Should monetary policy care about redistribution? Optimal fiscal and monetary policy with heterogeneous agents

François Le Grand   Alaïs Martin-Baillon   Xavier Ragot*

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Abstract

We derive optimal monetary and fiscal policies in a heterogeneous-agent economy with nominal frictions and aggregate shocks, and allowing for a rich set of fiscal tools. We first theoretically show that when linear taxes on capital and labor are available, there is no redistributive role for monetary policy: monetary policy solely implements price stability. Second, when fiscal tools are incomplete, we find that optimal deviation from price stability is quantitatively very small when the economy is running for a long time, i.e. in a timeless perspective. Redistribution is then mostly a matter of fiscal policy. When fiscal tools are missing, there can be a significant though temporary deviation from price stability when we consider a time-0 problem where the planner is not constrained by past commitment. We provide analytical results using an extended Lagrangian approach applied to incomplete-market models. Quantitative results are derived thanks to a truncated representation of incomplete-market models, which provides a relevant tractable environment.

Keywords: Heterogeneous agents, optimal Ramsey policies, monetary policy, fiscal policy.

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1 Introduction

Monetary policy generates redistributive effects through various channels that have been studied in a vast empirical and theoretical literature, reviewed below. However, it is not clear how these channels *should* change the conduct of monetary policy. It might be possible that monetary policy should take into account these effects to improve welfare, and thus participate in a function usually devoted to fiscal policy. On the contrary, monetary policy could focus only on monetary goals and let fiscal tools either dampen or strengthen the redistributive effects of monetary policy. To distinguish between these two claims, one must jointly solve for optimal fiscal and monetary policies in a realistic environment, where heterogeneity among agents generates a concern for redistribution.

To do so, we follow the so-called Bewley (1980) literature and assume incomplete insurance markets for idiosyncratic risks to be the main source of agent heterogeneity. This framework is known to be general enough to generate realistic income and wealth distributions. We further add nominal frictions, modeled as costly price adjustments. This environment has been named HANK following the seminal paper of Kaplan et al. (2018). Thanks to a methodological contribution explained below, we derive optimal monetary and fiscal policies with commitment when four fiscal instruments are available: a linear tax on capital income, a linear tax on labor income, lump-sum transfers and a riskless one-period public debt. We prove two main results.

First, we derive an equivalence result: We show that there is no redistributive role for monetary policy when the government can levy resources through linear capital and labor taxes. In this case, monetary policy solely aims at ensuring price stability in each period – as in any representative agent economy – and lets fiscal policy alone deal with redistribution. In this sense, there is a perfect dichotomy between the objectives of monetary and fiscal policies in such an economy. The redistributive role of monetary policy can stem only from missing fiscal instruments or, more precisely, from non-optimally time-varying capital or labor tax. These results can be understood thanks to the redistributive effects of monetary policy analyzed by Kaplan et al. (2018) or Auclert (2019) among others. Indeed, the effects that channel through changes in the real wage and in the real rate can be attained more efficiently by fiscal policy, as explained below.

Although monetary policy prescriptions are the same as in the representative agent economy, fiscal policy is very different, as there is an active role for redistribution over the business cycle. In particular, the optimal dynamics of capital tax and public debt strongly differ from those in a complete-market economy. Whereas the capital tax is very volatile in the complete-market economy, it is orders of magnitude less volatile in a quantitatively relevant incomplete-market setup. Indeed, agents then save for self-insurance motives, which makes capital taxation costlier. The implication of a less volatile capital tax is a more volatile public debt.

Second, we extend this analysis by considering various assumptions regarding the availability of fiscal instruments. Indeed, it may be argued that the political decision process making can slow down the reaction of taxes to the business cycle. We assume that the capital tax is not time-varying, which leaves a potential role for monetary policy to affect the ex-post real interest rate through inflation. Following the literature, we start with assessing the optimal inflation volatility when the economy is close to its long-run equilibrium, what is called the *timeless* perspective (see Woodford, 1999, for instance). As written by McCallum and Nelson (2000), this timeless perspective is the closest notion to "optimal policy making according to a rule". In this case, inflation volatility remains very small even when the capital tax remains constant. As a consequence, inflation is rarely used for redistributive purposes.

To generalize these results and compare them with the literature, we then analyze a *time-0 problem* in a monetary economy without capital, where the planner is not constrained by past commitments at the moment of the shock. Our equivalence result is still valid in this case: missing fiscal instruments are necessary to obtain a deviation from price stability. When taxes are not time-varying, we find that inflation significantly departs from price stability in the initial period. Inflation changes are front-loaded, as in the optimal capital tax literature (Chari and Kehoe, 1999, or Nuño and Thomas, 2020 for a recent reference). This experiment shows that our results are consistent with recent papers (in particular Acharya et al., 2020, Bhandari et al., 2020, and Nuño and Thomas, 2020).[1] As a consequence, deviation from price stability requires two conditions: missing fiscal instruments and a time-0 problem, where monetary authorities are not constrained by past commitments.

Deriving theoretical and quantitative results for optimal policies in incomplete-market economies with aggregate shocks is a challenging task. We perform our analysis thanks to two methodological contributions. First, we elaborate on the Lagrangian approach of Marcet and Marimon (2019), which appears particularly well-suited for HANK economies. We introduce the notion of *net social value of liquidity* for each agent, which considerably simplifies the algebra and interpretation. Second, to simulate the model, we use a truncated representation of incomplete insurance market economies that we apply here to a monetary economy. This theory of the truncation, also used in LeGrand and Ragot (2020) to study optimal unemployment benefits, constructs a consistent and accurate approximation of the economy where heterogeneity depends only on a finite but arbitrarily large number of past consecutive realizations of idiosyncratic risk. This allows us to estimate an empirically relevant social welfare function and to derive the dynamics of the model with a number of available planner's instruments. We check that the truncated approach generates accurate results by comparing its outcomes with other standard solution methods (Reiter, 2009; Boppart et al., 2018) and by comparing results regarding optimal policies with the ones found in the literature (Acharya et al., 2020; Bhandari et al., 2020; Nuño and Thomas, 2020).

[1] We also perform additional comparison exercises (on initial distribution, severity of credit constraint, or a time-varying idiosyncratic risk) and our results are consistent with those of the aforementioned papers.

Discussion of the related literature. First, our equivalence result must be related to the five transmission channels of monetary policy that have been identified in the heterogeneous-agent literature (Kaplan et al. 2018 and Auclert, 2019 among others). Monetary policy has direct effects, going through changes in real returns (Gornemann et al., 2016). The changes in returns generate a substitution effect, already present in the representative-agent new-Keynesian model (Woodford, 2003). Inflation also affects the real value of nominal assets, through a Fisher effect (Doepke and Schneider, 2006). Changes in real returns generate a wealth effect due to unhedged interest rate exposure, identified by Auclert (2019). In addition to these three direct channels, there are also two indirect effects due to the endogeneity of labor income and to the heterogeneous exposure to income variations (Coibion et al., 2017, Acharya and Dogra, 2020).

Our result states that fiscal policy can substitute for the five monetary channels without incurring the cost of price distortions. More precisely, capital tax is a sufficient instrument for the planner to reproduce any allocation that can be reached with the three direct channels, because it directly affects the return on capital. The labor tax is sufficient to replicate allocations reachable with the two indirect effects, because it directly affects labor income. Thus, linear taxes on capital and on labor are sufficient for monetary policy to optimally focus on price stability alone and to let fiscal policy deal with redistribution. The proof is thus similar to the one of Correia et al. (2008) and Correia et al. (2013), who consider consumption tax (and not capital tax) in representative-agent economies. Different monetary effects can be summarized by their impact on a distortion, for which a fiscal tool can have a direct effect.

This paper also connects to the literature investigating optimal policies with incomplete markets and heterogeneous agents. A first strand of this literature relies on tractable models featuring a simple distribution of wealth, which enables identifying the trade-offs faced by optimal policies. Challe (2020) solves for optimal monetary policy in a “zero-liquidity” environment with endogenous risk. Bilbiie and Ragot (2020) study optimal monetary policy in a tractable model with limited heterogeneity and money. Bilbiie (2020) analyzes a no-trade equilibrium with two types of agents. McKay and Reis (2020) solve for optimal simple fiscal rules (automatic stabilizers) in a tractable model considering exogenous monetary policy. In a time-0 experiment, they also find that there is a significant deviation from price stability when fiscal instruments are missing.

A second strand of the literature analyses optimal policies with more general distributions of wealth. This is especially true for the literature on optimal fiscal policy in incomplete-market and heterogeneous-agent models (Aiyagari et al., 2002; Werning, 2007; Bassetto, 2014; Açikgöz et al., 2018, or Dyrda and Pedroni, 2018 among others). In this strand, three recent papers study optimal monetary policy with incomplete insurance-markets: Acharya et al. (2020), Bhandari et al. (2020), and Nuño and Thomas (2020). Nuño and Thomas (2020) solve for optimal monetary policy under commitment in an economy with uninsurable idiosyncratic risk, nominal long-term bonds and costly inflation. They propose a methodology based on calculus of variation. They show that the optimal policy features inflation front-loading that can be sizable in a time-0 problem (especially when the initial distribution differs from the steady-state one). However, they also find that in a timeless perspective, optimal deviations from price stability remain limited. Acharya et al. (2020) solve for optimal monetary policy using the tractability of the CARA-normal environment without capital. They show that a countercyclical idiosyncratic risk creates a motive for redistribution and hence for monetary policy. They focus on a time-0 problem and deviation from price stability remains of small magnitude in their quantitative applications, except when the price-adjustment cost becomes very small. Bhandari et al. (2020) quantitatively solve for optimal monetary and fiscal policies in a new-Keynesian model with aggregate shocks. In a time-0 experiment, they report a significant deviation from price stability when fiscal instruments are missing and the initial distribution different from the steady-state one. They also quantitatively find that inflation volatility is low when time-varying capital taxes are introduced. In this literature, our contribution is to jointly characterize optimal public debt and fiscal-monetary policy dynamics in both the timeless and time-zero perspective, and in a setup with capital and occasionally binding credit constraints. We can also compare our results with previous findings by picking relevant calibration of our framework.

2 The environment

Time is discrete, indexed by $t \geqslant 0$. The economy is populated by a continuum of agents of size 1, distributed on a segment J following a non-atomic measure ℓ: $J(\ell) = 1$. Following Green (1994), we assume that the law of large numbers holds.

2.1 Risk

The only aggregate shock affects the technology level in the economy. We denote this risk by $(z_t)_{t\geq 0}$ and we assume that follows an AR(1) process follows: $z_t = \rho_z z_{t-1} + u_t^z$ with $\rho_z > 0$ the persistence parameter and the shock u_t^z being a white noise with a normal distribution $\mathcal{N}(0, \sigma_z^2)$. The economy-wide productivity, denoted $(Z_t)_{t\geq 0}$, is assumed to relate to z_t through: $Z_t = Z_0 e^{z_t}$. The history of aggregate risk up to period t is denoted z^t.

In addition to this aggregate shock, agents face an uninsurable idiosyncratic labor productivity shock $y \in \mathcal{Y}$. An agent i can adjust her labor supply, denoted by l_t^i and earns the before-tax hourly wage $\tilde{w}_t$ (which depends on the aggregate shock). Therefore, her total before-tax wage amounts to $y_t^i \tilde{w}_t l_t^i$. We assume that the productivity process is a first-order Markov chain with constant transition probabilities, denoted by $\Pi_{yy'}$. The share of agents with productivity y, denoted by S_y, is constant and equal to: $S_y = \sum_{\tilde{y}} \Pi_{\tilde{y}y} S_y$ for all $y \in \mathcal{Y}$. Finally, a history of productivity shocks up to date t is denoted by $y^t = \{y_0, \ldots, y_t\}$. Using transition probabilities, we can compute the measure θ_t, such that $\theta_t(y^t)$ represents the share of agents with history y^t in period t.

2.2 Preferences

In each period, the economy has two goods: a consumption good and labor. Households are expected utility maximizers and they rank streams of consumption $(c_t)_{t\geq 0}$ and of labor $(l_t)_{t\geq 0}$ according to a time-separable intertemporal utility function equal to $\sum_{t=0}^{\infty}\beta^t U(c_t, l_t)$, where $\beta \in (0,1)$ is a constant discount factor and $U(c,l)$ is an instantaneous utility function. As is standard in this class of models, we focus on the case where U is a Greenwood-Hercowitz-Huffman (GHH) utility function, exhibiting no wealth effect for the labor supply. For any consumption c and labor supply l, the instantaneous utility $U(c,l)$ can be expressed as:[2]

$$U(c,l) = u\left(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}\right),$$

where $\varphi > 0$ is the Frisch elasticity of labor supply, $\chi > 0$ scales labor disutility, and $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously derivable, increasing, and concave, with $u'(0) = \infty$.

2.3 Production

The consumption good Y_t is produced by a unique profit-maximizing representative firm that combines intermediate goods $(y_t^f(j))_j$ from different sectors indexed by $j \in [0,1]$ using a standard Dixit-Stiglitz aggregator with ε an elasticity of substitution, denoted ε:

$$Y_t = \left[\int_0^1 y_t^f(j)^{\frac{\varepsilon-1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

For any intermediate good $j \in [0,1]$, the production $y_t^f(j)$ is realized by a monopolistic firm. The profit maximization for the firm producing the final good implies:

$$y_t^f(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon}Y_t, \text{ where } P_t = \left(\int_0^1 p_t(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}.$$

The quantity P_t is the price index of the consumption good. Intermediary firms are endowed with a Cobb-Douglas production technology and use labor and capital as production factors. The production technology involves that $\tilde{l}_t(j)$ units of labor and $\tilde{k}_t(j)$ units of capital are transformed into $Z_t\tilde{k}_t(j)^\alpha \tilde{l}_t(j)^{1-\alpha}$ units of intermediate good. At the equilibrium, this production will exactly cover the demand $y_t^f(j)$ for the good j, which will be sold with the real price $p_t(j)/P_t$. We denote as $\tilde{w}_t$ the real before-tax wage and $\tilde{r}_t^K$ the real before-tax net interest rate on capital – which are both identical for all firms. The capital depreciation is denoted $\delta > 0$. Since intermediate firms have market power and internalize it, the firm's objective is to minimize production costs, including capital depreciation, subject to producing the demand $y_t(j)$. The cost function $C(j)$ of firm j is therefore $C(j) = \min_{\tilde{l}_t(j),\tilde{k}_t(j)}\{(\tilde{r}_t^K + \delta)\tilde{k}_t(j) + \tilde{w}_t\tilde{l}_t(j)\}$, subject to

[2] Our results do not depend on this functional form. See Section 3.3 for a general function U.

$y_t^f(j) = Z_t \tilde{k}_t(j)^\alpha \tilde{l}_t(j)^{1-\alpha}$. Denoting $\zeta_t(j)$ the Lagrange multiplier of the production constraint, first-order conditions imply:

$$\tilde{r}_t^K + \delta = \zeta_t(j)\alpha \frac{y_t^f(j)}{\tilde{k}_t(j)} \text{ and } \tilde{w}_t = \zeta_t(j)(1-\alpha)\frac{y_t^f(j)}{\tilde{l}_t(j)}. \tag{1}$$

We deduce that the optimum features a common value, denoted by ζ_t, independent of firm type j. Indeed, we have:

$$\zeta_t = \frac{1}{Z_t}\left(\frac{\tilde{r}_t^K + \delta}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}. \tag{2}$$

The firm j's cost becomes then $C(j) = \zeta_t y_t^f(j)$, which is linear in the demand $y_t^f(j)$. Following the literature, we assume the presence of a subsidy τ^Y on the total cost, which will compensate for steady-state distortions, such that the total cost supported by firm j is $\zeta_t y_t^f(j)(1-\tau^Y)$. Integrating factor price equations (1) over all firms leads to:

$$K_{t-1} = \frac{1}{Z_t}\left(\frac{\tilde{r}_t^K + \delta}{\alpha}\right)^{\alpha-1} \left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha} Y_t \text{ and } L_t = \frac{1}{Z_t}\left(\frac{\tilde{r}_t^K + \delta}{\alpha}\right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha}\right)^{-\alpha} Y_t, \tag{3}$$

where Y_t is the total production, with which (3) verifies:

$$Y_t = Z_t K_{t-1}^\alpha L_t^{1-\alpha} = \frac{(\tilde{r}_t^K + \delta)K_{t-1} + \tilde{w}_t L_t}{\zeta_t}. \tag{4}$$

Finally, in this set-up, the usual factor price relationships do not hold but we still have:

$$\frac{K_{t-1}}{L_t} = \frac{\alpha}{1-\alpha}\frac{\tilde{w}_t}{\tilde{r}_t^K + \delta}. \tag{5}$$

In a real setup (featuring $\zeta_t = 1$ for all t), equations (2) and (5) fall back to the standard definitions of factor prices: $\tilde{r}_t + \delta = \alpha Z_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha-1}$ and $\tilde{w}_t = (1-\alpha)Z_t\left(\frac{K_{t-1}}{L_t}\right)^\alpha$.

In addition to the production cost, intermediate firms face a quadratic price adjustment cost *à la* Rotemberg (1982) when setting their price. Following the literature, the price adjustment cost is proportional to the magnitude of the firm's relative price change and equal to $\frac{\kappa}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2 Y_t$, where $\kappa \geq 0$. We can thus deduce the real profit, denoted $\Omega_t(j)$, at date t of firm j:

$$\Omega_t(j) = \left(\frac{p_t(j)}{P_t} - \left(\frac{\tilde{r}_t + \delta}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha} \frac{1-\tau^Y}{Z_t}\right)\left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t - \frac{\kappa}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2 Y_t - t_t^Y, \tag{6}$$

where t_t^Y is a lump-sum tax financing the subsidy τ^Y. Computing the firm j's intertemporal profit requires to define the firm's pricing kernel. In a heterogeneous agent economy, there is no obvious choice for the pricing kernel. We discuss this aspect in Section 2.7. For the moment, we assume only that the firm's j pricing kernel is independent of its type and we denote it by $\frac{M_t}{M_0}$. With this notation, the firm j's program consisting in choosing the price schedule $(p_t(j))_{t\geq 0}$ maximizing the intertemporal profit at date 0, can be expressed as: $\max_{(p_t(j))_{t\geq 0}} \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \frac{M_t}{M_0} \Omega_t(j)]$. Since this program yields a solution independent of the firm type j, all firms in the symmetric equilibrium will charge the same price: $p_t(j) = P_t$. Denoting the gross inflation rate as $\Pi_t = \frac{P_t}{P_{t-1}}$ and setting $\tau^Y = \frac{1}{\varepsilon}$ to obtain an efficient steady state, we obtain the standard equation characterizing the Phillips curve in our environment:

$$\Pi_t(\Pi_t - 1) = \frac{\varepsilon - 1}{\kappa} (\zeta_t - 1) + \beta \mathbb{E}_t \Pi_{t+1}(\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \frac{M_{t+1}}{M_t}. \tag{7}$$

The real profit, independent of firm's type, becomes then with $\pi_t = \Pi_t - 1$:

$$\Omega_t = \left(1 - \zeta_t - \frac{\kappa}{2} \pi_t^2\right) Y_t. \tag{8}$$

2.4 Assets

Agents have the possibility to trade two assets. The first one is nominal public debt, whose size is denoted by B_t at date t. Public debt is issued by the government and is assumed to be exempt of default risk. The nominal debt pays off a nominal gross and pre-tax interest rate that is predetermined. In other words, the interest rate between dates $t-1$ and t is known at $t-1$. We denote this (gross and before tax) interest rate by $\tilde{R}_{t-1}^{B,N}$. The associated real before-tax (gross) interest rate for public debt is $\tilde{R}_{t-1}^{B,N}/\Pi_t$, where Π_t is the gross inflation rate. Note that due to inflation, this ex-post real rate is not pre-determined anymore. The second asset is capital shares, which pay off a (net and before-tax) real interest rate $\tilde{r}_t^K$ – as introduced above.

