{x}, \overline{x}], (\overline{x} - x)1_{p > k_2} + (x - \frac{k_2 - p}{k_2 - k_1}\underline{x} - \frac{p - k_1}{k_2 - k_1}\overline{x})1_{k_2 \geq p \geq k_1} + (x - \underline{x})\ 1_{p < k_1} = 0 \right\}$ ($k_2 > k_1$) may clarify this point. $\phi(p) = \{\overline{x}\}$ for $p > k_2$; $\phi(p) = \{\frac{k_2 - p}{k_2 - k_1}\underline{x} + \frac{p - k_1}{k_2 - k_1}\overline{x}\}$ for $k_2 \geq p \geq k_1$ and $\phi(p) = \{\underline{x}\}$ for $p < k_2$. The set $\{(p, \phi(p))\,,\ p \in \mathbb{R}\}$ is closed.

holding bonds), or both types of agents trade bonds.

We finally define the correspondence $\psi : ((P,Q),b) \in \mathcal{F}(Y,D_p)\times\mathcal{F}(Y,D_b) \rightrightarrows (\psi^p(b),\psi^x(P,Q)) \in \mathcal{P}(\mathcal{F}(Y,D_p)\times\mathcal{F}(Y,D_b))$. Since ψ^p and ψ^x are non-empty, compact- and convex-valued and upper semi-continuous, ψ also is. The Kakutani's theorem then ensures the existence of a fixed point $((P^*,Q^*),b^*) \in (\psi^p(b^*),\psi^x(P^*,Q^*))$. It is then straightforward to check that this fixed-point defines a competitive equilibrium. Moreover, for this equilibrium sets D_p and D_b are well-defined.

We now check that unproductive agents are kept out of the financial market.

Second step: unproductive agents do not participate in security markets. First note that the fixed-point generates an equilibrium with endogenous bond market participation of productive type-1 and type-2 agents. However, we need to determine, under which conditions unproductive agents of both types choose not to trade any security.

Security zero-supplies. We first assume $V_X = V_B = 0$. No security is traded and security prices are given by:

$$P(z) = \beta(\alpha^1(z) + (1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1))E^z\left[(P(z')+y(z'))\right], \tag{30}$$

$$Q(z) = \beta(\alpha^2(z) + (1-\alpha^2(z))\frac{1}{\lambda^2}u'(\delta^2)). \tag{31}$$

The equilibrium existence conditions are as follows (here $\hat{z}$ is the former state, z the current one and z' the next one):

$$P(z)\frac{1}{\lambda^1}u'(\delta^1) > \beta E^z\left[(1-\rho^1(z,z') + \rho^1(z,z')\frac{1}{\lambda^1}u'(\delta^1))(P(z')+y(z'))\right], \tag{32}$$

$$Q(z)\frac{1}{\lambda^1}u'(\delta^1) > \beta(1 - E^z\left[\rho^1(z,z')\right] + E^z\left[\rho^1(z,z')\right]\frac{1}{\lambda^1}u'(\delta^1)), \tag{33}$$

$$Q(z)\frac{1}{\lambda^2}u'(\delta^2) > \beta(1 - E^z\left[\rho^2(z,z')\right] + E^z\left[\rho^2(z,z')\right]\frac{1}{\lambda^2}u'(\delta^2)). \tag{34}$$

First notice that condition (32) can be expressed using (30) as:

$$E^z\left[\left(\alpha^1(z) + (1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1))\frac{1}{\lambda^1}u'(\delta^1) - (1-\rho^1(z,z') + \rho^1(z,z')\frac{1}{\lambda^1}u'(\delta^1))\right)(P(z')+y(z'))\right] > 0.$$

For conditions (32)–(34) to hold, it is sufficient that using (30)–(31), we have:

$$(\alpha^1(z) + (1 - \alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1))\frac{1}{\lambda^1}u'(\delta^1) > 1 - E^z\left[\rho^1(z,z')\right] + E^z\left[\rho^1(z,z')\right]\frac{1}{\lambda^1}u'(\delta^1), \tag{35}$$

$$(\alpha^2(z) + (1 - \alpha^2(z))\frac{1}{\lambda^2}u'(\delta^2))\frac{1}{\lambda^1}u'(\delta^1) > 1 - E^z\left[\rho^1(z,z')\right] + E^z\left[\rho^1(z,z')\right]\frac{1}{\lambda^1}u'(\delta^1), \tag{36}$$

$$(\alpha^2(z) + (1 - \alpha^2(z))\frac{1}{\lambda^2}u'(\delta^2))\frac{1}{\lambda^2}u'(\delta^2) > 1 - E^z\left[\rho^2(z,z')\right] + E^z\left[\rho^2(z,z')\right]\frac{1}{\lambda^2}u'(\delta^2). \tag{37}$$

We can check that equations (35) and (37) can be seen as positivity inequalities of polynomial functions in $\frac{1}{\lambda^1}u'(\delta^1)$ and $\frac{1}{\lambda^2}u'(\delta^2)$ respectively. Each polynomial function admits one negative root and another root equal to 1. Both polynomials are thus always positive since $\frac{1}{\lambda^1}u'(\delta^1) > 1$ and $\frac{1}{\lambda^2}u'(\delta^2) > 1$ (see Assumption C). Conditions (35) and (37) therefore always hold. The condition (36) can similarly be written as a positivity inequality of a polynomial function in $\frac{1}{\lambda^1}u'(\delta^1)$ and $\frac{1}{\lambda^2}u'(\delta^2)$, which is increasing in both arguments. We therefore deduce that: (i) when $\frac{1}{\lambda^1}u'(\delta^1) \geq \frac{1}{\lambda^2}u'(\delta^2)$, condition (36) holds whenever condition (37) does and (ii) when $\frac{1}{\lambda^1}u'(\delta^1) \leq \frac{1}{\lambda^2}u'(\delta^2)$, condition (36) holds whenever condition (35) does. In consequence, condition (36) always holds.

We finally check that consumptions of productive (resp. unproductive) agents lie in the linear (resp. concave) part of the utility function. Since our equilibrium features limited-heterogeneity, there are only 4 different agents classes per type, each of which depends on the current and past productive status. For instance, $c^{i,pu}_{\hat{z},z}$ is the consumption of type-i agents, who are currently unproductive (in state z) but were productive in the previous period (in state $\hat{z}$). The consumption levels of the different classes ($i = 1, 2$) are:

$$c^{i,pp}_{\hat{z},z} = c^{i,up}_{\hat{z},z} = \omega^i(z),$$
$$c^{i,pu}_{\hat{z},z} = c^{i,uu}_{\hat{z},z} = \delta^i.$$

Assumption C readily implies that consumptions lie in the proper regions of the utility function.

The equilibrium always exists in zero volume.

Positive supply economy. We assume that $V_B > 0$, and $V_X > 0$. Security prices are:

$$P(z) = \beta E_z \left[\left(\alpha^1(z) + (1-\alpha^1(z)) \frac{1}{\lambda^1} u'(\delta^1 + b^1(z) + \frac{V_X}{\eta^1(z)}(P(z') + y(z'))) \right) (P(z') + y(z')) \right], \tag{38}$$

$$Q(z) = \beta E_z[\alpha^2(z) + (1-\alpha^2(z)) \frac{1}{\lambda^2} u'(\delta^2 + b^2(z))], \tag{39}$$

where the quantities b^1 and b^2 are determined by three cases (see definition (29) of ψ^x):

- $b^1(z) = 0$ and $b^2(z) = \frac{V_B}{\eta^2(z)}$ if $E_z[\alpha^2(z) + (1-\alpha^2(z))\frac{1}{\lambda^2}u'(\delta^2 + \frac{V_B}{\eta^2(z)})] \geq E_z[\alpha^1(z) + (1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1 + \frac{V_X}{\eta^1(z)}(P(z') + y(z')))]$: the equilibrium features complete market segmentation;