We assume that the whole public debt and the whole capital are held by a risk-neutral fund and that agents can trade shares of this fund. The interest rate paid by this fund to agents is denoted by $\tilde{r}_t$. The gain of this market arrangement is to allow for two different asset classes, without a portfolio choice (see Gornemann et al., 2016, Bhandari et al., 2020 among others). Our theoretical results do not depend on this assumption, as shown in Section 3.3.

The three interest rates, for public debt, capital and the fund, are connected by two different relationships. The first reflects the non-profit condition of the fund. We denote by A_t the total asset amount in the economy, equal to the sum of public debt and capital, which verifies $A_t = K_t + B_t$. Since the fund holds all the public debt and the capital and sell shares, its non-profit condition at date t implies:

$$\tilde{r}_t A_{t-1} = \tilde{r}_t^K K_{t-1} + \left(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1\right) B_{t-1}. \tag{9}$$

The second relationship is the no-arbitrage condition between public debt holdings and capital shares. This condition states that one unit of consumption invested in each of the two assets should yield the same expected return. Formally, this condition can be written as:

$$\mathbb{E}_t \left[\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}} \right] = \mathbb{E}_t \left[1 + \tilde{r}_{t+1}^K \right]. \tag{10}$$

Because of the fund intermediation, households make no actual portfolio choice and we will denote by a_t their holdings in fund claims. Agents face borrowing constraints, and their fund holdings must be higher than $-\bar{a} \leq 0$. Alternatively, this constraint states that agents cannot borrow more than the amount $\bar{a}$. In the rest of the paper, we will focus on the case where the credit limit is above the steady-state natural borrowing limit.[3]

2.5 Government, fiscal tools and monetary policy

In each period t, the government has to finance an exogenous and possibly stochastic public good expenditure $G_t \equiv G_t(z_t)$, as well as lump-sum transfers T_t, which will be optimally chosen. The latter transfers can be thought of as social transfers, which can contribute to generating progressivity in the overall tax system. Heathcote et al. (2017) have shown that such transfers are needed to properly replicate the US fiscal system. The government has several tools for financing these expenditures. First, the government can levy two distorting taxes. A first tax τ_t^K is based on payoffs of all interest-rate bearing assets. The second tax τ_t^L concerns labor income. Second, the government can also tax the firms' profits. Finally, the government can also issue a one-period riskless public nominal bond. To sum up, fiscal policy is characterized by four instruments $(\tau_t^L, \tau_t^K, T_t, B_t)_{t\geq 0}$ given an exogenous public spending stream $(G_t)_{t\geq 0}$.

After-tax quantities are denoted without a tilde. The real after-tax wage w_t, as well as the real after-tax interest rates r_t, r_t^K, and $R_t^{B,N}$ (for the fund, the capital and public debt, respectively) can therefore be expressed as follows:

$$w_t = (1 - \tau_t^L)\tilde{w}_t, \quad r_t = (1 - \tau_t^K)\tilde{r}_t, \tag{11}$$

$$r_t^K = (1 - \tau_t^K)\tilde{r}_t^K, \quad \frac{R_t^{B,N}}{\Pi_t} - 1 = (1 - \tau_t^K)(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1). \tag{12}$$

Taxes on asset-bearing assets are identical for all asset classes and are levied on real returns. The period t real return on the nominal interest rate $\tilde{R}_{t-1}^{B,N}$, set in period $t-1$ is affected by both the period-t inflation rate and the period-t capital tax (hence the notation $R_t^{B,N}$ in (12)).

Regarding firm taxation, we assume that the government fully taxes profits. This solution simplifies the distribution of firm profits among the population of heterogeneous agents. As labor and capital taxes are distorting, this profit taxation policy avoids further distortion.

The government uses its financial resources, made of labor and asset taxes, firm profits and

[3] Aiyagari (1994) discusses the relevant values of $\bar{a}$, called the natural borrowing limit in an economy without aggregate shocks. Shin (2006) provides a similar discussion in presence of aggregate shocks. A standard value in the literature is $\bar{a} = 0$, which ensures that consumption remains positive in all states of the world.

public debt issuance, to finance public good, lump-sum transfers and debt repayment:

$$G_t + \frac{\tilde{R}^{B,N}_{t-1}}{\Pi_t} B_{t-1} + T_t \leq \tau_t^L \tilde{w}_t L_t + \tau_t^K \left(\tilde{r}_t^K K_{t-1} + \left(\frac{\tilde{R}^{B,N}_{t-1}}{\Pi_t} - 1 \right) B_{t-1} \right)$$
$$+ \left(1 - \zeta_t - \frac{\kappa}{2}\pi_t^2\right) Y_t + B_t.$$

The expression of this government budget constraint can be simplified, following Chamley (1986). Using the relationships (1) and (4), as well as the definition of post-tax rates in equations (11) and (12), the governmental budget constraint becomes:

$$G_t + B_{t-1} + r_t \left(B_{t-1} + K_{t-1}\right) + w_t L_t + T_t = B_t + \left(1 - \frac{\kappa}{2}\pi_t^2\right) Y_t - \delta K_{t-1}. \tag{13}$$

Monetary policy consists in choosing the nominal interest rate $\tilde{R}_t^{B,N}$ on public debt (between t and $t+1$), and the inflation rate π_t. The choice of optimal monetary-fiscal policy is thus the choice of the path of the instruments $(\tau_t^L, \tau_t^K, T_t, B_t, \tilde{R}_t^{B,N}, \pi_t)_{t\geq 0}$. These instruments are not independent and are intertwined though the budget constraint of the government and the Phillips curve.

2.6 Agents' program, resource constraints and equilibrium definition

Each agent i is endowed at the date 0 by an initial wealth a_{-1} and an initial productive status y_0 drawn out of an initial distribution $\Lambda_0(a, y) : [-\bar{a}; \infty) \times \mathcal{Y} \to \mathbb{R}^+$. At future dates, her savings pay off the post-tax real interest rate r_t between period $t-1$ and period t but must remain greater than an exogenous threshold denoted $-\bar{a} \leq 0$. Formally, the agent's program can be expressed, for a given initial endowment a_{-1}^i and given initial shocks y_0^i and z_0, as:

$$\max_{\{c_t^i, l_t^i, a_t^i\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u \left(c_t^i - \chi^{-1} \frac{l_t^{i,1+1/\varphi}}{1+1/\varphi} \right), \tag{14}$$
$$c_t^i + a_t^i = (1 + r_t) a_{t-1}^i + w_t y_t^i l_t^i + T_t, \tag{15}$$
$$a_t^i \geq -\bar{a},\ c_t^i > 0,\ l_t^i > 0, \tag{16}$$

where $\mathbb{E}_0$ an expectation operator over both idiosyncratic and aggregate risk.

As we express households' budget constraint using post-tax prices (and not tax rates), it is useful to comment equation (15) to discuss the effect of fiscal and monetary policies in relationship to the literature. First, an unexpected change in capital tax affects period t income, proportionally to interest payment on past savings payoffs $\tilde{r}_t a_{t-1}^i$, as all capital incomes are taxed at the same rate, i.e. $r_t = (1 - \tau_t^K)\tilde{r}_t$. Past savings payoffs are the amount of unhedged interest rate exposures (UREs) at period $t-1$, using the wording of Auclert (2019). Second, an unexpected change in inflation affects the average return on the total portfolio, r_t, due to a

Fisher effect, which can be seen in the definition of the pre-tax return on the total portfolio $\tilde{r}_t$, in equation (9). This effect is proportional to the share of nominal assets in the total portfolio $\frac{B_{t-1}}{\Pi_t}$. Third, labor tax affects the post-tax wage rate, which generates a heterogeneous income effect and a heterogeneous labor-supply effect. Finally, a change in the lump-sum transfer uniformly affects total income without distortion.

At date 0, the agent decides her consumption $(c_t^i)_{t\geq 0}$, her labor supply $(l_t^i)_{t\geq 0}$, and her saving plans $(a_t^i)_{t\geq 0}$ that maximize her intertemporal utility (14), subject to the budget constraint (15) and the borrowing limit (16). These decisions are functions of the initial state (a_{-1}^i, y_0^i), of the history of the idiosyncratic shock $y^{i,t}$ and of the history of the aggregate shocks z^t. Thus, there exist sequences of functions, defined over $([-\bar{a}; +\infty[\times\mathcal{Y}) \times \mathcal{Y}^t \times \mathbb{R}^t$ and denoted by $(c_t, a_t, l_t)_{t\geq 0}$, such that agent's optimal decisions can be written as follows:[4]

$$c_t^i = c_t((a_{-1}^i, y_0^i), y^{i,t}, z^t), \qquad a_t^i = a_t((a_{-1}^i, y_0^i), y^{i,t}, z^t), \qquad l_t^i = l_t((a_{-1}^i, y_0^i), y^{i,t}, z^t). \tag{17}$$

In what follows, we will simplify notation and keep the i-index. For instance, we write c_t^i instead of $c_t((a_{-1}^i, y_0^i), y^{i,t}, z^t)$.

The first-order conditions (FOCs) corresponding to the agent's program (14)–(16) can be expressed as – by using the properties of the GHH utility function:

$$u'(c_t^i - \chi^{-1}\frac{l_t^{i,1+1/\varphi}}{1+1/\varphi}) = \beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{t+1}^i - \chi^{-1}\frac{l_{t+1}^{i,1+1/\varphi}}{1+1/\varphi})\Big] + \nu_t^i, \tag{18}$$

$$l_t^{i,1/\varphi} = \chi w_t y_t^i, \tag{19}$$

where the discounted Lagrange multiplier of the credit constraint of agent i can be written as $\nu_t^i = \nu_t((a_{-1}, y_0), y^{i,t}, z_t)$, similarly to optimal choices. This Lagrange multiplier is null when agent i is not credit-constrained.

We now express the economy-wide constraints:

$$\int_i a_t^i \ell(di) = A_t = B_t + K_t, \quad \int_i y_t^i l_t^i \ell\,(di) = L_t, \tag{20}$$

$$\int_i c_t^i \ell(di) + G_t + K_t = \left(1 - \frac{\kappa}{2}\pi_t^2\right) Y_t + K_{t-1} - \delta K_{t-1}, \tag{21}$$

which correspond to the clearing of the financial market, of the labor market and of the goods market, respectively.[5]

Equilibrium definition. Our market equilibrium definition can be stated as follows.

[4]The existence of such functions is proven in Miao (2006), Cheridito and Sagredo (2016), and Açikgöz (2016).

[5]It would be equivalent to use the sequential representation and to integrate over initial states (of measure Λ) and idiosyncratic histories (of measure θ). For instance, total savings can be written as: $\int_i a_t^i \ell(di) = \sum_{y^{i,t}\in\mathcal{Y}^t}\sum_{y_0\in\mathcal{Y}}\int_{a_{-1}\in[-\bar{a};+\infty)} a_t((a_{-1}, y_0), y^{i,t}, z^t)\theta_t(y^{i,t})\Lambda(da_{-1}, y_0) = A_t(z^t)$.

Definition 1 (Sequential equilibrium) *A sequential competitive equilibrium is a collection of individual functions $(c_t^i, l_t^i, a_t^i, \nu_t^i)_{t\geq 0, i\in\mathcal{I}}$, of aggregate quantities $(K_t, L_t, Y_t)_{t\geq 0}$, of price processes $(w_t, r_t, r_t^K, R_t^{B,N}, \tilde{w}_t, \tilde{r}_t, \tilde{r}_t^K, \tilde{R}_t^{B,N})_{t\geq 0}$, of fiscal policies $(\tau_t^K, \tau_t^L, B_t, T_t)_{t\geq 0}$, and of monetary policies $(\Pi_t)_{t\geq 0}$ such that, for an initial wealth and productivity distribution $(a_{-1}^i, y_0^i)_{i\in\mathcal{I}}$, and for initial values of capital stock and public debt verifying $K_{-1} + B_{-1} = \int_i a_{-1}^i \ell(di)$ and of the aggregate shock z_0, we have:*

1. *given prices, the functions $(c_t^i, l_t^i, a_t^i, \nu_t^i)_{t\geq 0, i\in\mathcal{I}}$ solve the agent's optimization program in equations (14)–(16);*
2. *financial, labor, and goods markets clear at all dates: for any $t \geq 0$, equations (20) and (21) hold;*
3. *the government budget is balanced at all dates: equation (13) holds for all $t \geq 0$;*
4. *factor prices $(w_t, r_t, r_t^K, R_t^{B,N}, \tilde{w}_t, \tilde{r}_t, \tilde{r}_t^K, \tilde{R}_t^{B,N})_{t\geq 0}$ are consistent with condition (5), restrictions (9) and (10), as well as with post-tax definitions (11) and (12);*
5. *the inflation path $(\Pi_t)_{t\geq 0}$ is consistent with the dynamics of the Phillips curve: at any date $t \geq 0$, equation (7) holds.*

2.7 The Ramsey problem

The goal of this paper is to determine the optimal fiscal policy that generates the sequential equilibrium-maximizing aggregate welfare, using an explicit aggregate welfare criterion. This is a difficult question, as the policy is composed of five instruments $(\tau_t^L, \tau_t^K, T_t, B_t, \tilde{R}_t^{B,N}, \pi_t)_{t\geq 0}$ which affect the saving decisions and the labor supplies of all agents, the capital stock, and the price dynamics. This fiscal policy consists of a path for transfers, labor and capital taxes, as well as a path for public debt, while the monetary policy consists of nominal interest rate and an inflation path. Interestingly, monetary policy has to balance the cost of output destruction (through price adjustment costs) and nominal debt monetization (as well as a more indirect role on mark-ups). The government has to select the competitive equilibrium associated with the greatest welfare, subject to a constraint of balanced governmental budget.

The aggregate welfare. We consider a general welfare function that depends on the weights on the utility of each agent. For the sake of generality, we assume that these weights are consistent with the sequential representation, and depend on initial conditions and an idiosyncratic and aggregate history. The weight of agent $i \in \mathcal{I}$ at date t is $\omega_t^i = \omega_t((a_{-1}, y_0), y^{i,t}, z^t)$, where the weights satisfy $1 = \int_i w_t^i \ell(di) = \sum_{y^{i,t}\in\mathcal{Y}^t} \sum_{y_0\in\mathcal{Y}} \int_{a_{-1}\in[-\bar{a};+\infty)} w_t((a_{-1}, y_0), y^{i,t}, z^t)\theta_t(y^{i,t})\Lambda(da_{-1}, y_0)$ for $t \geq 0$. These weights represent the relative importance of each agent in the planner's objective and will be calibrated in our quantitative exercise of Section 5, so as to match the US fiscal and monetary policies at the steady-state. We detail the calibration procedure in Section 4. Formally, the aggregate welfare criterion can be expressed as follows:

$$W_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i) \ell(di) \right]. \tag{22}$$

Choosing the pricing kernel. As explained above, in a heterogeneous-agent economy, there is no straightforward choice for the firm's pricing kernel. We here assume that firms' pricing kernel is defined based on the weighted marginal utilities of agents. More precisely, the pricing kernel M_t is set consistently with the aggregate welfare criterion (22):

$$M_t = \int_i \omega_t^i U_c(c_t^i, l_t^i) \ell(di). \tag{23}$$

We choose this pricing kernel to avoid any inefficiency in the financial sector that the planner would like to correct. As noted by others (Acharya and Dogra, 2020; Bhandari et al., 2020), the choice of the pricing kernel has, however, minor quantitative effects.[6]

The Ramsey program. We formalize the Ramsey program as follows:

$$\max_{\left(w_t, r_t, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^{B,N}, \tau_t^K, \tau_t^L, B_t, K_t, L_t, \Pi_t, (a_t^i, c_t^i, l_t^i, \nu_t^i)_i\right)_{t \geq 0}} W_0, \tag{24}$$

$$G_t + B_{t-1} + r_t \left(B_{t-1} + K_{t-1}\right) + w_t L_t + T_t = B_t + \left(1 - \frac{\kappa}{2} \pi_t^2\right) Y_t - \delta K_{t-1}, \tag{25}$$

$$\text{for all } i \in \mathcal{I}\text{: } a_t^i + c_t^i = (1 + r_t) a_{t-1}^i + w_t y_t^i l_t^i + T_t, \tag{26}$$

$$a_t^i \geq -\bar{a}, \ \nu_t^i (a_t^i + \bar{a}) = 0, \ \nu_t^i \geq 0, \tag{27}$$

$$U_c(c_t^i, l_t^i) = \beta \mathbb{E}_t \left[U_c(c_{t+1}^i, l_{t+1}^i)(1 + r_{t+1}) \right] + \nu_t^i, \tag{28}$$

$$l_t^{i, 1/\varphi} = \chi w_t y_t^i, \tag{29}$$

$$\Pi_t (\Pi_t - 1) = \frac{\varepsilon - 1}{\kappa} (\zeta_t - 1) + \beta \mathbb{E}_t \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \frac{M_{t+1}}{M_t}, \tag{30}$$

$$K_t + B_t = \int_i a_t^i \ell(di), \ L_t = \int_i y_t^i l_t^i \ell\left(di\right), \tag{31}$$

$$r_t = (1 - \tau_t^K) \frac{\tilde{r}_t^K K_{t-1} + \left(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1 \right) B_{t-1}}{A_{t-1}}, \tag{32}$$

and subject to several other constraints (which are not reported here for space constraints): the definition (2) of ζ_t, the definition (4) of Y_t, the definitions (11) and (12) of after-tax wage w_t, the no-arbitrage constraint (10), the factor-price relationship (5), the pricing kernel definition (23), and the positivity of labor and consumption choices, and initial conditions.

[6] Acharya and Dogra (2020), Acharya et al. (2020) and Bhandari et al. (2020) consider a risk-neutral pricing kernel.

The constraints in the Ramsey program include: the governmental and individual budget constraints (25) and (26), individual credit constraint (and related constraints on ν_t^i) (27), Euler equations for consumption and labor (28) and (29), the Phillips curve (30), market clearing conditions for financial and labor markets (31), and the zero profit condition for the fund (32). We have modified the zero-profit condition (9) to express it as a function of r_t, $\tilde{R}_t^{B,N}$, and $\tilde{r}_t^K$, which enables us to drop $\tilde{r}_t$, r_t^K, and $R_t^{B,N}$ from the planner's program. To simplify the derivation of first-order conditions, we use some aspects of the methodology of Marcet and Marimon (2019) which is sometimes called the Lagrangian method (Golosov et al., 2016), applied to incomplete-market environments. We denote by $\beta^t \lambda_t^i \omega_t^i$ the Lagrange multiplier of the Euler equation of agent i at date t. Similarly, we denote by $\beta^t \gamma_t$ the Lagrange multiplier of equation (30) of the Phillips curve. The Lagrange multiplier of the government budget constraint is $\beta^t \mu_t$.

As a final remark, the Ramsey program can be written in a recursive form. The state space for the planner and all agents is actually very large. It is the joint distribution over past value of Lagrange multipliers, wealth, productivity levels and the current aggregate state. This inclusion of past values of Lagrange multipliers, which makes the problem difficult, stems from the commitment of the planner not to surprise agents, such that their Euler equations hold. We skip this representation as the discussion of first-order conditions in the sequential representation is more intuitive.