- $b^1(z) = \frac{V_B}{\eta^1(z)}$ and $b^2(z) = 0$ if $E_z[\alpha^2(z) + (1-\alpha^2(z))\frac{1}{\lambda^2}u'(\delta^2)] \leq E_z[\alpha^1(z) + (1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1 + \frac{V_X}{\eta^1(z)}(P(z') + y(z')) + \frac{V_B}{\eta^1(z)})]$: the equilibrium also features complete market segmentation;

- $b^1(z) = \frac{V_B - \eta^2 b^2(z)}{\eta^1(z)}$ and $b^2(z)$ solves $E_z[\alpha^2(z) + (1-\alpha^2(z))\frac{1}{\lambda^2}u'(\delta^2 + b^2(z))] = E_z[\alpha^1(z) + (1-\alpha^1(z))\frac{1}{\lambda^1}u'(\delta^1 + \frac{V_X}{\eta^1(z)}(P(z') + y(z')) + \frac{V_B - \eta^2(z)b^2(z)}{\eta^1(z)})]$: both agents types trade bonds.

Since prices and bond quantities depend on the security supplies V_X and V_B (in addition to other model parameters), equilibrium existence conditions can be expressed as $\Theta(V_X, V_B) > 0$, where:

$$\Theta(V_X, V_B) = \left(\begin{array}{c} P(z)\frac{1}{\lambda^1}u'(\delta^1 + b^1(\hat{z}) + \frac{V_X}{\eta^1(\hat{z})}(P(z) + y(z)))\ldots \\ \ldots - \beta E_z\left[(1 - \rho^1(z,z') + \rho^1(z,z')\frac{1}{\lambda^1}u'(\delta^1))(P(z') + y(z'))\right] \\ Q(z)\frac{1}{\lambda^1}u'(\delta^1 + b^1(\hat{z}) + \frac{V_X}{\eta^1(\hat{z})}(P(z) + y(z)))\ldots \\ \ldots - \beta(1 - E^z\left[\rho^1(z,z')\right] + E^z\left[\rho^1(z,z')\right]\frac{1}{\lambda^1}u'(\delta^1)) \\ Q(z)\frac{1}{\lambda^2}u'(\delta^2 + b^2(\hat{z}))\ldots \\ \ldots - \beta(1 - E^z\left[\rho^2(z,z')\right] + E^z\left[\rho^2(z,z')\right]\frac{1}{\lambda^2}u'(\delta^2)) \end{array} \right)_{(\hat{z},z)\in Z^2} > 0. \tag{40}$$

Since the set Z is of cardinal n, $\Theta(V_X, V_B) \in \mathbb{R}^{3n^2}$. Note that $\Theta(V_X, V_B) > 0$ means that every component of $\Theta(V_X, V_B)$ is strictly positive.

We define

$$\mathcal{V}_\Lambda = \left\{(V_X, V_B) \in (\mathbb{R}^+)^2 | \Theta(V_X, V_B) > 0\right\}. \tag{41}$$

The zero supply part implies that $\mathcal{V}_\Lambda$ is not empty and, by continuity, includes an open set (of $(\mathbb{R}^+)^2$ endowed with the Euclidean norm) containing $(0,0)$. In other words, there exist $\overline{V}_X^\Lambda > 0$ and $\overline{V}_B^\Lambda > 0$, such that for all $0 \leq V_X \leq \overline{V}_X^\Lambda$ and $0 \leq V_B \leq \overline{V}_B^\Lambda$, $(V_X, V_B) \in \mathcal{V}_\Lambda$.