3 An equivalence result

This section presents our main equivalence result and analyzes fiscal and monetary policies in different institutional setups. The analysis is greatly simplified if one introduces a new concept, the *social valuation of liquidity for agent* i denoted by ψ_t^i, and formally defined as:

$$\psi_t^i \equiv \underbrace{\omega_t^i U_c(c_t^i, l_t^i)}_{\text{direct effet}} - \underbrace{U_{cc}(c_t^i, l_t^i)\left(\lambda_t^i - (1+r_t)\lambda_{t-1}^i\right)}_{\text{effect on saving}} \tag{33}$$
$$- \underbrace{\left(\left(\gamma_t - \gamma_{t-1}\right)\Pi_t\left(\Pi_t - 1\right) - \frac{\varepsilon - 1}{\kappa}\gamma_t\left(\zeta_t - 1\right)\right) Y_t \omega_t^i U_{cc}(c_t^i, l_t^i)}_{\text{effect on pricing kernel}}.$$

The valuation ψ_t^i measures the benefit – from the planner's perspective – of transferring an extra unit of consumption to agent i, valued with weight ω_t^i. This can be seen as the planner's version of the marginal utility of consumption for the agent.[7] As can be seen in equation (33), this valuation consists of three terms. The first is the marginal utility of consumption $\omega_t^i U_c(c_t^i, l_t^i)$, which can be seen as the private valuation of liquidity for agent i. The second and third terms can be seen as the internalization, by the planner, of the economy-wide externalities of this extra

[7]To simplify the notation, we keep the index i, but the sequential representation can be derived along the lines of equation (17).

consumption unit. More precisely, the second term in (33) takes into consideration the impact of the extra unit consumption on saving incentives from periods $t-1$ to t and from periods t to $t+1$. An extra consumption unit makes the agent more willing to smooth out her consumption between periods t and $t+1$ and thus makes her Euler equation more "binding". This more "binding"constraint reduces the utility by the algebraic quantity $U_{cc}(c_t^i, l_t^i)\lambda_t^i$, where λ_t^i is the Lagrange multiplier of the agent's Euler equation at date t. The extra consumption unit at t also makes the agent less willing to smooth her consumption between periods $t-1$ and t and therefore "relaxes" the constraint of date $t-1$. This is reflected in the quantity λ_{t-1}^i.

These first two effects are present both in a real and a nominal framework. This is not the case of the third effect, which is specific to the nominal framework and vanishes in the real one (when $\Pi_t = \zeta_t = 1$ for all t). This effect, due to the third and last term of equation (33), reflects how the extra consumption unit affects the pricing kernel of the agent and thereby the valuation of monopoly profits.

In addition to ψ_t^i, another key quantity is the Lagrange multiplier, μ_t, on the governmental budget constraint. The quantity μ_t represents the marginal cost in period t of transferring one extra unit of consumption to households. Therefore, the quantity $\psi_t^i - \mu_t$ can be interpreted as the "net" valuation of liquidity: this is from the planner's perspective, the benefit of transferring one extra unit of consumption to agent i, net of the governmental cost. We thus define:

$$\hat{\psi}_t^i = \psi_t^i - \mu_t. \tag{34}$$

The interpretation of first-order conditions is greatly clarified by expressing them using $\hat{\psi}_t^i$ rather than the multiplier on Euler equations, λ_t^i.

3.1 The flexible-price economy

Our main result below is that the planner reproduces the flexible-price allocations, if it is possible to choose capital and labor taxes. We thus first analyze the flexible-price allocation, in which the price adjustment cost is $\kappa = 0$.[8] The Phillips curve does not constrain the planner's choices and its associated Lagrange multiplier is $\gamma_t = 0$. We can therefore follow here Chamley (1986) and express all the planner's constraints in post-tax price. The Ramsey equilibrium can thus be derived using a narrower set of variables, which simplifies the algebra. Taxes are then recovered from the allocation: The before-tax rates $\tilde{w}_t$ and $\tilde{r}_t^K$ can be deduced from equations (2) and (5) with $\zeta_t = 1$. Taxes τ^L and τ^K are obtained from the relationships between pre-tax and post-tax rates (11) and (12). Finally, the nominal rate $\tilde{R}_{t-1}^{B,N}$ can be deduced from relationship (32).

The resolution of the Ramsey program in Appendix A.2 shows that a solution to Ramsey program is characterized by: (i) a Euler-like equation for each individual valuation $\hat{\psi}_t^i$ and (ii)

[8]Within the literature on optimal fiscal policy in heterogeneous agent models (Werning, 2007; Bassetto, 2014; Açikgöz et al., 2018; or Dyrda and Pedroni, 2018), one of our contributions is to introduce the concept of net social value of liquidity, which eases the interpretation of the planner's first-order conditions and numerical simulations.

four first-order conditions related to the planner's four instruments.

First, the set of first-order conditions with respect to individual savings for non-credit-constrained agents can be written as:

$$\hat{\psi}_t^i = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{t+1}^i \right]. \tag{35}$$

Constrained agents i have no Euler equation, as $a_t^i = -\bar{a}$, as a consequence $\lambda_t^i = 0$. Equation (35) states that the net social value of liquidity should be smoothed out over time. It can be interpreted as a Euler-like equation for the planner and generalizes the standard individual Euler equation by taking into account the externalities of saving choices on interest rate.

The second first-order condition concerns the role of public debt and is written as follows:

$$\mu_t = \beta \mathbb{E}_t \left[(1 + \tilde{r}_{t+1}^K) \mu_{t+1} \right]. \tag{36}$$

We recall that μ_t is the Lagrange multiplier on the governmental budget constraint – which therefore represents the shadow cost of one unit of consumption for the planner. Equation (36) therefore states that the shadow cost of resources should be smoothed out over time. Again, as with equation (35), equation (36) can be interpreted as a Euler-like equation, with one difference: the interest rate is the before-tax interest rate on capital ($\tilde{r}^K$) instead of the post-tax interest rate of the fund (r) in equation (35). This condition was already discussed by Aiyagari (1995).

The third first-order condition deals with the post-tax interest rate r_t and can be written as:

$$\int_i \hat{\psi}_t^i a_{t-1}^i \ell(di) = -\int_i \lambda_{t-1}^i U_c(c_t^i, l_t^i) \ell(di). \tag{37}$$

In absence of any side effect, the planner would choose the interest rate so as to set to zero the aggregate net value of liquidity among all agents – weighted by agents' asset holdings, such that $\int_i \hat{\psi}_t^i a_{t-1}^i \ell(di) = 0$, or equivalently to equalize the social liquidity valuation to its cost: $\int_i \psi_t^i a_{t-1}^i \ell(di) = \mu_t \int_i a_{t-1}^i \ell(di)$. However, this is not possible since the planner needs to account for the side effect of r_t on the savings incentives, through the Euler equation. This effect is proportional to the shadow cost of the Euler equation. Note that the sign of this shadow cost depends on the planner's perception of the savings quantity in the economy. It is positive when the planner perceives excess savings in the economy, and negative the other way around (see LeGrand and Ragot, 2020 for a lengthier discussion). In consequence, for instance, when there is an excess quantity of savings in the economy, the total net valuation of liquidity is negative.

The fourth condition regarding the post-tax wage rate w_t is:

$$\int_i \hat{\psi}_t^i y_t^i l_t^i \ell(di) = \varphi \mu_t \left(L_t - (1 - \alpha) \frac{Y_t}{w_t} \right). \tag{38}$$

Similarly to equation (37), in absence of side effect for the wage rate, the planner would like to set the aggregate net liquidity value – weighted by individual labor supply in efficient terms – to zero. However, this is not possible, since the planner has also to take into account the distortions implied by wage variations on total labor supply and relatedly on output. These distortions are proportional to the labor elasticity φ and vanish when labor supply is inelastic ($\varphi = 0$).

Finally, the fifth condition regarding the lump-sum transfer T_t is:

$$\int_i \hat{\psi}_t^i \ell(di) = 0. \tag{39}$$

Since there are no distortions implied by the lump-sum transfer, it is set such that the redistributive effect is null.[9]

As a summary, the $\hat{\psi}_t^i$ measure the net gain of transferring resources to agent i. The first-order conditions (37)–(39) equalize the total gain for all agents (weighted by the relevant fiscal base) to the cost of general-equilibrium distortions, for each fiscal instrument. It is interesting to connect these first-order conditions with concepts used in the empirical literature on the evaluation of public policy, as in Hendren and Sprung-Keyser (2020). They define the marginal value of public fund (MVPF) for each instrument as the net total gain minus the general distortions costs. With this wording, our first-order conditions state that the planner sets to 0 the MVPF for each fiscal instrument.

3.2 Characterization of the steady-state capital tax

Before stating the equivalence result, we characterize the steady-state equilibrium capital tax in this economy. This additional result is useful to clarify the existence conditions for equilibria with positive steady-state capital taxes.[10] Using the first-order conditions of Section 3.1, we derive theoretical implications about the steady-state optimal fiscal policy. To denote steady-state variables, we simply drop the subscript t. Our results are gathered in the following proposition.

Proposition 1 (Steady-state) *In the interior steady-state of the Ramsey equilibrium:*

1. *the marginal productivity of capital is pinned down by the discount factor β:*

$$1 + F_K(K, L) = 1 + \tilde{r}_K = \frac{1}{\beta}, \tag{40}$$

2. *the capital tax τ^K is positive and proportional to the aggregate credit-constraint severity:*

$$\tau^K = \frac{\int_i \nu^i \ell(di)}{(1-\beta) \int_i U_c(c^i, l^i) \ell(di)}, \tag{41}$$

where t ν^i is the Lagrange multiplier of credit constraint for agent i.

[9] We did not impose any sign restriction on the lump-sum transfer T_t. In the quantitative analysis of Section 5, T_t will be positive.

[10] Straub and Werning (2020) show that these conditions are not obvious in a Chamley-Judd framework with complete markets. We here show that a simple characterization can be obtained with occasionally binding credit constraints

The proof is in Appendix B. The first item in equation (40) of Proposition 1 is a direct implication of the public debt Euler equation (36). As a consequence, the marginal productivity of capital – and therefore the before-tax interest rate on capital ($\tilde{r}^K$) – is determined by the discount factor β only, as originally explained by Aiyagari (1995). This important restriction is the so-called *modified golden rule*.

To the best of our knowledge, the second item in Proposition 1 is new in the literature. The capital tax is positive and that its value is determined by the severity of credit constraints. The quantity $\int_i \nu^i \ell(di)$ can indeed be interpreted as the aggregate shadow price of credit constraints in the economy.[11] The intuition for this result is that when credit constraints are binding for some agents, credit-constrained agents cannot borrow as much as they would like to, while non-constrained agents save too much to self-insure, as identified by Woodford (1990). Both effects contribute to the creation of an oversupply of liquidity. To correct this oversupply, the government raises the capital tax, which decreases the post-tax interest rate and thus the incentives to save.[12]

A zero capital tax would correspond to credit constraints that do not bind for any agent – for instance, if they are chosen to be below the natural borrowing limit, as defined by Aiyagari (1994). However, in that case a stationary equilibrium cannot exist since it would imply $\beta(1 + r) = 1$ (where r is the agents' saving rate) and the result of Chamberlain and Wilson (2000) would apply. Conversely, as soon as a positive mass of agents is credit constrained, capital taxes are positive, the post-tax return on capital is below $1/\beta$, and the existence of a steady-state distribution follows from Huggett (1993).[13]

In the quantitative exercise of Section 5, we will estimate the Pareto weights for the empirical average capital tax to be optimal at the steady state.

3.3 The equivalence result

As a preliminary remark, observe that our monetary economy features two market imperfections. The first imperfection is the imperfect competition between firms, which can yield a price markup ζ_t strictly above one. The second imperfection is the Rotemberg inefficiency, which prevents firms from setting their prices at no cost. The two imperfections are complementary. Indeed, in the absence of Rotemberg inefficiency (i.e., $\kappa = 0$), firms' profit maximization yields $\zeta_t = 1$ and the markup inefficiency vanishes, as can be seen from the Phillips curve in equation (7). Conversely, in the absence of imperfect competition, we have $\zeta_t = 1$ and the Phillips curve implies

[11] The denominator in equation (41) is indeed bounded away from zero because the economy is finite, i.e., not all marginal utilities can be simultaneously zero.

[12] This logic is discussed in Dávila et al. (2012). This paper differs from ours, because the authors analyze the constrained optimality of the capital stock in a situation where the planner can change agents' saving decisions without distortion, while we focus on distorting fiscal instruments.

[13] Aiyagari (1995) also finds that the capital tax is positive in a similar environment without aggregate shocks. See also Yared (2013) for a formal proof that a positive mass of credit-constrained agents can be optimal. Our contribution is to connect the capital tax rate to the severity of credit constraints.

the Rotemberg inefficiency has no role to play. The planner's objective – in a monetary setup – therefore includes minimizing the impact of these two inefficiencies.

We now show that linear taxes on capital and labor are two sufficient tools to offset these two inefficiences along the business cycle, even when agents are heterogeneous. To do so, we first solve for the optimal monetary and fiscal policies when the government has access to a full set of fiscal tools. This program can be written as:

$$\max_{\left(w_t,r_t,\tilde{w}_t,\tilde{r}_t^K,B_t,T_t,K_t,L_t,\Pi_t,(a_t^i,c_t^i,l_t^i)_i\right)_{t\geq 0}} W_0, \tag{42}$$

subject to the same equations as in the Ramsey program (24. Three observations can be made. First, we have dropped the taxes τ_t^K and τ_t^L from the Ramsey program since, as in the flexible-price case, they can be substituted by post-tax rates r_t, r_t^K and w_t (and recovered from allocation). Second, as in the flexible-price case again, the pre-tax nominal rate $\tilde{R}_t^{B,N}$, along with constraints (9) and (10), are also dropped since they do not play any role (and also recovered from allocation). Finally, the before-tax rates $\tilde{w}_t$ and $\tilde{r}_t^K$ play a role only in the markup coefficient ζ_t of equation (2) and in the factor price equation (5). The planner has thus two independent instruments ($\tilde{r}_t^K$ or $\tilde{w}_t$ on one side and Π_t on the other side) to address the two monetary frictions of the economy. The planner can thus set $\tilde{r}_t^K$ or $\tilde{w}_t$ such that the markup inefficiency vanishes (i.e., such that $\zeta_t = 1$). The gross inflation rate is then set to 1 at all dates, so as to neutralize the Rotemberg inefficiency. The program, with the full set of tools, can be expressed as $\max_{(w_t,r_t,B_t,K_t,L_t,(a_t^i,c_t^i,l_t^i)_i)_{t\geq 0}} W_0$, subject to exactly the same constraints as in the flexible-price economy (because $\Pi_t = \zeta_t = 1$ at all dates). This thus leads to the same allocation as in the real economy. We summarize this first result in the following proposition.

Proposition 2 (An equivalence result) *When both labor and capital taxes are available, the government exactly reproduces the flexible-price allocation and net inflation is null at all periods.*

Proposition 2, formally proven in Appendix C.1, states that monetary policy is irrelevant to manage inequality over the business cycle, as long as capital and labor taxes are available instruments. Before discussing Proposition 2 further, one should note that it actually holds in a much more general framework. The result can indeed be generalized to an environment where:

1. The period utility is general (and not GHH).

2. There is no fund and households actually choose their asset portfolio, investing without intermediation in the real capital stock and in the nominal public debt.

3. There is a separate capital income tax for each of the two interest-bearing assets (capital and public debt).

In this more general setup, Proposition 2 is still valid. The proof is in Appendix C.2. The generalization thus states that if a capital tax for each asset and a labor tax are available instruments, then nominal prices are constant over the business cycle. A noteworthy aspect of our extension is that if we allow for an actual portfolio choice, the equivalence result requires one planner's instrument per asset choice in addition to the labor tax. In our benchmark economy, only capital and labor taxes are needed for the equivalence result of Proposition 2 to hold. It does not rely on other planner's instruments, such as lump-sum transfers or public debt. In Appendix C.2, we also derive the planner's first-order conditions, which allows us to identify how wealth effects on the labor supply (due to a non-GHH utility function) affect the conditions presented in Section 3.1.

Following the analysis of Kaplan et al. (2018) and Auclert (2019), monetary policy has direct effects, due to price changes and indirect effects through general-equilibrium feedback. Proposition 2 states that the effects achieved by monetary policy can be achieved by labor and capital taxation. Loosely speaking, on the one hand, outcomes of the direct effects can be replicated by the linear capital tax, which globally affects the return on all savings. On the other hand, general equilibrium effects, affecting the real wage, can be replicated by the linear labor tax, which creates a wedge between the marginal labor cost of the firm, determining their pricing decision, and the labor income of households determining their labor supply decisions.

Finally, the equivalence result holds from time-0 onwards. As a consequence, it is valid both in a time-0 perspective, when instruments are chosen at the initial period before convergence to a long-run equilibrium and in a timeless perspective, when the economy is running for a long period, such that the effect of initial conditions has vanished. We study both of these cases quantitatively in Section 6.

The equivalence result of Proposition 2 is in the same vein as Correia et al. (2008) and Correia et al. (2013) who also show that one can recover price stability if the planer has access to a time-varying consumption tax. The inclusion of capital taxes (instead of consumption tax) allows us to connect our result to the literature on optimal capital taxation.[14]

3.4 The economy with missing tools: Missing capital taxes

We now investigate how monetary policy is affected by missing fiscal instruments. We assume here that the capital tax is constant and fixed at its optimal steady-state value τ^K_{SS}. We perform the same exercise for missing labor taxes in Appendix D.3, but this case is less interesting as we find that optimal labor taxes barely move along the business cycle.[15]

[14] In addition, the inclusion of capital tax may be more relevant quantitatively, at least in the US (see Trabandt and Uhlig, 2011).

[15] It could be possible to consider a vast number of incomplete sets of planner's instruments (actually, $2^m - 1$, where m is the number of instruments). Fixing capital tax may be the most natural option, since it can be argued that this tax is set over the medium run, but not varying in the short run. Capital tax is a relatively recent tool, introduced in 1921 in the US (Auten, 1999).

With fixed capital tax, the planner's program can be written as:

$$\max_{(w_t, r_t, \tilde{R}_t^{B,N}, \tilde{w}_t, \tilde{r}_t^K, \tau_t^L, T_t, B_t, K_t, L_t, \Pi_t, (a_t^i, c_t^i, l_t^i)_i)_{t\geq 0}} W_0,$$

subject to same equations as in case with the full set of instruments (Section 3.3), as well as to the additional constraint $\tau_t^K = \tau_{SS}^K$. The main difference with the previous case is that we cannot choose $\tilde{w}_t$ and $\tilde{r}_t^K$ to set $\zeta_t = 1$ and fully offset markup inefficiency. Indeed, because of the fixed capital tax rate and the factor price relationship (5), the pre-tax rate $\tilde{r}_t^K$ affects both the markup ζ_t and the fund interest rate r_t. Nominal inefficiencies cannot be removed, and consequently, the inflation rate cannot be set to 1. The algebra is thus much more involved. We derive the first-order conditions in Appendix A.3 and discuss here the main results.

Compared to the case where all instruments are available, we have two additional Lagrange multipliers: one, denoted by Υ_t on the no-arbitrage condition (10), and another one, denoted by Γ_t on the zero-profit condition (32) of the fund. The first-order condition for inflation is:

$$\mu_t \kappa \left(\Pi_t - 1\right) = -\left(\gamma_t - \gamma_{t-1}\right)\left(2\Pi_t - 1\right) M_t + \left(\Gamma_t (1 - \tau_{SS}^K) B_{t-1} - \Upsilon_{t-1}\right) \frac{\tilde{R}_{t-1}^{B,N}}{Y_t \Pi_t^2}, \tag{43}$$

where we recall that M_t is the pricing kernel. The left side captures the cost of an increase in inflation in terms of output destruction. The right-hand side is the benefit of an increase in current inflation in terms of the Phillips curve's relaxation and of nominal interest rate.

Second, the choice of individual savings a_t^i yields:

$$\hat{\psi}_t^i = \overbrace{\beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{t+1}^i\right]}^{=\text{extended real effect}} + \overbrace{\beta \mathbb{E}_t \left[\Gamma_{t+1} \left(r_{t+1} - (1 - \tau_{SS}^K) \left(\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}} - 1\right)\right)\right]}^{=\text{wedge rate effect}}, \tag{44}$$

which is similar to the real case, except for two differences. First, the net social value of liquidity $\hat{\psi}_t^i$ now includes a term related to the variation of the pricing kernel due to one extra unit of consumption (see the second line in expression (33)). This is why we refer to this term as an "extended real effect". Second, the fixed capital tax implies a wedge in interest rates coming from the fund's no profit condition. We refer to this aspect as a "wedge rate effect".

Third, the choice of public debt yields the following first-order condition:

$$\mu_t = \overbrace{\beta\mathbb{E}_t\left[\mu_{t+1}\left(1-\delta+\zeta_{t+1}^{-1}(\tilde{r}_{t+1}+\delta)\left(1-\frac{\kappa}{2}\left(\Pi_{t+1}-1\right)^2\right)\right)\right]}^{=\text{extended real effect}} \tag{45}$$
$$\underbrace{-\,\alpha\beta\mathbb{E}_t\left[\left(\left(\gamma_{t+1}-\gamma_t\right)\Pi_{t+1}\left(\Pi_{t+1}-1\right)+\frac{\varepsilon-1}{\kappa}\gamma_{t+1}\right)\frac{Y_{t+1}}{K_t}M_{t+1}\right]}_{=\text{Phillips curve effect}}$$
$$+\underbrace{\frac{\varepsilon-1}{\kappa}\beta\mathbb{E}_t\left[\gamma_{t+1}\frac{\zeta_{t+1}Y_{t+1}}{K_t}M_{t+1}\right]}_{=\text{markup effect}}+\underbrace{\beta(1-\tau_{SS}^K)\mathbb{E}_t\left[\Gamma_{t+1}\left(\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}}-1-\tilde{r}_{t+1}^K\right)\right]}_{=\text{wedge rate effect}}.$$

The first line states that public debt enables the planner to smooth out the cost of liquidity for the government. This is similar to the flexible-price economy, except that the smoothing also needs to account for nominal effects, and in particular the presence of the markup and of the price adjustment cost, as well as the wedge between the capital rate and the public debt rate. When monetary imperfections go away ($\zeta_t = 1$ and $\Pi_t = 1$), we exactly fall back on the smoothing effect of the real economy (equation (36)), up to the wedge rate effect. The second line is related to the impact of public debt on the Phillips curve through capital crowding-out and output mitigation. The third term comes from effect on public debt of the markup inefficiency – through capital crowding-out and interest rate. Finally, the fourth term is related to the wedge rate effect, stemming from the difference in rates between public debt and capital.

The first-order condition related to the interest rate r_t is:

$$\int_i \hat{\psi}_t^i a_{t-1}^i \ell(di) = -\overbrace{\int_i \lambda_{t-1}^i U_c(c_t^i, l_t^i)\ell(di)}^{=\text{extended real effect}} - \overbrace{\frac{\varepsilon-1}{\kappa}\gamma_t\frac{Y_t\zeta_t}{r_t}M_t}^{=\text{markup effect}} + \overbrace{\Gamma_t A_{t-1}}^{=\text{rate setting}}. \tag{46}$$

We can parallel the interpretation of the flexible-price case. Here, a second side effect is present and is related to the impact of r_t on price mark-up. Due to the lack of appropriate instruments, the planner has to set the post-tax interest rate r_t to manage at the same time the aggregate net benefit of liquidity and mitigates the consequences of the markup inefficiency. The constraint related to the relationship between the rates of the fund, of public debt and capital is also present.

Finally, equation (39) for T_t being unchanged, the choice of w_t yields the following FOC:

$$\int_i \omega_t^i \hat{\psi}_t^i y_t^i l_t^i \ell(di) = \overbrace{\varphi\mu_t\left(L_t-(1-\alpha)\varphi\frac{Y_t}{w_t}\left(1-\frac{\kappa}{2}\left(\Pi_t-1\right)^2\right)\right)}^{=\text{extended real effect}} \tag{47}$$
$$+\underbrace{\frac{(1-\alpha)\varphi}{w_t}\left(\left(\gamma_t-\gamma_{t-1}\right)\Pi_t\left(\Pi_t-1\right)+\frac{\varepsilon-1}{\kappa}\gamma_t\right)Y_tM_t}_{=\text{Phillips curve effect}}.$$

The interpretation of equation (47) is similar to that of equation (46). Due to missing instruments, setting the post-tax wage rate must also account for the effect related to nominal rigidities through the Phillips curve – in addition to the effect on labor supply and output already present in the real case. Note that when labor is inelastic ($\varphi = 0$), these two effects are absent, since labor supply – and thereby output – remain unaffected by wage variations. All in all, the planner sets the post-tax wage to pursue both a real and a nominal objective: manage the aggregate net benefit of liquidity (while internalizing the possible effects on aggregate labor supply) and mitigate the nominal inefficiencies. These characterizations will be used in the simulation exercises below.

4 Truncating the model

The previous Ramsey problem cannot be solved with current simulation techniques. First reason, there is no general algorithm to find the steady-state allocation. Indeed, the Ramsey equilibrium is now a joint distribution across wealth and Lagrange multipliers, which is a high-dimensional object. In addition, the planner's instruments depend on the dynamics of this joint distribution in a non-obvious way. For this reason, the methods using perturbation or simulation methods around a well-identified steady-state cannot be used (as Reiter, 2009, Boppart et al., 2018, Bayer et al. (2019) or Auclert et al., 2019). Moreover, optimizing over simple rules in the spirit of Krusell and Smith (1998) may generate non-optimal rules as we could ignore the variables underlying the rule for each instrument.

We provide a method to determine the steady-state allocation and to derive equations that can be perturbated to simulate the dynamics for small aggregate shocks, elaborating on LeGrand and Ragot (2020).[16] The general idea of the method relies on the "truncation" of idiosyncratic histories. We construct an aggregation of the Bewley model in which agents having the same history over last N periods (where N is a fixed horizon) are aggregated into a single "agent". By construction, this aggregate agent has the same wealth as all agents having this history for the last N periods. The heterogeneity in the original Bewley model among agents with the same history of length N, is captured in our aggregate model through additional parameters (the so-called "ξs").[17]

This construction results in a so-called truncated model. In the absence of aggregate shocks, the truncated model is an exact aggregation of the underlying Bewley model and does not involve any simplifying assumption – thanks to the "ξs". In the presence of aggregate shocks, the truncated model can be simulated using standard perturbation methods and these allocations can be shown to converge toward the allocations of the full-fledged model when the length of

[16] In LeGrand and Ragot (2020), we derive convergence properties and provide accuracy results. Bhandari et al. (2020) is the only other method to compute Ramsey allocations with aggregate shocks in general cases. However, to the best of our knowledge, it can be implemented only when credit constraints are not binding in equilibrium.

[17] Our aggregation method is related to the one of Werning (2015), except that Werning (2015) captures the heterogeneity through a change in the discount factor, while we do it through the ξs. This choice simplifies the solution of the Ramsey program, as all agents have the same discount factor.

the truncation increases. Finally, the truncated model makes it possible to solve for the Ramsey program.

In the quantitative Section 5, we show that the truncated economy is an accurate approximation for relatively short truncation lengths by comparing outcomes to those of standard simulation techniques (Reiter, 2009; Boppart et al., 2018).

We detail in the remainder of this section the truncation method in our monetary setup – which is however not necessary to understand the quantitative results of Section 5.

4.1 The aggregation procedure

As explained in Section 2.6, the sequential representation characterizes the equilibrium of the full-fledged model by sequences of functions depending on the full aggregate and idiosyncratic histories of agents. Our aggregation procedure involves expressing the model based on so-called *truncated histories*, which are N-length vectors $y^N = (y_{-N+1}, \ldots, y_0)$ representing agents' idiosyncratic histories over the last N periods. The quantity $N \geq 0$ is the truncation length. Loosely speaking, we can represent the truncated history of an agent i whose idiosyncratic history is y^t as:

$$y^t = \{\underbrace{\ldots, y_{-N-2}, y_{-N-1}, y_{-N}}_{\xi_{y^N}}, \underbrace{y_{-N+1}, \ldots, y_{-1}, y_0}_{=y^N}\}, \tag{48}$$

where the parameter ξ_{y^N} captures the residual heterogeneity for the truncated history y^N (as further discussed below). We now turn to the different steps of the aggregation procedure.

First, the measure of agents with truncated history y^N, denoted by S_{y^N}, can be computed as $S_{y^N} = \sum_{y^N \in \mathcal{Y}^N} S_{\hat{y}^N} \Pi_{\hat{y}^N y^N}$, where $\Pi_{\hat{y}^N y^N}$ is the probability to switch from $\hat{y}^N$ at $t-1$ to y^N at t. It is equal to $\Pi_{\hat{y}^N y^N} = 1_{y^N \succeq \hat{y}^N} \Pi_{\hat{y}_0^N y_0^N}$, and thus to the probability to switch from current state $\hat{y}_0^N$ to current state y_0^N if y^N is a possible continuation of $\hat{y}^N$ (denoted by $y^N \succeq \hat{y}^N$).

Second, the model aggregation implies to assign consumption, saving and labor choices to groups of agents with the same truncated history. For the sake of simplicity, we will write *truncated history* for "the group of agents sharing the same truncated history". Take the case of a generic variable, denoted by $X_t(y^t, z^t)$, where we make the dependence in y^t and z^t explicit. The quantity assigned to truncated history y^N is denoted by X_{t,y^N} and equal to the average value of X among agents with truncated history y^N. Formally:

$$X_{t,y^N} = \frac{1}{S_{t,y^N}} \sum_{y^t \in \mathcal{Y}^t | (y^t_{t-N+1}, \ldots, y^t_t) = y^N} X_t(y^t, z^t) \theta_t(y^t), \tag{49}$$

where $\theta_t(y^t)$ is recalled to be the measure of agents with history y^t. Definition (49) can be applied to the average consumption, the end-of-period saving, the labor supply and the credit-constraint Lagrange multiplier respectively and lead to the quantities c_{t,y^N}, a_{t,y^N}, l_{t,y^N}, and ν_{t,y^N}.

Third, we compute the aggregate beginning-of-period wealth. Applying definition (49) to period-$(t-1)$ end-of-period wealth requires to account that agents transit from one truncated history at $t-1$ to another one at t. The beginning-of-period wealth $\tilde{a}_{t,y^N}$ for truncated history y^N becomes:

$$\tilde{a}_{t,y^N} = \sum_{\hat{y}^N \in \mathcal{Y}^N} \frac{S_{t-1,\hat{y}^N}}{S_{t,y^N}} \Pi_{t,\hat{y}^N,y^N} a_{t-1,\hat{y}^N}. \tag{50}$$

Fourth, the aggregation of the different equations characterizing the equilibrium is rather straightforward except for Euler equations – which involve non-linear marginal utilities. Indeed, the marginal utility of consumption aggregation ($u'(c_{t,y^N})$) is different from the aggregation of marginal utility (u'_{t,y^N}): $u'(c_{t,y^N}) \neq u'_{t,y^N}$. The ratio of these two scalars will be denoted by ξ_{t,y^N}, which guarantee that Euler equations hold with aggregate consumption levels.[18]

4.2 The truncated model

We can now proceed with the aggregation of the full-fledged model. First, the aggregation of individual budget constraints (15) using equations (49) and (50) yields the following equation:

$$a_{t,y^N} + c_{t,y^N} = w_t y_0^N l_{t,y^N} + (1+r_t)\tilde{a}_{t,y^N} + T_t, \text{ for } y^N \in \mathcal{Y}^N. \tag{51}$$

The aggregation of Euler equations for consumption (18) and labor (19) yields:

$$\xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}) = \beta \mathbb{E}_t \left[(1+r_{t+1}) \sum_{\tilde{y}^N \succeq y^N} \Pi_{t+1,y^N \tilde{y}^N} \xi_{\tilde{y}^N} U_c(c_{t+1,\tilde{y}^N}, l_{t+1,\tilde{y}^N}) \right] + \nu_{t,y^N}, \tag{52}$$

$$l_{t,y^N}^{1/\varphi} = \chi w_t y_0^N. \tag{53}$$

The system consisting of equations (51)–(53) is an exact aggregation of the full-fledged model with aggregate shocks in terms of truncated idiosyncratic histories. This system characterizes the dynamics of the aggregated variables c_{t,y^N}, a_{t,y^N}, l_{t,y^N} and ν_{t,y^N} without any approximation.

Finally, we express market clearing conditions (20) using aggregated variables:

$$K_t = \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} a_{t,y^N}, \quad L_t = \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} y_{y^N} l_{t,y^N}. \tag{54}$$

The parameters ξs that appear in the aggregated Euler equations (52) are key in our method. We show how to compute them using steady-state allocations and how these computations can used to simulate the model in the presence of aggregate shocks.

Steady-state and computation of the ξs. At the steady state, the computations of the parameters ξs can be done based on allocations. Indeed, we can compute the stationary

[18] Because the labor supply Euler equation is linear in productivity (see equation (19)), the aggregation of this equation is straightforward.

wealth distribution of the full-fledged model (using the individual equations) and identify credit-constrained histories. We can then compute aggregate (steady-state) allocations, c_{y^N}, a_{y^N}, l_{y^N} and ν_{y^N}, using equations (49) and (50). Then, consumption Euler equations (52) can be inverted to compute the preference parameters $(\xi_{y^N})_{y^N \in \mathcal{Y}^N}$. Actually, this computation involves only standard linear algebra and we provide a closed-form expression for the ξs – see equation (123) in Appendix E.1.

The truncated model in the presence of aggregate shocks. To use our truncation method in the presence of aggregate shocks, two further assumptions are needed.

Assumption A *We make the following two assumptions.*

1. *The preference parameters $(\xi_{y^N})_{y^N}$ remain constant and equal to their steady-state values.*
2. *The set of credit-constrained histories, denoted by $\mathcal{C} \subset \mathcal{Y}^N$, is time-invariant.*

The resulting model (i.e., the aggregated model with Assumption A) is called the truncated model. We therefore use the ξs twice: (i) once exactly to estimate them using the steady-state allocation; (ii) once approximately to simulate the model in the presence of aggregate shocks.[19]

Finally, two properties are worth mentioning. First, by construction of the ξs, the allocations of the full-fledged Bewley model and of the truncated equilibria coincide with each other at the steady state. Second, truncated equilibrium allocations (in the presence of aggregate shocks) can be proved to converge to those of the full-fledged equilibrium (and the ξs to 1), when the truncation length N becomes increasingly long. Furthermore, from a quantitative perspective, Section 5 shows that the ξs are an efficient tool to capture the heterogeneity within truncated histories, even when the truncation length is not too large.

4.3 Ramsey program

Thanks to its finite state-space representation, the truncated model makes it possible to solve the Ramsey program in the presence of aggregate shocks, which is a challenging task.[20] If we denote by $(\omega_{y^N})_{y^N \in \mathcal{Y}^N}$ the Pareto weights associated to history y^N, the Ramsey program in the truncated economy can be expressed as follows.

$$\max_{(w_t, r_t, \tilde{r}_t^K, \tilde{R}_t^{B,N}, \tau_t^K, \tau_t^L, B_t, T_t, K_t, L_t, \Pi_t, (a_{t,y^N}, c_{t,y^N}, l_{t,y^N})_{y^N})_{t \geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \sum_{y^N \in \mathcal{Y}^N} S_{y^N} \omega_{y^N} \xi_{y^N} U(c_{t,y^N}, l_{t,y^N}) \right], \quad (55)$$

[19] Assuming that the ξ_{y^N} are constant in the dynamics is equivalent to assuming that the distribution of agents within truncated history is constant. This has the same spirit as assuming that the distribution within bins of wealth is uniform in the histogram method of Reiter (2009), among others.

[20] Note that we derive the first-order conditions of the truncated model, and we do not truncate the first-order conditions of the full-fledged Ramsey model. This ensures numerical stability, as the truncated model is "well-defined". It can be indeed considered as a model with limited insurance. See LeGrand and Ragot (2020) for this expression and for the convergence of the FOCs of the truncated model toward the ones of the initial model.

subject to aggregate Euler equations (52) and (53), aggregate budget constraint (51), aggregate market clearing conditions (54), credit constraints $a_{t,y^N} \geq -\bar{a}$, as well as constraints that were already present in the full-fledged Ramsey program: the governmental budget constraint (25), the Phillips curve (30), the definition (2) of ζ_t, the one (4) of Y_t, the ones (11) and (12) of after-tax rates r_t, r_t^K, $R_t^{B,N}$ and w_t, the zero profit condition for the fund (9), the no-arbitrage constraint (10), and the relationship (5) between factor prices. The only difference is that truncated pricing kernel is now $M_t = \sum_{y^N \in \mathcal{Y}^N} S_{y^N} \xi_{y^N} \omega_{y^N} U_c(c_{t,y^N}, l_{t,y^N})$.

As we did in Section 2.7, it is possible to use the tools of Marcet and Marimon (2019) to rewrite the Ramsey program. The truncation adds no complexity to the formulation of the planner's objective. First-order conditions can similarly be derived as in the general case and we obviously have the same equivalence results. The first-order conditions in the three cases (flexible-price economy, no time-varying capital tax) are similar to the ones of Section 2.7 and we do not repeat them for the sake of conciseness. These equations can be found in Appendix D.[21]

Computing the Pareto weights $(\omega_{y^N})_{y^N \in \mathcal{Y}^N}$. We estimate Pareto weights, such that the first-order conditions of the Ramsey program (55) are fulfilled at the steady-state for the actual US tax system.[22] More precisely, we choose Pareto weights such that the steady-state values capital and labor taxes, as well as public debt and lump-sum transfers are equal to their empirical counterparts (see Section 5.1 below for these values). However, the estimation involves Y^N Pareto weights (where $Y = Card\,\mathcal{Y}$ and Y^N amounts to 3125 in our quantitative exercise), while there are only three constraints. The problem is therefore underidentified and there is no uniqueness of such Pareto weights. To circumvent this difficulty, we choose among the admissible weights those which minimize the distance to the equi-weighted case (where each agent in every truncated history has the same weight, which is the Utilitarian social welfare function). Formally, the weights solve $\min_{(\omega_{y^N})} \left\| (\omega_{y^N})_{y^N} - \mathbf{1}_{y^N} \right\|_2$ subject to $\sum_{y^N} \omega_{y^N} = 1$ and such that planner's first-order conditions hold.[23] As we show in Appendix E.2, this computation pins down to simple matrix algebra.

[21] A final aspect regarding the truncated Ramsey program is that it solutions can be shown to converge to the solutions of the full-fledged Ramsey program (if they exist), when the truncation length N becomes infinitely long. See LeGrand and Ragot (2020, Proposition 5). This convergence property is the parallel of the convergence result regarding allocations of the competitive equilibrium.