We now turn to the consumption expression. The consumption levels of the different classes ($i = 1, 2$) can be expressed as follows

$$c_{\hat{z},z}^{i,pp} = \omega^i(z)(1-\tau(z)) + \left(P(z)\left(\frac{V_X}{\eta^1(\hat{z})} - \frac{V_X}{\eta^1(z)}\right) + y(z)\frac{V_X}{\eta^1(\hat{z})} - \chi^1\right) 1_{i=1} + b^i(\hat{z}) - Q(z)b^i(z), \tag{42}$$

$$c_{\hat{z},z}^{i,up} = \omega^i(z)(1-\tau(z)) - \left(P(z)\frac{V_X}{\eta^1(z)} + \chi^1\right) 1_{i=1} - Q(z)b^i(z), \tag{43}$$

$$c_{\hat{z},z}^{i,pu} = \delta^i + (P(z) + y(z))\frac{V_X}{\eta^1(\hat{z})} 1_{i=1} + b^i(\hat{z}), \tag{44}$$

$$c_{\hat{z},z}^{i,uu} = \delta^i. \tag{45}$$

where taxes are given by $\tau(z) = \frac{(1-Q(z))V_B}{\omega^1(z)\eta^1(z)+\omega^2(z)\eta^2(z)}$. The vector of consumptions is denoted $C(V_X, V_B) = [c_{\hat{z},z}^{1,pp}, c_{\hat{z},z}^{1,up}, c_{\hat{z},z}^{2,pp}, c_{\hat{z},z}^{2,up}, c_{\hat{z},z}^{1,pu}, c_{\hat{z},z}^{1,uu}, c_{\hat{z},z}^{2,pu}, c_{\hat{z},z}^{2,uu}]_{(\hat{z},z)\in Z^2}$ and depends on V_B and V_X. The space of admissible consumptions is $\Gamma = \left([c_4^*, c_5^*]^2 \times [c_2^*, c_3^*]^2 \times [0, c_1^*]^4\right)^{n^2}$. We now define

$$\mathcal{V}_\Gamma = \left\{(V_X, V_B) \in (\mathbb{R}^+)^2 | C(V_X, V_B) \in \Gamma\right\}. \tag{46}$$

As for $\mathcal{V}_\Lambda$, we know, from the zero supply part, that $\mathcal{V}_\Gamma$ is not empty and by continuity that an open set containing $(0,0)$ is included in $\mathcal{V}_\Lambda$.

We conclude with defining the set $\mathcal{V}_1$ containing all volumes for which the equilibrium, where only type-1 agents trade stocks, exists:

$$\mathcal{V}_1 = \mathcal{V}_\Gamma \cap \mathcal{V}_\Gamma. \tag{47}$$

Previous remarks imply that $\mathcal{V}_1$ is non-empty and includes an open set containing $(0,0)$.

C Proof of Proposition 2

From Proposition 1 and in particular from its proof above, it is straightforward to deduce that there exist two distinct subsets $I_i \subset \{1, \ldots, n\}$ ($i = 1, 2$), characterizing the states of the world in which only type-i agents trade bond, such that the $4 \times n$ variables $(b_k^1, b_k^2, P_k, Q_k)_{k=1,\ldots,n}$ characterizing the equilibrium are given by the $4 \times n$ equations (12)–(17).

For the equilibrium to exist, we need to check two sets of conditions. The first one concerns the states of world, in which productive agents of a given type are excluded from bond markets. The second one concerns all states of the world, since unproductive agents of both types are permanently excluded from both financial markets.

In the states of the world I_2 where only type-2 agents trade bonds, type-1 agents are excluded due to too high bond prices and the following inequality has to hold for all $k = 1, \ldots, n$:

$$Q_k > \beta \sum_{j=1}^{n} \pi_{kj} (\alpha_k^1 \lambda^1 + (1 - \alpha_k^1) \frac{1}{\lambda^1} u'(\delta^1 + (P_j + y_j) \frac{V_X}{\eta^1})), \text{ for } k \in I_2. \tag{48}$$

By the same token, in the states of the world I_1, when only type-1 agents trade bonds, type-2 agents are excluded and the following inequality has to hold for all $k = 1, \ldots, n$:

$$Q_k > \beta(\alpha_k^2 \lambda^2 + (1 - \alpha_k^2) \frac{1}{\lambda^2} u'(\delta^2)), \ k \in I_1. \tag{49}$$