[22] We actually follow here the methodology of the inverse optimal taxation literature, which estimates social welfare functions that are consistent with observed fiscal systems (see Bargain and Keane, 2010; Bourguignon and Amadeo, 2015; Heathcote and Tsujiyama, 2017; Chang et al., 2018, among others). This strategy ensures that we investigate the dynamics around a quantitatively relevant steady state.

[23] In the previous expression, $\|\cdot\|_2$ denotes the Euclidean norm, $(\omega_{y^N})_{y^N}$ is the vector of Pareto weights, and $\mathbf{1}_{y^N}$ the vector of masses of agents with history y^N.

5 Quantitative assessment

We now solve for optimal policies over the business cycle in a quantitatively relevant environment. As we consider various environments to identify the economic mechanisms, we start here with presenting a roadmap of our quantitative exercise.

1. We simulate the dynamics of the model assuming that the actual average US fiscal system corresponds to the model optimal steady state.[24] To do so, we calibrate the model to reproduce a relevant wealth distribution for the average US fiscal system (Section 5.1) and then we estimate Pareto weights for the optimal steady-state fiscal system to be the actual average one in the US (Section 5.2).[25]

2. We check the accuracy of our method by comparing it to other simulation techniques in Section 5.2.

3. We then solve for the optimal policy in the timeless perspective: we simulate the optimal policy when the economy is close to its long-run steady state and is hit by aggregate shocks.

4. We simulate the model with the full set of instruments (capital and labor taxes, public debt, lump-sum transfers), and check that inflation is constant. We compare the model outcomes with those of a complete-market economy in Section 5.3 to understand how incomplete insurance markets affect the dynamics of optimal fiscal policy.

5. We simulate the model when capital taxes do not vary along the business cycle in Section 5.4 and analyze the residual redistributive role of monetary policy.

6. We solve the time-0 program in a monetary model without capital and compare our results to the literature in Section 6. The optimal policy is computed in initial periods, when the planner is not constrained by past commitments and when the economy starts converging to a long-run equilibrium.

5.1 The calibration and steady-state distribution

Preferences. The period is a quarter. The discount factor is $\beta = 0.99$ and the period utility function $\log(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi})$. The Frisch elasticity of labor supply is set to $\varphi = 0.5$, which is the value recommended by Chetty et al. (2011) for the intensive margin in heterogeneous-agent

[24] We thus investigate the optimal *dynamics* of taxes, which is a different question from the optimal long-run value of these taxes

[25] The well-defined optimal debt level comes from the incomplete-market structure with binding credit constraints, where the Ricardian equivalence does not hold. When credit constraints are not binding and lump-sum taxes are available, the level of public debt is not well determined (Bhandari et al., 2017a and Bhandari et al., 2020), as in the representative agent economy (Aiyagari et al., 2002). See also Aiyagari and McGrattan (1998) or Açikgöz et al. (2018) for contributions in the same environment.

models. The scaling parameter is set to $\chi = 0.068$, which implies normalizing the aggregate labor supply, defined in (20), to 1/3.

Technology and TFP shock. The production function is Cobb-Douglas: $Y = ZK^{\alpha}L^{1-\alpha}$. The capital share is set to $\alpha = 36\%$ and the depreciation rate to $\delta = 2.5\%$, as in Krueger et al. (2018) among others. The TFP process is a standard AR(1) process, with $Z_t = \exp(z_t)$ and $z_t = \rho_z z_{t-1} + \varepsilon_t^z$, where $\varepsilon_t^z \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_z^2)$. We use the standard values $\rho_z = 0.95$ and $\sigma_z = 0.31\%$ to obtain a deviation of the TFP shock z_t equal to 1% at a quarterly frequency (see Den Haan, 2010 for instance).

Idiosyncratic risk. We use a standard productivity process: $\log y_t = \rho_y \log y_{t-1} + \varepsilon_t^y$,with $\varepsilon_t^y \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_y^2)$. We calibrate a persistence of the productivity process $\rho_y = 0.99$ and a standard deviation of $\sigma_y = 0.14$. These values are in line with empirical estimates.[26] This process generates a realistic empirical pattern for wealth. First, the Gini coefficient of the wealth distribution amounts to 0.77, in line with the data (see below). Second, the model implies an average wealth-to-GDP ratio of 11.8 and an average capital-to-GDP ratio of 2.5. These two values are in line with their empirical counterparts. Finally, the Rouwenhorst (1995) procedure is used to discretize the productivity process into 5 idiosyncratic states with a constant transition matrix.

Taxes and government budget constraint. Fiscal parameters are calibrated based on the computations of Trabandt and Uhlig (2011), who use the methodology of Mendoza et al. (1994) on public finance data prior to 2008. This approach consists in computing a linear tax on capital and on labor, as well as lump-sum transfers that are consistent with the governmental budget constraint. Their estimations for the US in 2007 yield a capital tax (including both personal and corporate taxes) of 36%, a labor tax of 28% and lump-sum transfers equal to 8% of the GDP. This affine structure (lump sum transfers and linear marginal tax rates) is often used in the literature because it enables to properly reproduce the amount of redistribution in the US, as shown for instance by Heathcote et al. (2017) and Dyrda and Pedroni (2018).

This fiscal system generates two untargeted outcomes. First, it implies a public debt-to-GDP ratio equal to 60%, which is very close to the value of 63% estimated by Trabandt and Uhlig (2011). Second, it also implies a public spending-to-GDP ratio equal to 12.4%. This value is consistent with other quantitative investigations (Bhandari et al., 2017b), even though a little bit low compared to the postwar value, which has decreased to 14.1% in 2017, from 17% in the 1970s.

[26]For instance, Krueger et al. (2018) estimate a more general process with an additional transitory process. The implied AR(1) process generates a standard deviation equal to $\sigma^y = 0.13$.

Monetary parameters. The monetary friction is captured by two parameters. The first is the elasticity of substitution across goods ε. The second is the price adjustment cost κ, in the Rotemberg representation. We follow the literature and set $\varepsilon = 6$ and $\kappa = 100$ (see Bilbiie and Ragot, 2020 for a discussion and references).

Table 1 provides a summary of the model parameters.

Parameter	Description	Value
	Preference and technology	
β	Discount factor	0.99
α	Capital share	0.36
δ	Depreciation rate	0.025
$\bar{a}$	Credit limit	0
χ	Scaling param. labor supply	0.068
φ	Frisch elasticity labor supply	0.5
	Shock process	
ρ_z	Autocorrelation TFP	0.95
σ_z	Standard deviation TFP shock	0.31%
ρ_y	Autocorrelation idio. income	0.99
σ_y	Standard dev. idio. income	14%
	Tax system	
τ^K	Capital tax	36%
τ^L	Labor tax	28%
T	Transfer over GDP	8%
B/Y	Public debt over yearly GDP	60%
G/Y	Public spending over yearly GDP	12.4%
	Monetary parameters	
κ	Price adjustment cost	100
ε	Elasticity of sub.	6

Table 1: Parameter values in the baseline calibration. See text for descriptions and targets.

Steady-state equilibrium distribution. We first simulate a Bewley model (i.e., without aggregate shocks). In Table 2, we report the wealth distribution generated by the model and compare it to the empirical distribution. We compute a number of standard statistics – listed in the first column – including the quintiles, the Gini coefficient, and 95-100 intercentiles.

The empirical wealth distribution reported in the second and third columns of Table 2 is computed using two sources, the PSID for the year 2006 and the SCF for the year 2007. The fourth column reports the wealth distribution generated by our model. Overall, the distribution of wealth generated by the model is quite similar for the two replacement rates and is close to the data. In particular, the model does a good job in matching the wealth distribution with a high Gini of 0.77. The concentration of wealth at the top 1% of the distribution is higher in the data than in the model. It is known that additional model features must be introduced to match the high wealth inequality in the US, such as heterogeneous discount rates, as in Krusell and Smith (1998), or entrepreneurship, as in Quadrini (1999).

	Data		Model
Wealth statistics	PSID, 06	SCF, 07	
Q1	−0.9	−0.2	0.0
Q2	0.8	1.2	0.1
Q3	4.4	4.6	3.5
Q4	13.0	11.9	15.1
Q5	82.7	82.5	81.3
Top 5%	36.5	36.4	37.8
Top 1%	30.9	33.5	10.7
Gini	0.77	0.78	0.77

Table 2: Wealth distribution in the data and in the model.

5.2 Truncation and estimating Pareto weights

We now construct the truncated model. We use a truncation length of $N = 5$. This implies that we consider $Y^N = 5^5 = 3125$ different truncated histories.

Computing the ξs. For each history, we compute the coefficients $(\xi_{y^N})_{y^N}$ such that the truncated allocations are equal to the truncation of allocations in the Bewley model (see Section 4). We find that the standard deviation across histories is $std(\xi_{y^N}) = 0.13$.

Comparing with other solution techniques. We choose the number of truncated histories such that the simulation outcomes using our truncation method are close to other simulation methods. The two standard alternative solution methods are the histogram method developed by Rios-Rull (2001), Reiter (2009), and Young (2010) and IRFs analysis by Boppart et al. (2018) and Auclert et al. (2019). We simulate the model with the previous calibration, assuming no public spending with three different methods. The results are provided in Appendix F, and are shown to be very close. For instance, the standard deviation of GDP is 1.71% with Reiter's method and 1.64% for our truncation method. See also LeGrand and Ragot (2020) for the same exercise in a different environment.

Estimating Pareto weights. We estimate the value of Pareto weights, such that the first-order conditions of the planner are fulfilled at the steady state for the actual US tax system. See Section 4.3 for the formal algebraic characterization. We estimate 5^5 different Pareto weights, due to the truncation with $N = 5$. We find that weights are increasing with productivity. The weight of agents with the highest productivity level is approximately twice that of agents with the lowest productivity level.

5.3 Dynamic of the fiscal system with a complete set of instruments

We now present the dynamics of the fiscal system after a technology shock in the incomplete-market economy (henceforth, IM stands for incomplete market). We solve the model when the planner has the full set of fiscal instruments (τ^K, τ^L, T, and B). We know from Proposition 2 that inflation does not play any role in this case, and the allocation is the same as the flexible-price one. To understand the effect of heterogeneity in this environment, we compare the IRFs in the IM economy with those of the complete-market economy (henceforth, CM stands for complete-market).

In the CM economy, the households' side is represented by a representative agent.[27] This environment is studied in a vast literature (Chari and Kehoe, 1999; Aiyagari et al., 2002; Farhi, 2010; Bhandari et al., 2017a, among others). Our calibration of the CM economy relies on the same parameters as those of Table 1, ensuring that the steady-state GDP and aggregate consumption are the same in the CM and IM economies.[28] Results are plotted in Figure 1. Each panel of Figure 1 reports the proportional change for the variable under consideration, in percentage points, except for tax rates, for which the absolute variation is reported. For instance, Panel 1 reports a persistent fall in TFP for 100 periods, after a fall of 1% on impact.

Overall, the comparison of the dynamics of aggregate quantities (Consumption, Panel 2; Capital, Panel 3; GDP, Panel 4) shows that the IM and CM economies exhibit very similar behavior along those dimensions. The main difference between the two economies concerns fiscal instruments. The capital tax is very volatile and labor taxes are very smooth in the CM economy, which is a standard result in this literature (Chari and Kehoe, 1999 and Farhi, 2010). The planner uses the capital tax to front-load all adjustments, such that the public debt jumps on a path consistent with zero tax on both capital and labor. As the public debt is negative, the decrease in public debt means that the planner actually further accumulates assets to pay for public spending, because it needs to compensate for lower governmental revenues (due to the negative shock). This very high capital tax volatility is known to be an unappealing feature of the CM economy.

[27]In our setup, this corresponds to the limit case when the truncation length is set to $N = 1$ and $\xi_{y^N} = 1$.

[28]In the CM economy, the standard outcome is that the government ends up holding a negative debt (i.e., it holds a part of the capital stock) in order to reduce distortionary taxation. This economy is of interest to compare the variations of the policy instruments along the business cycle.

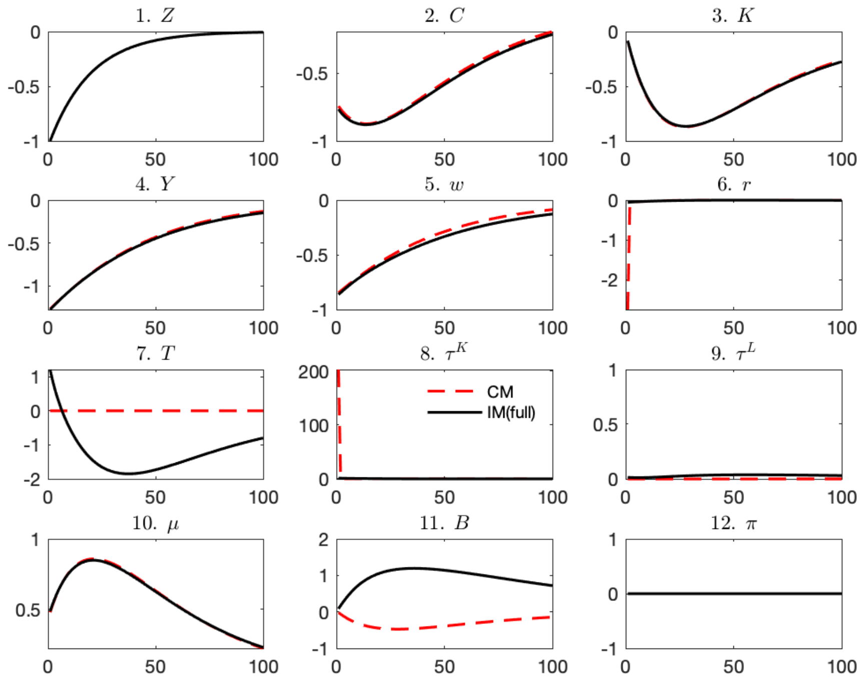

Figure 1: Comparison between the complete-market economy (red dashed line) and the incomplete-market economy with all instruments (black solid line).

In the IM economy, the fiscal policy sharply differs. First, the capital tax is much less volatile. It is actually 100 times less volatile. This result shows that incomplete markets contribute to solving the capital tax volatility puzzle. Indeed, due to precautionary saving motive, it is very costly for the planner to raise very sharply the capital tax and thereby wipe out agents' savings. A consequence of this very moderate increase in the capital tax is that public debt increases (and public debt is actual debt in the IM economy) to smooth out the impact of the negative TFP shock on public finance. The governmental budget adjustment is actually performed by a moderate increase in the labor tax (around 0.1%) for quite a long period. Finally, inflation remains constant and null, as is consistent with Proposition 2.

5.4 Dynamics of the fiscal system without time-varying capital tax

We now turn to the dynamics of the IM economy, when the capital tax is not time-varying and is set to its steady-state value. IRFs are reported in Figure 2 and compared to those of the IM economy with the full set of instruments. As in Figure 1, the dynamics of aggregate quantities (aggregate consumption, capital stock and GDP, in panels 2, 3 and 4, respectively) have a very similar pattern in the two IM economies. The most salient difference is that keeping a constant

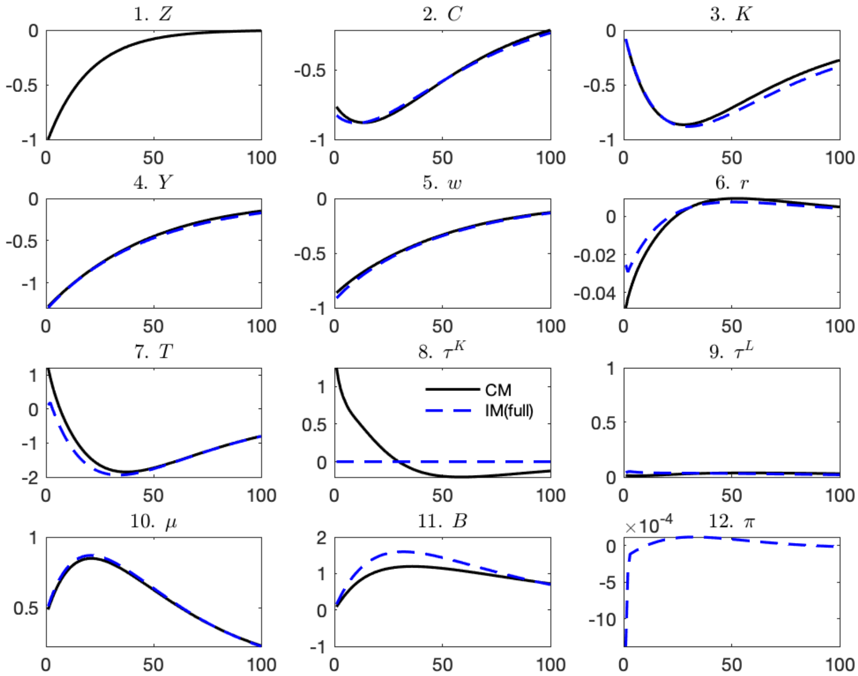

Figure 2: Comparison between the IM economy with all instruments (black solid line) and the IM economy with a fixed capital tax (blue dashed line).

capital tax generates a public debt that is more countercyclical as well as a slightly higher labor tax. Indeed, due to the distortions implied by the labor tax, its increase remains very limited and smoothed out through time, which is permitted by a higher public debt. Regarding inflation (Panel 12), it can be observed that it barely moves, even when the capital tax is constant.[29]

These findings are confirmed by second-order moments that can be found in Table 3. More precisely, we report in Table 3 the unconditional first- and second-order moments for several key variables, in three economies. The first is the representative agent economy (CM). The second is the IM economy with the full set of instruments (Full). The third is the economy without capital taxes (No cap. tax), discussed in Section 5.4.

For each variable, we report the steady-state value (labeled "Mean") and the normalized standard deviation, equal to the standard deviation divided by the mean (labeled "Std"), except for taxes for which the standard deviation is reported. The second part of the table reports correlations. First, we can observe that Table 3 confirms the IRFs regarding aggregate variables. Their mean and standard deviations are very close in the CM and IM economies. Second, the main difference, as for IRFs again, lies in the volatility of taxes and of the behavior of public debt, which significantly differs. Finally, we observe that inflation volatility is low even when

[29]Nuño and Thomas (2020) find a similar result in a different environment in the same timeless perspective.

capital tax is not time-varying when $\kappa = 100$ (third column).[30]

		CM	Full	No cap.tax
C	Mean	0.7543	0.7542	0.7542
	Std	0.0259	0.0266	0.0269
K	Mean	11.0557	11.0536	11.0535
	Std	0.0268	0.0270	0.0288
Y	Mean	1.1760	1.1759	1.1759
	Std	0.0264	0.0268	0.0274
L	Mean	0.3334	0.3334	0.3334
	Std	0.0088	0.0094	0.0098
τ^K	Mean	0.0009	0.3600	0.3600
	Std	0.8855	0.0145	0.0000
τ^L	Mean	0.0000	0.2800	0.2800
	Std	0.0000	0.0016	0.0015
B	Mean	-10.9327	2.8435	2.8424
	Std	0.0146	0.0462	0.0541
T	Mean	0.0000	0.0941	0.0941
	Std	0.0000	0.0610	0.0637
π	Mean	0.0000	0.0000	0.0000
	Std	0.0000	0.0000	0.0007
Correlations				
$corr(\tau^K, Y)$		-0.2085	-0.4868	0.0000
$corr(\tau^L, Y)$		0.9273	-0.6374	-0.9249
$corr(B, Y)$		-0.8349	-0.7592	-0.8291
$corr(T, Y)$		0.0000	0.6523	0.7796
$corr(C, Y)$		0.9673	0.9691	0.9755
$corr(Y, Y_{-1})$		0.9776	0.9781	0.9785
$corr(B, B_{-1})$		0.9992	0.9996	0.9994

Table 3: First- and second-order moments for key variables, in the four economies (CM: complete market; Full: IM with full fiscal set; No cap. tax: IM with fixed capital tax. See text for details.

From IRFs and second-order moments, we conclude that, compared to CM, the IM economy implies a sharp reduction in the volatility of the capital tax, as well as a countercyclical and more volatile public debt. In addition, although inflation could be a partial substitute for the missing capital tax, the associated distortions (on savings and output destruction) are too high for the planner to actively rely on inflation for realistic price-adjustment costs.

[30]We must implement unrealistically low price adjustment cost to obtain a high optimal inflation volatility. For instance, we find substantial inflation volatility for $\kappa = 0.1$, but this is not the case for $\kappa = 10$ or higher.

6 When is inflation optimal? Timeless and time-0 problem

We conduct here a thorough comparison of our results to those of the literature regarding optimal monetary policy in HANK models. Indeed, Acharya et al. (2020), Bhandari et al. (2020), and Nuño and Thomas (2020) find significant deviation from price stability in some cases. As we will see, this is not related to different resolution techniques, but to the assumption about available fiscal instruments and to the investigation of a time-0 rather than a timeless problem.

As a preliminary remark, the results of Proposition 2 are valid in all periods. As a consequence, deviation from price stability is obtained only for missing fiscal tools (capital or labor tax), both in the timeless and the time-0 perspectives. To compare our results with those of the literature, we focus on a no-capital economy, which is a special case of the previous setup. Furthermore, in our time-0 exercise, we allow the initial wealth distribution to differ from the steady-state one, as in Bhandari et al. (2020). Income volatility can also be time-varying, as in Acharya et al. (2020).

6.1 Setup and calibration

We consider the same model as in Section 5, with the following calibration, to simplify the comparison with the literature. First, we have $\alpha = 0$ (no capital) and $\tau_t^K = \tau_t^L = B_t = G_t = 0$ in all periods (no fiscal system, as in the literature). The governmental budget constraint implies that the lump-sum transfers are equal to the firms' monopoly profits: $T_t = (1 - \zeta_t - \frac{\kappa}{2}\pi_t^2)Y_t$, as assumed by Acharya et al. (2020), and Bhandari et al. (2020) in their baseline model.[31] This specification means that transfers can vary along the business cycle but they are null at the steady state, since the steady state still features zero inflation ($\pi = 0$) and no distortion ($\zeta = 1$). The monetary parameters driving the cost of inflation are $\varepsilon = 10$ and $\kappa = 30$. This generates a slope of the Phillips curve that is comparable to the high value of Acharya et al. (2020).

The credit limit $\bar{a}$ is set equal to six times the steady-state quarterly wage w which implies that the share of credit-constrained agents amounts to 18% in the long run, which is a realistic number (Jappelli, 1990). Because the credit limit is not null and the total net supply of debt is still zero, some agents will be net borrowers, while others will be net lenders. Unexpected inflation thus redistributes wealth from creditors and lenders (which is the so-called Fisher effect).

We assume that the initial (nominal) wealth distribution differs from the steady-state one, as in Bhandari et al. (2020). More precisely, the initial distribution is a mean-preserving transformation of the steady-state distribution: the standard deviation of the initial distribution is 1% higher than the steady-state one. This implies that the economy features higher wealth inequality in the initial period than in the long run – which generates an additional motive for redistribution.

[31]More specifically, Bhandari et al. (2020) assume that agents may have some equity but they cannot trade it. As a consequence, agents can receive different shares of aggregate dividends. We thus assume that agents hold the same amount of equity.

The other preference parameters (β, δ, χ and φ) and parameters driving the shock process are set to the same values as in our baseline calibration of Table 1.

6.2 Resolution and results

We consider two different Ramsey problems: (i) the timeless perspective that we considered in Section 5; (ii) the time-0 problem, also considered in the literature.

Resolution. In both cases, we consider a truncation length set to $N = 5$ and we check that results are not overly affected by the length of the truncation. In our baseline simulation, the economy is hit at time 0 by an unexpected negative productivity shock of -1% with an autocorrelation of 0.95. Because we consider an initial distribution that differs from the steady-state one, we cannot rely on a perturbation resolution. We compute a transition path under perfect foresight by solving a non-linear system (MIT shock). We detail the strategy in Appendix G.1. In the case of the timeless perspective, we set the initial values of Lagrange multipliers for agents' Euler equations and for the Phillips curve to their steady-state values. In the case of the time-0 problem, we set these initial values to zero, reflecting the fact that the planner at time 0 is not constrained by past commitments.

Timeless perspective. The results are plotted in Figure 3, where we report the dynamics of the aggregate shock Z, of inflation π and of aggregate consumption C. The red-dashed line corresponds to the timeless perspective. It can be observed that inflation barely moves in initial periods and in the dynamics, while aggregate consumption incurs a sharp decline, as almost no redistribution occurs across agents. Even with a total absence of fiscal system, and stronger motives for redistribution (due to the initial wealth distribution), this empirical exercise confirms our result of Section 5: in a timeless perspective, the quantitative role of inflation is very limited. This finding confirms in a more general setting the claim of Nuño and Thomas (2020) stating that the time-0 problem is necessary to generate a significant deviation from price stability.

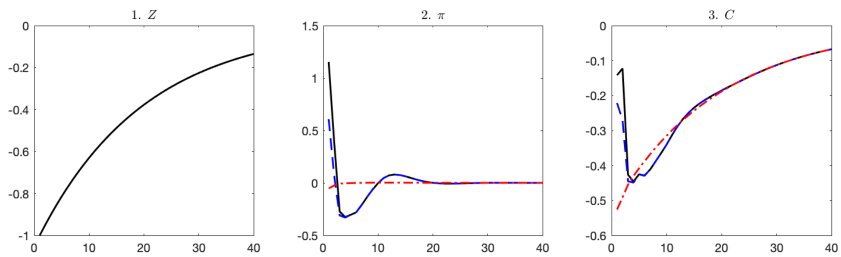

Figure 3: Dynamics of the economy after a negative technology shock (in percent). Z and C are in proportional deviation to steady-state values, while π is reported in level deviation.

Time-0 problem. The results are also plotted in Figure 3. The black solid line corresponds to the same calibration as in the timeless perspective, but for a time-0 problem. The paths are very different in timeless and time-0 economies. In time-0 economy, inflation increases by more than 1% on impact – but we do not observe any persistent deviation from zero inflation. This redistributes wealth from wealth-rich agents to wealth-poor agents, as the real interest rate falls unexpectedly on impact (Fisher effect). This wealth transfer towards agents with high marginal propensity to consume (in particular credit-constrained agents) contributes to limit the decrease in aggregate consumption, compared to the timeless case.

To better understand the role of the different elements, we have also conducted some comparative static exercise. Numerical results are provided in Appendix G. As a summary, our findings corroborate the existing literature. More precisely, the increase in inflation on impact is higher when: (i) the cost of inflation (κ) diminishes (as in Acharya et al., 2020), (ii) the idiosyncratic risk is countercyclical (as in Acharya et al., 2020 also), and (iii) when the credit limit $\bar{a}$ becomes looser (agents can borrow more). This last effect is reminiscent of the importance of the Fisher effect pointed out in Nuño and Thomas (2020) and also connects with the results Bhandari et al. (2020), where credit constraints do not bind.

To conclude this comparison section, when fiscal tools are missing, inflation is used to transfer wealth across agents but only when the inflation can be used as a surprise. We do not observe any persistent deviation from zero inflation since inflation is used only in the initial periods.

7 Conclusion

We derive optimal fiscal-monetary policy with commitment in an economy with incomplete insurance markets, nominal frictions, and aggregate shocks. We find that market incompleteness considerably reduces the volatility of capital tax, but increases the counter-cyclicality of public debt after technology shocks. In the long run (in a timeless perspective), optimal monetary policy has little role for redistribution, even when the capital tax remains fixed along the business cycle.

Departing from the timeless perspective and considering a time-0 problem can generate significant deviations from price stability but only when taxes are constrained to remain constant. However, this situation generates only a temporary deviation from price stability, and no persistent deviation ever occurs in our experiments.

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Appendix

A Computing the FOCs of the full-fledged Ramsey program

This section is organized in three parts. In Section A.1, we transform the Ramsey program by including in the planner's objective the Euler equations. In Section A.2, we derive the FOCs of the Ramsey program in the flexible-price economy, while in Section A.3, we derive those when the capital tax is fixed to its steady-state value.

A.1 Transforming the Ramsey program

The objective of the Ramsey program (22)–(32) can be rewritten as:

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U_t^i \ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \lambda_t^i \omega_t^i \left(U_{c,t}^i - \nu_t^i - \beta \mathbb{E}_t \left[(1+r_{t+1}) U_{c,t+1}^i \right] \right) \ell(di)$$
$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t \left(\Pi_t \left(\Pi_t - 1 \right) Y_t M_t - \frac{\varepsilon - 1}{\kappa} \left(\zeta_t - 1 \right) Y_t M_t - \beta \mathbb{E}_t \left[\Pi_{t+1} \left(\Pi_{t+1} - 1 \right) Y_{t+1} M_{t+1} \right] \right),$$

where we define $U_t^i = U(c_t^i, l_t^i)$, and $U_{c,t}^i = U_c(c_t^i, l_t^i)$ (and later, $U_{l,t}^i = U_l(c_t^i, l_t^i)$, $U_{cc,t}^i = U_{cc}(c_t^i, l_t^i)$, and $U_{cl,t}^i = U_{cl}(c_t^i, l_t^i)$). In the previous expression, we have used the fact that when the credit constraint is binding for agent i at period t, ($\nu_t^i > 0$), then there is no Euler equation to consider ($\lambda_t^i > 0$). With $\gamma_{-1} = 0$, we obtain after some manipulations:

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U_t^i \ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\omega_t^i \lambda_t^i - (1+r_t) \lambda_{t-1}^i \omega_{t-1}^i \right) U_{c,t}^i \ell(di) \tag{56}$$
$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left((\gamma_t - \gamma_{t-1}) \Pi_t \left(\Pi_t - 1 \right) - \frac{\varepsilon - 1}{\kappa} \gamma_t \left(\zeta_t - 1 \right) \right) Y_t M_t,$$

Using (56), the Ramsey program (24)–(31) can now be expressed as: max J on $\big(w_t, r_t, \tilde{R}_t^{B,N}, \tilde{w}_t, \tilde{r}_t^K, \tau_t^K, \tau_t^L, B_t, T_t, K_t, L_t, \Pi_t, (a_t^i, c_t^i, l_t^i)_i\big)_{t\geq 0}$, subject to the same set of constraints, except the individual Euler equations for consumption (28) and the Phillips curve (30). Note that we could also write the Ramsey program using the sequential representation (as in equations (17)), but at the cost of tedious notation.

A.2 First-order conditions for the flexible-price economy

The flexible-price allocation is the solution of $\max_{\left(w_t, r_t, B_t, K_t, L_t, T_t, (a_t^i, c_t^i, l_t^i)_i\right)} J$, subject to:

$$l_t^i = \left(\chi w_t y_t^i \right)^{\varphi}, \; c_t^i = \chi^{\varphi} \left(w_t y_t^i \right)^{\varphi+1} + (1+r_t) a_{t-1}^i - a_t^i,$$
$$B_t + Y_t - \delta K_{t-1} = G_t + B_{t-1} + r_t \left(B_{t-1} + K_{t-1} \right) + w_t L_t + T_t, \text{ and } K_t = \int_i a_t^i \ell(di) - B_t,$$

with $\kappa = 0$ and $\gamma_t = 0$ for all t, $\zeta_t = 1$, $\Pi_t = 1$ for all t (because prices are flexible).

Derivative with respect to w_t: the labor tax. We have:

$$0 = \int_i \omega_t^i \left(U_{c,t}^i \frac{\partial c_t^i}{\partial w_t} + U_{l,t}^i \frac{\partial l_t^i}{\partial w_t} \right) \ell(di) + \mu_t \left(\frac{\partial Y_t}{\partial w_t} - L_t - w_t \frac{\partial L_t}{\partial w_t} \right)$$
$$- \int_i \left(\lambda_t^i - (1+r_t)\lambda_{t-1}^i \right) \left(U_{cc,t}^i \frac{\partial c_t^i}{\partial w_t} + U_{cl,t}^i \frac{\partial l_t^i}{\partial w_t} \right) \ell(di).$$

We have $U_{l,t}^i = -w_t y_t^i U_{c,t}^i$ and $U_{c,t}^i \frac{\partial c_t^i}{\partial w_t} + U_{l,t}^i \frac{\partial l_t^i}{\partial w_t} = U_{c,t}^i \left(\frac{\partial c_t^i}{\partial w_t} - w_t y_t^i \frac{\partial l_t^i}{\partial w_t} \right) = U_{c,t}^i \chi^{\varphi} w_t^{\varphi} \left(y_t^i\right)^{\varphi+1}$. Similarly, $U_{cc,t}^i \frac{\partial c_t^i}{\partial w_t} + U_{cl,t}^i \frac{\partial l_t^i}{\partial w_t} = U_{cc,t}^i \chi^{\varphi} w_t^{\varphi} \left(y_t^i\right)^{\varphi+1}$. Furthermore:

$$\frac{\partial L_t}{\partial w_t} = \int_i y_t^i \frac{\partial l_t^i}{\partial w_t} \ell(di) = \varphi \int_i y_t^i \frac{l_t^i}{w_t} \ell(di) = \frac{\varphi}{w_t} L_t, \text{ and } \frac{\partial Y_t}{\partial w_t} = (1-\alpha) \frac{\varphi}{w_t} Y_t.$$

Using the definition of $\hat{\psi}^i$ in (34), we obtain:

$$\mu_t \varphi \left(L_t - (1-\alpha) \frac{Y_t}{w_t} \right) = \int_i \hat{\psi}_t^i y_t^i l_t^i \ell(di).$$

Derivative with respect to r_t: the capital tax. Using $\frac{\partial c_t^i}{\partial r_t} = a_{t-1}^i$ and $\frac{\partial l_t^i}{\partial r_t} = 0$, we have:

$$0 = \int_i (\omega_t^i U_{c,t}^i - (\lambda_t^i - (1+r_t)\lambda_{t-1}^i) U_{cc,t}^i) a_{t-1}^i \ell(di) + \int_i \lambda_{t-1}^i U_{c,t}^i \ell(di) - \mu_t (K_{t-1} + B_{t-1}).$$

We therefore obtain:

$$\int_i \lambda_{t-1}^i U_{c,t}^i \ell(di) + \int_i \hat{\psi}_t^i a_{t-1}^i \ell(di) = 0. \tag{57}$$

Derivative with respect to B_t: the public debt. We get: $0 = \mu_t - \beta \mathbb{E}_t \left[\mu_{t+1}(1-\delta) \right] + \beta \mathbb{E}_t \mu_{t+1} \frac{\partial Y_{t+1}}{\partial B_t}$. Using (3), we have: $\frac{K_{t-1}}{L_t} = \frac{\alpha}{1-\alpha} \frac{\tilde{w}_t}{\tilde{r}_t^K + \delta}$ and $1 = \frac{1}{Z_t} \left(\frac{\tilde{r}_t^K + \delta}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha} \right)^{1-\alpha}$. We deduce that: $\frac{\partial Y_{t+1}}{\partial B_t} = -(\tilde{r}_{t+1}^K + \delta)$ and obtain $0 = \mu_t - \beta \mathbb{E}_t \left[\mu_{t+1}(1 - \delta + \tilde{r}_{t+1}^K + \delta) \right]$, or:

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + \tilde{r}_{t+1}^K \right) \right]. \tag{58}$$

Derivative with respect to a_t^i: the net saving of consumers. We have, since $\frac{\partial l_t^i}{\partial a_t^i} = 0$:

$$\beta \mathbb{E}_t \left[\mu_{t+1} (r_{t+1} + \delta - \frac{\partial Y_{t+1}}{\partial a_t^i}) \right] = \int_i \omega_t^i U_{c,t}^i \frac{\partial c_t^i}{\partial a_t^i} \ell(di) - \int_i \left(\lambda_t^i - (1+r_t)\lambda_{t-1}^i \right) U_{cc,t}^i \frac{\partial c_t^i}{\partial a_t^i} \ell(di)$$
$$+ \beta \mathbb{E}_t \int_i \omega_t^i U_{c,t+1}^i \frac{\partial c_{t+1}^i}{\partial a_t^i} \ell(di) - \int_i \left(\lambda_t^i - (1+r_t)\lambda_{t-1}^i \right) U_{cc,t+1}^i \frac{\partial c_{t+1}^i}{\partial a_t^i} \ell(di).$$

This yields using $\frac{\partial c_t^i}{\partial a_t^i} = -1$, $\frac{\partial c_{t+1}^i}{\partial a_t^i} = 1 + r_{t+1}$, and $\frac{\partial Y_{t+1}}{\partial a_t^i} = \tilde{r}_{t+1}^K + \delta$:

$$\psi_t^i = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \psi_{t+1}^i \right] + \beta \mathbb{E}_t \left[\mu_{t+1} \left(\tilde{r}_{t+1}^K - r_{t+1} \right) \right].$$

Observing that $\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + \tilde{r}_{t+1}^K \right) \right]$ (equation (58)), we obtain using the definition of $\hat{\psi}^i$:

$$\hat{\psi}_t^i = \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{t+1}^i \right]. \tag{59}$$

Derivative with respect to T_t. We obtain:

$$\int_i \hat{\psi}_t^i \ell(di) = 0. \tag{60}$$

A.3 First-order conditions for the economy with fixed capital taxes

Because pre- and post-tax rates cannot be set independently, nominal frictions cannot be suppressed in this setup. We have two additional Lagrange multipliers: (i) $\beta^t \Upsilon_t$ on the no-arbitrage condition (10), and (ii) $\beta^t \Gamma_t$ on the zero-profit condition (32) of the fund. In this context, the planner's objective can be written as:

$$\begin{aligned}
J = {} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i) \ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_t^i - (1 + r_t) \lambda_{t-1}^i) U_c(c_t^i, l_t^i) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_t - \gamma_{t-1}) \Pi_t (\Pi_t - 1) Y_t \int_i \omega_t^i U_{c,t}^i \ell(di) + \frac{\varepsilon - 1}{\kappa} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t (\zeta_t - 1) Y_t \int_i \omega_t^i U_{c,t}^i \ell(di) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(B_t - (1 - \delta) B_{t-1} - G_t - T_t + (1 - \frac{\kappa}{2} (\Pi_t - 1)^2) Y_t - (r_t + \delta) \int_i a_{t-1}^i \ell(di) - w_t L_t \right) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t-1} \Upsilon_{t-1} (\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - (1 + \tilde{r}_t^K)) + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Gamma_t (r_t A_{t-1} - (1 - \tau_{SS}^K) (\tilde{r}_t^K K_{t-1} + (\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1) B_{t-1}) \Big).
\end{aligned}$$

Derivative with respect to $\tilde{R}_t^{B,N}$. We obtain:

$$(1 - \tau^K) \mathbb{E}_t \left[\frac{\Gamma_{t+1}}{\Pi_{t+1}} \right] B_t = \Upsilon_t \mathbb{E}_t \left[\frac{1}{\Pi_{t+1}} \right]. \tag{61}$$