Regarding unproductive agents, type-1 (unproductive) agents are excluded from both stock and bond markets. The two following inequalities therefore need to hold for all $k, h = 1, \ldots, n$:

$$P_k \frac{1}{\lambda^1} u'(\delta^1 + b_h^1 + \frac{V_X}{\eta_h^1}(P_k + y_k)) > \beta \sum_{j=1}^{n} \pi_{kj} (1 - \rho_{kj}^1 + \rho_{kj}^1 \frac{1}{\lambda^1} u'(\delta^1))(P_j + y_j), \tag{50}$$

$$Q_k \frac{1}{\lambda^1} u'(\delta^1 + b_h^1 + \frac{V_X}{\eta_h^1}(P_k + y_k)) > \beta \sum_{j=1}^{n} \pi_{kj} (1 - \rho_{kj}^1 + \rho_{kj}^1 \frac{1}{\lambda^1} u'(\delta^1)). \tag{51}$$

Unproductive type-2 agents cannot participate to stock markets. For them to be excluded from bond markets, the following inequality needs to hold for all $k, h = 1, \ldots, n$:

$$Q_k \frac{1}{\lambda^2} u'(\delta^2 + b_h^2 + \frac{V_X}{\eta_h^2}(P_k + y_k)) > \beta \sum_{j=1}^{n} \pi_{kj} (1 - \rho_{kj}^2 + \rho_{kj}^2 \frac{1}{\lambda^2} u'(\delta^2)). \tag{52}$$

D Proof of propositions in Section 3

D.1 Proof of Proposition 3

Since dividends are IID, stock prices are constant. Provided that condition (11) holds, the Euler equation for the stock implies:

$$P^{ZV} = \frac{\beta(\alpha^1 + (1-\alpha^1)\frac{u'(\delta^1)}{\lambda^1})}{1-\beta(\alpha^1 + (1-\alpha^1)\frac{u'(\delta^1)}{\lambda^1})} E^{\tilde{z}}[y(\tilde{z})], \tag{53}$$

where $E^{\tilde{z}}[\cdot]$ is the expectation with respect to $\tilde{z}$. Type-2 agents are trading riskless bonds, while the bond price is too expensive for type-1 agents, i.e.:

$$Q^{ZV} = \beta\left(\alpha^2 + (1-\alpha^2)\frac{u'(\delta^2)}{\lambda^2}\right), \tag{54}$$

$$Q^{ZV} > \beta\left(\alpha^1 + (1-\alpha^1)\frac{u'(\delta^1)}{\lambda^1}\right), \tag{55}$$

where condition (55) holds thanks to condition (18). The zero supply economy therefore features full market segmentation, where type-1 agents hold stocks, while type-2 agents hold bonds. This equilibrium always exists from Proposition 1.

From price expressions (53) and (54), we deduce the equity premium of equation (20).

D.2 Proof of Proposition 4

Because the dividend process is IID, stock and bond prices, as well as bond holdings, are constant. The Euler equations for both securities become:

$$P^{PV} = \beta E^{\tilde{z}}\left[\left(\alpha^1 + (1-\alpha^1)\frac{u'(\delta^1 + (P_{t+1}+y_{t+1})\frac{V_X}{\eta^1} + b^1)}{\lambda^1}\right)(P^{PV} + y(\tilde{z}))\right],$$

$$Q^{PV} = \beta(\alpha^2 + (1-\alpha^2)\frac{u'(\delta^2 + b^2)}{\lambda^2}).$$

We solve for the price expression in the neighborhood of zero volumes. We assume that $0 < V_X \ll 1$ and $0 < V_B \ll 1$. Since bonds cannot be short-sold, we also have $0 \leq b^i \ll 1$. We obtain $P^{PV} \approx P^{ZV} + \pi_x V_X + \pi_b b^1$,[28] where P^{ZV} defined in equation (53) is the stock