Derivative with respect to $\tilde{r}_t^K$. Using the definition of ζ_t in equation (2), we obtain: $Y_t \frac{\partial \zeta_t}{\partial \tilde{r}_t^K} = \frac{1}{\alpha Z_t} (\frac{K_{t-1}}{L_t})^{1-\alpha} Z_t K_t^\alpha L_t^{1-\alpha} = \frac{K_{t-1}}{\alpha}$. This yields the following first-order condition:

$$\Upsilon_{t-1} + \Gamma_t (1 - \tau_{SS}^K)(A_{t-1} - B_{t-1}) = \frac{\varepsilon - 1}{\alpha \kappa} \gamma_t K_{t-1} M_t.$$

Derivative with respect to Π_t. We have:

$$0 = \mu_t \kappa \left(\Pi_t - 1\right) + \left(\gamma_t - \gamma_{t-1}\right)\left(2\Pi_t - 1\right) M_t + \left(\Gamma_t \left(1 - \tau^K\right) B_{t-1} - \Upsilon_{t-1}\right) \frac{\tilde{R}^{B,N}_{t-1}}{Y_t \Pi_t^2}. \tag{62}$$

Derivative with respect to r_t. We obtain:

$$0 = \int_i \hat{\psi}_t^i a_{t-1}^i \ell(di) + \int_i \lambda_{t-1}^i \omega_{t-1}^i U_{c,t}^i \ell(di) + \Gamma_t A_{t-1}. \tag{63}$$

Derivative with respect to w_t. Using the definitions (1) and (20) of Y_t and L_t, as well as individual budget constraint (26), we obtain:

$$\frac{\partial L_t}{\partial w_t} = \int_i y_t^i \frac{\partial l_t^i}{\partial w_t} \ell(di) = \varphi \int_i y_t^i \frac{l_t^i}{w_t} \ell(di) = \frac{\varphi}{w_t} L_t,$$
$$U_{cc}(c_t^i, l_t^i) \frac{\partial c_t^i}{\partial w_t} + U_{cl}(c_t^i, l_t^i) \frac{\partial l_t^i}{\partial w_t} = U_{cc,t}^i y_t^i l_t^i, \text{ and } \frac{\partial Y_t}{\partial w_t} = (1-\alpha) \frac{\varphi}{w_t} Y_t.$$

The definition (2) of ζ_t also implies: $\frac{\partial \zeta_t}{\partial w_t} = -(1-\alpha)\zeta_t \frac{\varphi}{w_t}$. Using previous computations and the definition of $\hat{\psi}$, we obtain after some algebra:

$$\begin{aligned} \mu_t \varphi \left(L_t - (1-\alpha) \frac{Y_t}{w_t} \left(1 - \frac{\kappa}{2} \left(\Pi_t - 1\right)^2\right)\right) &= \int_i \hat{\psi}_t^i y_t^i l_t^i \ell(di) \\ - \frac{(1-\alpha)\varphi}{w_t} \left(\left(\gamma_t - \gamma_{t-1}\right) \Pi_t \left(\Pi_t - 1\right) + \frac{\varepsilon - 1}{\kappa} \gamma_t \right) & Y_t \int_i \omega_t^i U_{c,t}^i \ell(di). \end{aligned} \tag{64}$$

Derivative with respect to B_t: the public debt. The financial market clearing condition (20), $Y_t = Z_t K_{t-1}^\alpha L_t^{1-\alpha}$ implies: $\frac{\partial Y_{t+1}}{\partial B_t} = -\alpha \frac{Y_{t+1}}{K_t} = -\zeta_{t+1}^{-1}(\tilde{r}_{t+1}^K + \delta)$, where the second equality comes from equations (1)–(2). It also implies: $\zeta_t = \frac{1}{\alpha Z_t}\left(\tilde{r}_{t+1}^K + \delta\right)\left(\frac{\int_i a_{t-1}^i \ell(di) - B_{t-1}}{L_t}\right)^{1-\alpha}$ and $\frac{\partial \zeta_{t+1}}{\partial B_t} = -(1-\alpha)\frac{\zeta_{t+1}}{K_t}$. We obtain after some manipulation:

$$\begin{aligned} \mu_t = {} & \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 - \delta + \zeta_{t+1}^{-1} (\tilde{r}_{t+1}^K + \delta) \left(1 - \frac{\kappa}{2} \left(\Pi_{t+1} - 1\right)^2 \right)\right)\right] \\ & - \alpha \beta \mathbb{E}_t \left[\left(\left(\gamma_{t+1} - \gamma_t\right) \Pi_{t+1} \left(\Pi_{t+1} - 1\right) + \frac{\varepsilon - 1}{\kappa} \gamma_{t+1} \right) \frac{Y_{t+1}}{K_t} M_{t+1} \right] \\ & + \frac{\varepsilon - 1}{\kappa} \beta \mathbb{E}_t \left[\gamma_{t+1} \frac{\zeta_{t+1} Y_{t+1}}{K_t} M_{t+1} \right] + \beta \left(1 - \tau^K\right) \mathbb{E}_t \left[\Gamma_{t+1} \left(\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}} - 1 - \tilde{r}_{t+1}^K \right)\right]. \end{aligned} \tag{65}$$

Derivative with respect to a_t^i: the net saving of consumers. Note that using individual budget constraints (26), we have: $\frac{\partial c_t^i}{\partial a_t^i} = -1$, and $\frac{\partial c_{t+1}^i}{\partial a_t^i} = 1 + r_{t+1}$. Furthermore, using equations (2)–(5) and the financial market clearing condition (20), we obtain: $\frac{\partial Y_{t+1}}{\partial a_t^i} = \alpha \frac{Y_{t+1}}{K_t} = \zeta_{t+1}^{-1}(\tilde{r}_{t+1}^K + \delta)$, and $\frac{\partial \zeta_{t+1}}{\partial a_t^i} Y_{t+1} = \frac{1-\alpha}{\alpha}(\tilde{r}_{t+1}^K + \delta)$. We obtain then the following first-order condition for unconstrained agents $i$which yields, by difference with (65):

$$\hat{\psi}_t^i = \beta\mathbb{E}_t\left[(1+r_{t+1})\hat{\psi}_{t+1}^i\right] + \beta\mathbb{E}_t\left[\Gamma_{t+1}(r_{t+1} - (1-\tau_{SS}^K)(\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}} - 1))\right]. \tag{66}$$

Derivative with respect to T_t.

$$\int_i \hat{\psi}_t^i \ell(di) = 0. \tag{67}$$

B Proof of Proposition 1

First-order conditions (36) immediately imply equation (40) at the steady state, since the governmental budget constraint must be binding ($\mu \neq 0$). Summing individual consumption Euler equations (28) for all $i \in \mathcal{I}$ and substituting for the expression (11) of r:

$$\int_i U_c(c^i, l^i)\ell(di) = \beta(1 + (1-\tau^K)\tilde{r})\int_i U_c(c^i, l^i)\ell(di) + \int_i \nu^i \ell(di). \tag{68}$$

The no-profit condition (9) of the fund and the no-arbitrage condition (10 at the steady state, we deduce that $\tilde{r}_K = \tilde{r}$. This implies with equation (40) that $\beta(1+\tilde{r}) = \beta(1+\tilde{r}_K) = 1$. Plugging this result in (68), we obtain:

$$\int_i U_c(c^i, l^i)\ell(di) = (1 - \tau^K(1-\beta))\int_i U_c(c^i, l^i)\ell(di) + \int_i \nu^i \ell(di),$$

which yields equation (41) since $\beta \in (0,1)$ and $\int_i U_c(c^i, l^i)\ell(di) > 0$ (otherwise we would have $U_c(c^i, l^i) = 0$ for all i and would imply an infinite consumption for all agents).

C The equivalence result

This section focuses on the proof of the equivalence result, stated in Proposition 2, and on its generalization discussed in Section C.2.

C.1 Proof of Proposition 2

The proof follows the spirit of Correia et al. (2008). The first observation is that when there is a full tax system, pre- and post-tax rates are independent from each other. Second, pre-tax rates do not play any role in the planner's constraints. In particular, the governmental budget constraint (81), the individual budget constraints (26), and the constraints on Euler equations (28)–(29) depend only on the post-tax rates r_t and w_t. So pre-tax rates can be removed from the planner's program and we can impose another constraint without loss of generality. We therefore set:

$$\tilde{w}_t = (1-\alpha) Z_t^{\frac{1}{1-\alpha}} \left(\frac{\tilde{r}_t^K + \delta}{\alpha} \right)^{-\frac{\alpha}{1-\alpha}}, \tag{69}$$

which with equations (2) implies: $\zeta_t = 1$. This then leads with (1) to $\tilde{r}_t + \delta = \alpha Z_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha-1}$ and $\tilde{w}_t = (1-\alpha) Z_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha}$ (which are obviously compatible with (69). Pre-tax rates will be determined by equilibrium capital and total labor supply.

We can further set $\Pi_t = 1$, which with $\zeta_t = 1$, implies that the Phillips curve holds at all dates. The pre-tax nominal interest rate will be determined by the no-arbitrage condition (10):

$$\tilde{R}_t^{B,N} = \mathbb{E}_t \left[1 + \tilde{r}_{t+1}^K \right].$$

Finally, capital and labor taxes will be computed using allocations as:

$$\tau_t^K = 1 - \frac{r_t A_{t-1}}{\tilde{r}_t^K K_{t-1} + \left(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1 \right) B_{t-1}}, \text{ and } \tau_t^L = 1 - \frac{w_t}{\tilde{w}_t}.$$

With these elements, the Ramsey program can be written exactly as in the flexible-price economy, which concludes the proof.

C.2 Generalizing our equivalence result

In this section, we generalize our equivalence result regarding monetary policy to an economy with a non-separable utility function, an actual portfolio choice of agents (no fund intermediation), and the possibly for agents to borrow in nominal terms. The rest of the economy (production, risk structure) is unchanged compared to the main text.

C.2.1 Setup

We assume in this section that the agent can directly trade real and nominal assets, without the intermediary of a fund. Real assets are capital shares and are prevented from being sold short. Nominal assets can be held in positive or negative quantities – and as such constitute savings or private debt. We assume that there exists an enforcement technology that makes private nominal debt fully riskless: Nominal savings can be indistinctly invested in private or public debt.

Government, fiscal tools and monetary policy. As in the main text, the government has to finance a public spending G_t using public and monetary policies. The fiscal policy consists of a lump-sum transfer T_t, public debt B_t, a labor income tax τ_t^L. The main difference is that nominal and real assets are taxed differently. The capital tax, bearing on real assets, is denoted τ_t^K, while the tax on nominal assets is denoted τ_t^B. After-tax quantities, still denoted without a tilde, are defined as follows:

$$w_t = (1-\tau_t^L)\tilde{w}_t,\ r_t^K = (1-\tau_t^K)\tilde{r}_t^K,\ \frac{R_t^{B,N}}{\Pi_t} - 1 = (1-\tau_t^B)(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1). \tag{70}$$

Note that as in the main text the pre-tax nominal rate is pre-determined (in period $t-1$ for the rate between $t-1$ and t), while the post-tax rate is determined in the period. A remark is in order regarding taxation of nominal assets. We have made the simplest assumption to show that our equivalence result holds in the presence of nominal liabilities. Private and public nominal debts are perfectly fungible: they are both traded at the same pre-tax interest rate and both taxed identically (and thus traded at the same ex-post interest rate). In consequence, the amount of private debt is fully consolidated (negative holdings exactly offset positive holdings) and does not influence the governmental budget constraint. It would be possible to allow for a more complex mechanism where the government taxes private nominal debt at a different rate. Introducing this additional instrument would therefore not affect our neutrality result.

We now turn to the governmental budget constraint. Again assuming that the government fully taxes the profit of all firms, we obtain for the date-t constraint:

$$G_t + \frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t}B_{t-1} + T_t \leq \tau_t^L\tilde{w}_tL_t + \tau_t^K\tilde{r}_t^KK_{t-1} + \tau_t^B(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1)B_{t-1} + (1-\zeta_t - \frac{\kappa}{2}\pi_t^2)Y_t + B_t.$$

Using the relationship (1) stating that $\zeta_tY_t = (\tilde{r}_t^K + \delta)K_{t-1} + \tilde{w}_tL_t$, as well as the definition of post-tax rates in equations (70), the governmental budget constraint can be simplified into:

$$G_t + \frac{R_t^{B,N}}{\Pi_t}B_{t-1} + r_t^KK_{t-1} + w_tL_t + T_t = B_t + \left(1 - \frac{\kappa}{2}\pi_t^2\right)Y_t - \delta K_{t-1}. \tag{71}$$

As in the main text, monetary policy consists in choosing the nominal interest rate $\tilde{R}_{t-1}^{B,N}$, as well as the inflation rate π_t.

Agents' program and resource constraints. We consider an agent $i \in \mathcal{I}$. She is endowed with a per period utility $U(c,l)$ over consumption c and labor supply l. We consider here the general case, where the utility function is non-separable in consumption and labor. The intertemporal utility remains however time-additive with a constant discount factor $\beta \in (0,1)$.

The agent can save in real assets, paying off the post-tax rate r_t^K. Her real asset holding, denoted by $a_t^{K,i}$, is constrained to be nonnegative: $a_t^{K,i} \geq 0$. She can also save in nominal assets whose post-tax rate is $\frac{R_t^{B,N}}{\Pi_t}$ in real terms. Her nominal savings $a_t^{B,i}$ must remain greater than an exogenous threshold denoted $-\bar{a} \leq 0$. Agents can thus borrow in nominal terms and as already explained, private debt is assumed to be riskless. The agent's program becomes:

$$\max_{\{c_t^i, l_t^i, a_t^{K,i}, a_t^{B,i}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t^i, l_t^i), \tag{72}$$

$$c_t^i + a_t^{K,i} + a_t^{B,i} = (1 + r_t^K) a_{t-1}^{K,i} + \frac{R_t^{B,N}}{\Pi_t} a_{t-1}^{B,i} + w_t y_t^i l_t^i, \tag{73}$$

$$a_t^{K,i} \geq 0,\ a_t^{B,i} \geq -\underline{a},\ c_t^i > 0,\ l_t^i > 0. \tag{74}$$

Agent i maximizes her intertemporal utility (equation (72)) subject to per period budget constraints (equation (73)), as well as positivity and asset-specific borrowing constraints (equation (74)). This yields the following first-order conditions:

$$U_c(c_t^i, l_t^i) = \beta \mathbb{E}_t \left[(1 + r_{t+1}^K) U_c(c_{t+1}^i, l_{t+1}^i) \right] + \nu_t^{K,i}, \tag{75}$$

$$U_c(c_t^i, l_t^i) = \beta \mathbb{E}_t \left[\frac{R_{t+1}^{B,N}}{\Pi_{t+1}} U_c(c_{t+1}^i, l_{t+1}^i) \right] + \nu_t^{B,i}, \tag{76}$$

$$-U_l(c_t^i, l_t^i) = w_t y_t^i U_c(c_t^i, l_t^i), \tag{77}$$

where we denote by $\nu_t^{K,i}$ and $\nu_t^{B,i}$ the Lagrange multipliers on the real and nominal credit constraints, respectively. Equation (75) is the Euler equation for real asset holdings, while equation (76) is the Euler equation for nominal ones. These two consumption Euler equations reflect the agents' portfolio choice. Finally, equation (77) is the labor Euler equation – which is not linear in labor anymore (unlike (19) in the GHH case).

We now turn to market clearing conditions. We now have two market clearing conditions, for real and nominal assets respectively:

$$\int_i a_t^{K,i} \ell(di) = K_t, \text{ and } \int_i a_t^{B,i} \ell(di) = B_t. \tag{78}$$

The clearing equations for the goods and labor markets are unchanged compared to the baseline:

$$\int_i c_t^i \ell(di) + G_t + K_t = \left(1 - \frac{\kappa}{2}\pi_t^2\right) Y_t + K_{t-1} - \delta K_{t-1}, \text{ and } \int_i y_t^i l_t^i \ell\,(di) = L_t. \tag{79}$$

The competitive equilibrium definition can now be stated as follows.

Definition 2 (Sequential equilibrium) *A sequential competitive equilibrium is a collection of individual allocations* $\left(c_t^i, l_t^i, a_t^{K,i}, a_t^{B,i}\right)_{t\geq 0, i\in\mathcal{I}}$*, of aggregate quantities* $(K_t, L_t, Y_t)_{t\geq 0}$*, of price processes* $(w_t, r_t^K, R_t^{B,N}, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^{B,N})_{t\geq 0}$*, of fiscal policies* $(\tau_t^K, \tau_t^B, \tau_t^L, B_t)_{t\geq 0}$*, and of monetary policies* $(\Pi_t)_{t\geq 0}$ *such that, for an initial wealth distribution* $\left(a_{-1}^{K,i}, a_{-1}^{B,i}\right)_{i\in\mathcal{I}}$*, and for initial values of capital stock* $K_{-1} = \int_i a_{-1}^{K,i} \ell(di)$*, of public debt* $B_{-1} = \int_i a_{-1}^{B,i} \ell(di)$ *and of the aggregate shock* z_{-1}*, we have:*

1. *given prices, individual strategies* $\left(c_t^i, l_t^i, a_t^{K,i}, a_t^{B,i}\right)_{t\geq 0, i\in\mathcal{I}}$ *solve the agent's optimization program in equations (72)–(74);*

2. *real and nominal financial markets as well as labor and goods markets clear at all dates: for any* $t \geq 0$*, equations (78)–(79) hold;*

3. *the government budget is balanced at all dates: equation (71) holds for all* $t \geq 0$*;*

4. *factor prices* $(w_t, r_t^K, R_t^{B,N}, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^{B,N})_{t\geq 0}$ *are consistent with condition (5), as well as with post-tax definitions (70);*

5. *the inflation path* $(\Pi_t)_{t\geq 0}$ *is consistent with the dynamics of the Phillips curve: at any date* $t \geq 0$*, equation (7) holds.*

Note that some elements (such as the Phillips curve) remain unchanged compared to the baseline economy.