[28] The approximation sign $\approx$ refers to a first order development with respect to security volumes. It should be understood as $\ldots = \ldots + o(V_X, V_B)$. We assume that both volumes have the same

price in zero volume and where:

$$\pi_x(1-\beta\psi^1) = \beta(1-\alpha^1)\frac{u''(\delta^1)}{\lambda^1}E^{\tilde{z}}\left[(P^{ZV}+y(\tilde{z}))^2\right], \tag{56}$$

$$\pi_b(1-\beta\psi^1) = \beta(1-\alpha^1)\frac{u''(\delta^1)}{\lambda^1}E^{\tilde{z}}\left[P^{ZV}+y(\tilde{z})\right], \tag{57}$$

$$\text{with: } \psi^i = \alpha^i + (1-\alpha^i)\frac{1}{\lambda^i}u'(\delta^i),\ i=1,2. \tag{58}$$

For the bond, we obtain $Q^{PV} \approx Q^{ZV} + \beta(1-\alpha^2)\frac{u''(\delta^2)}{\lambda^2}b^2$ for type-2 agents, where Q^{ZV} defined in equation (54) is the bond price in zero volume. From these equations, one gets equation (21).

For type-1 agents, we have $Q^{PV} \gtrsim \beta\psi^1 + \beta(1-\alpha^1)\frac{u''(\delta^1)}{\lambda^1}(b^1 + E^{\tilde{z}}[P^{ZV}+y(\tilde{z})])$. If type-1 agents do not participate to the bond market, the previous inequality is strict and we have $b^1 = 0$ and $b^2 = \frac{V_B}{\eta^2}$. If type-1 agents trade bonds, the previous inequality is an equality and noticing that $b^1 = \frac{V_B}{\eta^1} - \frac{\eta^2}{\eta^1}b^2$, we deduce the bond expressions (23) and (24). Because of condition (18), type-2 agents cannot be credit-constrained. Otherwise, we would have $(1-\alpha^1)\frac{u''(\delta^1)}{\lambda^1}(\frac{V_B}{\eta^1} + \frac{V_X}{\eta^1}E^{\tilde{z}}[P^{ZV}+y(\tilde{z})]) > \psi^2 - \psi^1 > 0$, contradicting positive volumes. We derive then from bond and stock prices the equity premium in (20).

E Data Appendix

We consider the dataset used by Heathcote, Perri and Violante (2010). To measure the consumption of non-durable and services, we use the sum of expenditures on non-durable goods, including: the vehicle services and other vehicle expenses (insurance, maintenance, etc.), the housing services, the rent paid, other lodging expenses, household equipment and entertainment. These items are deflated using the CPI. This measure corresponds to the variable *ndpnd*0 in Heathcote et al. (2010). We use the weights given in the CEX to define in each quarter the bottom 50% and the top 50% of households in the consumption distribution.

To compute the volatility of consumption growth for a given group in each quarter, we use the variance of the consumption growth rate between quarter t and quarter $t+1$ among all households belonging to said group at date t (regardless the household's group in $t+1$). We then compute the average variance per group over the time period.

F Description of the calibration algorithm

We describe here the algorithm of Section 4 that we use to minimize the distance between the 6 moments generated by the model and their empirical counterparts and that allows us to calibrate our model through the simulated method of moments. We denote $\chi_v = [\alpha^1, \alpha^2, \omega^2, \delta^1, \delta^2, \omega_G^1] \in \mathbb{R}_+^6$ the vector of model parameters we have to compute. We start from an initial guess vector χ_v^0.

1. We compute the six moments $\widetilde{T}^0$ generated by the model when parameters are equal to χ_v^0. We compute the score $S^0 = (\widetilde{T}^0 - T)\Omega(\widetilde{T}^0 - T)'$, where $\Omega = I_{6\times 6}$ is the weight matrix and T is the vector of empirical moments we match.

2. We construct the hyper-cube of the $2^6 = 64$ neighbors of χ_v^0 by considering marginal increase or decrease in each parameter: $\chi_v^i = [\chi_{v,1} \pm \epsilon, \ldots, \chi_{v,6} \pm \epsilon]$ $(i = 1, \ldots, 64)$ where we set $\epsilon = 10^{-3}$.

3. For every vector χ_v^i, we compute the moments generated by the model $\widetilde{T}^i$ $(i = 1, \ldots, 64)$. We then also compute the related score $S^i = (\widetilde{T}^i - T)\Omega(\widetilde{T}^i - T)'$ for $i = 1, \ldots, 64$.

4. If $S^0 \leq S^i$ for all $i = 1, \ldots, 64$, we stop the algorithm and we have just found a minimum. We then set our model parameters equal to χ_v^0. If not, we start the algorithm in step 1 with the new initial value $\chi_v^0 = \chi_v^{i_{\min}}$, where $i_{\min} = \arg\min_i \ S_i$.

This algorithm generates a path in $\mathbb{R}_+^6$ converging towards a (local) minimum. We try different starting points χ_v^0 to find a global minimum.

G The model without participation costs

Following the same steps as in the paper, we deduce the structure of the model without participation costs. At the equilibrium, both agents types may trade or not bonds and stocks. In particular, stock holdings x_k^1 and x_k^2 in state k are determined by Euler equations. There exist sets $I_i^B, I_i^X \subset \{1, \ldots, n\}$ $(i = 1, 2)$, such that the $6 \times n$ variables $(b_k^1, b_k^2, x_k^1, x_k^2, P_k, Q_k)_{k=1,\ldots,n}$ defining the equilibrium are given by the following $6 \times n$

equations ($i = 1, 2$):

$$P_k = \beta \sum_{j=1}^{n} \pi_{kj} (\alpha_k^i + (1 - \alpha_k^i) \frac{1}{\lambda^i} u'(\delta^i + (P_j + y_j) x_k^i + b_k^i))(P_j + y_j), \ k \in \{1, \ldots, n\} - I_i^X, \tag{59}$$

$$Q_k = \beta \sum_{j=1}^{n} \pi_{kj} (\alpha_k^i + (1 - \alpha_k^i) \frac{1}{\lambda^i} u'(\delta^i + (P_j + y_j) x_k^i + b_k^i)), \ k \in \{1, \ldots, n\} - I_i^B, \tag{60}$$

$$V_X = \eta_k^i \, x_k^i \text{ and } 0 = x_k^j, \ k \in I_i^X, \ i \neq j = 1, 2, \tag{61}$$

$$V_X = \eta_k^1 \, x_k^1 + \eta_k^2 \, x_k^2, \ k \in \{1, \ldots, n\} - I_1^X - I_2^X, \tag{62}$$

$$V_B = \eta_k^i \, b_k^i \text{ and } 0 = b_k^j, \ k \in I_i^B, \ i \neq j = 1, 2, \tag{63}$$

$$V_B = \eta_k^1 \, b_k^1 + \eta_k^2 \, b_k^2, \ k \in \{1, \ldots, n\} - I_1^B - I_2^B. \tag{64}$$

The set I_i^X $i = 1, 2$ gathers states of the world, in which type-i agents do not trade stocks. The set I_i^B has the same meaning for bond market. The sets I_1^B, I_2^B on one side and I_1^X, I_2^X on the other side must be disjoint. This means that there should not exist a state of the world, in which no one is trading bond or stocks. Note that since our equilibrium features security prices that only depend on the current state of the world, the sets I_i^B, I_i^X are not time-dependent.

As in the core of the paper, several inequalities have to hold for the above equations to define a small-trade equilibrium. These inequalities guarantee that: (i) productive agents who do not trade a given security do not want to do so (i.e., this implies inequalities similar to (48)–(48)), and (ii) that unemployed agents do not want to trade (i.e., this implies inequalities similar to (50)–(52)). For the sake of conciseness, we do not report here these inequalities, which are rather straightforward to deduce from the previous equilibrium but rather lengthy to write down.

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