C.2.2 Ramsey program

The Ramsey program consists in determining the competitive equilibrium, as defined in Definition 2, that maximizes the aggregate welfare (equation (22) again). The Ramsey program can formally be expressed as:

$$\max_{\left(w_t,\tilde{w}_t,r_t^K,R_t^{B,N},\tilde{r}_t^K,\tilde{R}_t^{B,N},\tau_t^K,\tau_t^B,\tau_t^L,B_t,K_t,L_t,\Pi_t,(a_t^{K,i},a_t^{B,i},c_t^i,l_t^i)_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega_t^i U(c_t^i,l_t^i)\ell(di)\right], \tag{80}$$

$$G_t + \frac{R_t^{B,N}}{\Pi_t}B_{t-1} + r_t^K K_{t-1} + w_t L_t + T_t = B_t + \left(1 - \frac{\kappa}{2}\pi_t^2\right)Y_t - \delta K_{t-1}, \tag{81}$$

$$\text{for all } i \in \mathcal{I}\text{: } c_t^i + a_t^{K,i} + a_t^{B,i} = (1 + r_t^K)a_{t-1}^{K,i} + \frac{R_t^{B,N}}{\Pi_t}a_{t-1}^{B,i} + w_t y_t^i l_t^i, \tag{82}$$

$$a_t^{K,i}, \nu_t^{K,i}, \nu_t^{B,i} \geq 0 \text{ and } a_t^{B,i} \geq -\bar{a}, \tag{83}$$

$$a_t^{K,i}\nu_t^{K,i} \geq 0 \text{ and } \left(a_t^{B,i} + \bar{a}\right)\nu_t^{B,i} \geq 0, \tag{84}$$

$$U_c(c_t^i, l_t^i) = \beta\mathbb{E}_t\left[(1 + r_{t+1}^K)U_c(c_{t+1}^i, l_{t+1}^i)\right] + \nu_t^{K,i}, \tag{85}$$

$$U_c(c_t^i, l_t^i) = \beta\mathbb{E}_t\left[\frac{R_{t+1}^{B,N}}{\Pi_{t+1}}U_c(c_{t+1}^i, l_{t+1}^i)\right] + \nu_t^{B,i}, \tag{86}$$

$$U_l(c_t^i, l_t^i) = -w_t y_t^i U_c(c_t^i, l_t^i), \tag{87}$$

$$\Pi_t(\Pi_t - 1) = \frac{\varepsilon - 1}{\kappa}(\zeta_t - 1) + \beta\mathbb{E}_t\Pi_{t+1}(\Pi_{t+1} - 1)\frac{Y_{t+1}}{Y_t}\frac{M_{t+1}}{M_t}, \tag{88}$$

$$K_t = \int_i a_t^{K,i}\ell(di),\ B_t = \int_i a_t^{B,i}\ell(di),\ L_t = \int_i y_t^i l_t^i \ell(di), \tag{89}$$

and subject to several other constraints (that are not reported here for sake of conciseness): the definition (2) of ζ_t, the definition (70) of after-tax rates, the relationship (5) between factor prices, the pricing kernel definition (23), and the positivity of labor and consumption choices, and initial conditions. The interpretation of this Ramsey program is similar to the one in the baseline economy, except that it needs to be adapted to the agent's portfolio choice involving real and nominal assets, as well as to the non-separability of the utility function. This has consequence for individual and governmental budget constraints, Euler equations and market clearing conditions.

As in the baseline case, we can integrate Euler equations into the objective. We denote by $\beta^t\omega_t^i\lambda_t^{K,i}$, $\beta^t\omega_t^i\lambda_t^{B,i}$, and $\beta^t\omega_t^i\lambda_t^{l,i}$ the Lagrange multipliers of the Euler equations (85)–(87) of agent i at date t. We also denote $\beta^t\gamma_t$ the Lagrange multiplier of the Phillips curve at date t. Similarly to the baseline case, the objective of the Ramsey program can be rewritten as:

$$
\begin{aligned}
J = &\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i \omega_t^i U_{t,i}\ell(di) + \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i \lambda_t^{l,i}\omega_t^i\left(U_{l,t}^i + w_t y_t^i U_{c,t}^i\right)\ell(di)\\
&- \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i (\lambda_t^{K,i} - \lambda_{t-1}^{K,i}(1+r_t^K))U_{c,t}^i\ell(di) - \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i (\lambda_t^{B,i} - \lambda_{t-1}^{B,i}\frac{R_t^{B,N}}{\Pi_t})U_{c,t}^i\ell(di)\\
&- \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(\gamma_t - \gamma_{t-1})\Pi_t\left(\Pi_t - 1\right)Y_tM_t + \frac{\varepsilon-1}{\kappa}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma_t\left(\zeta_t - 1\right)Y_tM_t,
\end{aligned}
$$

subject to the same set of constraint as in the initial program (80)–(89).

To interpret Ramsey first-order conditions, we define the social value of liquidity, which we still denote by ψ_t^i, in this economy as:

$$
\begin{aligned}
\psi_t^i = &\omega_t^i U_{c,t}^i - \left(\lambda_t^{K,i} - (1+r_t^K)\lambda_{t-1}^{K,i}\right)U_{cc,t}^i - \left(\lambda_t^{B,i} - R_t^{B,N}\lambda_{t-1}^{B,i}\right)U_{cc,t}^i \\
&+ \lambda_t^{l,i}\left(U_{cl,t}^i + w_t y_t^i U_{cc,t}^i\right) - \left(\left(\gamma_t - \gamma_{t-1}\right)\Pi_t\left(\Pi_t - 1\right) - \frac{\varepsilon-1}{\kappa}\gamma_t\left(\zeta_t - 1\right)\right)Y_t\omega_t^i U_{cc,t}^i.
\end{aligned} \tag{90}
$$

Compared to the baseline economy, the quantity ψ_t^i features additional terms corresponding to the two assets (instead of one) and the wealth effect implied by the non-GHH utility function.

Remark 1 (GHH) *When the utility function is of the GHH form as in the baseline economy, we have $U_{cl,t}^i + w_t y_t^i U_{cc,t}^i = 0$ and the expression of ψ_t^i is exactly the one of equation (33) for the baseline economy. Equation (90) should be seen as generalization of (33) rather than a new definition.*

C.2.3 First-order conditions for the flexible-price

We now turn to the first-order conditions of the Ramsey program in the real economy (that features $\kappa = 0$, $\zeta_t = 1$, and $\Pi_t = 1$ at all dates t). The Ramsey program can be written as:

$$
\max_{\left(w_t, r_t^K, R_t^{B,N}, (a_t^{K,i}, a_t^{B,i}, c_t^i, l_t^i)_i\right)} J, \tag{91}
$$

subject to budget constraints (81) and (82), Euler equations (85)–(87), and aggregation equations (89). As in the baseline economy, the Ramsey program is expressed in post-tax rates.

Compared to the GHH case, the non-separable utility function leads to the introduction of an additional first-order condition – with respect to labor supply l_t^i, which is:

$$-\psi_t^{l,i} = \psi_t^i w_t y_t^i - \mu_t \left(L_t - (1-\alpha) \frac{Y_t}{w_t} \right) \frac{w_t y_t^i}{L_t}, \tag{92}$$

where $\psi_t^{l,i}$ is the social valuation of an extra unit of labor supplied by agent i, defined as:

$$\psi_t^{l,i} = \omega_t^i U_{l,t}^i - \left(\lambda_t^{K,i} - (1+r_t^K)\lambda_{t-1}^{K,i} \right) U_{cl,t}^i - \left(\lambda_t^{B,i} - R_t^{B,N} \lambda_{t-1}^{B,i} \right) U_{cl,t}^i + \lambda_t^{l,i} \left(U_{ll,t}^i + w_t y_t^i U_{cl,t}^i \right),$$

which is very similar to the expression of ψ_t^i – but based on the marginal disutility of labor, $U_{l,t}^i$, instead of the marginal utility of consumption, $U_{c,t}^i$.

In other words, equation (92) can thus be seen as a generalized Euler equation for labor (87), as seen from planner's perspective. It therefore includes the effect of labor supply on aggregate production and on governmental budget constraint. Marginal utilities for consumption and labor are also replaced by ψ and ψ^l respectively.

The first-order condition with respect to post-tax wage is:

$$\int_i \omega_t^i \hat{\psi}_t^i y_t^i l_t^i \ell(di) = - \int_i \lambda_t^{l,i} y_t^i U_{c,t}^i \ell(di), \tag{93}$$

where the aggregate net liquidity value, weighted by individual labor supply in efficient terms, is set equal to individual labor distortions, represented by the Lagrange multiplier, $\lambda_t^{l,i}$.

First-order conditions with respect to real and nominal post-tax rates can be expressed:

$$\int_i \hat{\psi}_t^i a_{t-1}^{B,i} \ell(di) = - \int_i \lambda_{t-1}^{B,i} U_{c,t}^i \ell(di), \tag{94}$$

$$\int_i \hat{\psi}_t^i a_{t-1}^{K,i} \ell(di) = - \int_i \lambda_{t-1}^{K,i} U_{c,t}^i \ell(di). \tag{95}$$

Both equations are very similar to each other as well as to the one with respect to the fund rate in the baseline economy.

Finally the first-order conditions with respect to nominal and real savings are:

$$\hat{\psi}_t^i = \beta \mathbb{E}_t \left[R_{t+1}^{B,N} \hat{\psi}_{t+1}^i \right], \tag{96}$$

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + \tilde{r}_{t+1}^K \right) \right] + \beta \mathbb{E}_t \left[(1 + r_{t+1}^K - R_{t+1}^{B,N}) \hat{\psi}_{t+1}^i \right]. \tag{97}$$

These equations provide Euler-like equations for the net liquidity cost $\hat{\psi}_t^i$ and the Lagrange multiplier, μ_t, for the governmental budget constraint.

Remark 2 (GHH) *When the utility function is of the GHH form, we have* $\psi_t^{l,i} + \psi_t^i w_t y_t^i =$

$\lambda_t^{l,i}(U^i_{cl,t}w_t y^i_t + U^i_{ll,t}) = -\lambda_t^{l,i}(\varphi l^i_t)^{-1} w_t y^i_t U^i_{c,t}$. Equation (92) is then equivalent to:

$$\lambda_t^{l,i} U^i_{c,t} = -\varphi \mu_t \left(L_t - (1-\alpha)\frac{Y_t}{w_t} \right) \frac{l^i_t}{L_t}.$$

The first-order condition (93) on labor becomes thus, using (79):

$$\int_i \omega^i_t \hat{\psi}^i_t y^i_t l^i_t \ell(di) = \varphi \mu_t \left(L_t - (1-\alpha)\frac{Y_t}{w_t} \right), \tag{98}$$

which is exactly the first-order condition on w_t in the baseline economy ($\hat{\psi}^i_t$ definitions coincide in both economies).

Other first-order conditions are not affected beyond the generalization of ψ^i and $\hat{\psi}^i$ (and the difference in market structure).

Remark 3 (The fund) *When agents do not form any explicit portfolio choice and invest only through a fund, there is only first-order equation with respect to savings. Equations (94) and (95) collapse and reduce to equation (37) in the baseline case.*

Second, we also have $1 + r^K_{t+1} = R^{B,N}_t$ because of the absence of specific nominal instruments for the planner. First-order conditions are then identical to (35) and (36) in the baseline model.

All previous remarks show that our new set of first-order conditions can be seen as a generalization of our first-order conditions in the baseline economy.

C.2.4 A nominal economy with a full set of instruments

We consider a nominal economy with $\kappa > 0$, and possibly ζ_t and Π_t different from one. In that case, the Ramsey program can be written as:

$$\max_{\left(w_t, r^K_t, \tilde{w}_t, \tilde{r}^K_t, R^{B,N}_t, \Pi_t, (a^{K,i}_t, a^{B,i}_t, c^i_t, l^i_t)_i\right)} J, \tag{99}$$

subject to the same equations as in the real economy case (without $\kappa \neq 0$), as well as the Phillips curve (88) and the factor price equations (2) and (5). The same mechanism as in the baseline economy is at stake here and the proof of Section C.1 could be reproduced in a straightforward way. The pre-tax instruments $\tilde{w}_t$ and $\tilde{r}_t$ intervene only in the markup coefficient ζ_t of equation (2) and in the capital-to-labor ratio equation (5). The planner can thus set $\zeta_t = 1$ using one pre-tax instrument, and then let the capital-to-labor ratio determine the other pre-tax instrument. When only one instrument is available, ζ_t cannot be set to one without constraining the capital-to-labor ratio. Furthermore, once ζ_t is set to one, inflation has no role anymore due to the availability of a tax on nominal assets, which makes inflation a redundant instrument. Inflation can thus be set to 1, as in the real case. In consequence, we can state the following result.

Proposition 3 (An equivalence result) *When labor, nominal and real capital taxes are available, the government exactly reproduces the real-economy allocation.*

Note that the result would not hold anymore if the nominal tax is removed as an independent instrument and for instance set equal to the real capital tax. In that case inflation would be used to partly substitute for this absence of specific nominal instrument.

D Truncated model

We derive the planner's first-order conditions for truncated economies. Section D.1 assumes a full set of fiscal tools, while Section D.2 assumes fixed capital tax.

D.1 Program in the economy with full set of fiscal tools

D.1.1 Program formulation

We take advantage of the equivalence result to simplify the program expression, which is:

$$\max_{((a_{t,y^N},c_{t,y^N},l_{t,y^N})_{y^N\in\mathcal{Y}^N},w_t,r_t,B_t)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty}\beta^t \sum_{y^N\in\mathcal{Y}} \Big[S_{t,y^N}\Big(\omega_{y^N}\xi_{y^N}U(c_{t,y^N},l_{t,y^N}) - \Big(\lambda_{t,y^N} - \tilde{\lambda}_{t,y^N}(1+r_t)\Big)\xi_{y^N}U_c(c_{t,y^N},l_{t,y^N})\Big)\Big],$$

$$\text{s.t. } G_t + B_{t-1} + r_t\left(B_{t-1}+K_{t-1}\right) + w_tL_t + T_t = B_t + K_{t-1}^{\alpha}L_t^{1-\alpha} - \delta K_{t-1},$$

$$\tilde{\lambda}_{t,y^N} = \frac{\sum_{\tilde{y}^N\in\mathcal{Y}^N} S_{t-1,\tilde{y}^N}\lambda_{t-1,\tilde{y}^N}\Pi_{t,\tilde{y}^N,y^N}}{S_{t,y^N}}, \tag{100}$$

$$c_{t,y^N} + a_{t,y^N} = w_t l_{t,y^N} y_{y^N} + (1+r_t)\,\tilde{a}_{t,y^N} + T_t, \tag{101}$$

$$a_{t,y^N} \geq 0 \text{ and } \tilde{a}_{t,y^N}, = \sum_{\tilde{y}^N\in\mathcal{Y}^N} \Pi_{\tilde{y}^N y^N,t}\frac{S_{t-1,\tilde{y}^N}}{S_{t,y^N}}a_{t-1,\tilde{y}^N}, \tag{102}$$

$$l_{t,y^N} = (\chi y_{y^N} w_t)^{\phi}. \tag{103}$$

D.1.2 First-order conditions

We define the net social value of liquidity for history y^N similarly to definitions (33) and (34):

$$\hat{\psi}_{t,y^N} = \omega_{y^N}\xi_{y^N}U_c(c_{t,y^N},l_{t,y^N}) - \Big(\lambda_{t,y^N} - \tilde{\lambda}_{t,y^N}(1+r_t)\Big)\xi_{y^N}U_{cc}(c_{t,y^N},l_{t,y^N}) - \mu_t.$$

The first-order conditions on a_{t,y^N}, w_t, r_t, are, respectively:

$$\hat{\psi}_{t,y^N} = \beta \mathbb{E}_t\Big[(1+r_{t+1}) \sum_{\tilde{y}^N \in \mathcal{Y}^N} \Pi_{t,y^N\tilde{y}^N} \hat{\psi}_{t+1,\tilde{y}^N}\Big] \text{ if } \nu_{y^N} = 0 \text{ and } \lambda_{t,y^N} = 0 \text{ otherwise,} \tag{104}$$

$$\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} y_{y^N} l_{t,y^N} = \varphi \mu_t (L_t - (1-\alpha)\frac{Y_t}{w_t}), \tag{105}$$

$$\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} \tilde{a}_{t,y^N} = - \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \tilde{\lambda}_{t,y^N} \xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}), \tag{106}$$

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1}(1+\tilde{r}^K_{t+1})\right] \text{,and } \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} = 0. \tag{107}$$

D.2 Program in the economy without time-varying capital tax

D.2.1 Program formulation

We can rely on Section A.3 to obtain the expression of the truncated Ramsey program in absence of time-varying capital taxes. The program is:

$$\max_{\left((a_{t,y^N}, c_{t,y^N}, l_{t,y^N})_{y^N \in \mathcal{Y}^N}, w_t, r_t, B_t\right)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{y^N \in \mathcal{Y}} \Big[S_{t,y^N} \Big(\omega_{y^N} \xi_{y^N} U(c_{t,y^N}, l_{t,y^N})$$
$$- (\lambda_{t,y^N} - \tilde{\lambda}_{t,y^N}(1+r_t)) \xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}) \Big) \Big],$$

subject to truncated-history constraints:

$$c_{t,y^N} + a_{t,y^N} = w_t (\chi y_0^N w_t)^{\phi} y_{y^N} + (1+r_t)\tilde{a}_{t,y^N} + T_t,$$
$$\tilde{a}_{t,y^N} = \sum_{\tilde{y}^N \in \mathcal{Y}^N} \Pi_{t,\tilde{y}^N y^N} \frac{S_{t-1,\tilde{y}^N}}{S_{t,y^N}} a_{t-1,\tilde{y}^N},$$

to aggregate constraints:

$$G_t + r_t (B_{t-1} + K_{t-1}) + w_t L_t = B_t - B_{t-1} - T_t + \left(1 - \kappa \frac{\pi_t^2}{2}\right) K_{t-1}^{\alpha} L_t^{1-\alpha} - \delta K_{t-1},$$
$$\Pi_t(\Pi_t - 1) = \frac{\varepsilon - 1}{\kappa}(\zeta_t - 1) + \beta \mathbb{E}_t \Pi_{t+1}(\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \frac{M_{t+1}}{M_t},$$
$$\zeta_t = \frac{1}{\alpha Z_t} (\tilde{r}_t^K + \delta) \left(\frac{K_{t-1}}{L_t}\right)^{1-\alpha},$$
$$A_t = K_t + B_t = \sum_{y^N \in \mathcal{Y}} S_{t,y^N} a_{t,y^N}, \; L_t = \sum_{y^N \in \mathcal{Y}} S_{t,y^N} y_0^N l_{t,y^N},$$

and to interest rate constraints:

$$(r_t - (1-\tau^K_{SS})\tilde{r}^K_t)A_{t-1} = (1-\tau^K)\left(\frac{\tilde{R}^{B,N}_{t-1}}{\Pi_t} - 1 - \tilde{r}^K_t\right)B_{t-1}, \tag{108}$$

$$\mathbb{E}_t\left[\frac{R^{B,N}_t}{\Pi_{t+1}}\right] = \mathbb{E}_t\left[1+\tilde{r}^K_{t+1}\right]. \tag{109}$$

D.2.2 First-order conditions

We denote by $\beta^t\Gamma_t$ and $\beta^t\Upsilon_t$ the Lagrange multipliers on (108) and (109), respectively. We define the pricing kernel M_t as follows:

$$M_t = \sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\xi_{y^N}\omega_{y^N}U_c(c_{t,y^N}, l_{t,y^N}).$$

Derivative with respect to $\tilde{R}^{B,N}_t$.

$$(1-\tau^K_{SS})\mathbb{E}_t\left[\frac{\Gamma_{t+1}}{\Pi_{t+1}}\right]B_t = \Upsilon_t\mathbb{E}_t\left[\frac{1}{\Pi_{t+1}}\right]. \tag{110}$$

Derivative with respect to $\tilde{r}^K_t$.

$$\Upsilon_{t-1} + \Gamma_t(1-\tau^K_{SS})\left(A_{t-1}-B_{t-1}\right) = \frac{\varepsilon-1}{\alpha\kappa}\gamma_t K_{t-1}M_t.$$

Derivative with respect to Π_t.

$$0 = \mu_t\kappa\left(\Pi_t-1\right) + \left(\gamma_t-\gamma_{t-1}\right)\left(2\Pi_t-1\right)M_t + \left(\Gamma_t\left(1-\tau^K\right)B_{t-1} - \Upsilon_{t-1}\right)\frac{\tilde{R}^{B,N}_{t-1}}{Y_t\Pi_t^2}. \tag{111}$$

Derivative with respect to r_t.

$$\sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\hat{\psi}_{t,y^N}\tilde{a}_{t,y^N} = -\sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\tilde{\lambda}_{t,y^N}\xi_{y^N}U_c(c_{t,y^N}, l_{t,y^N}) - \Gamma_t A_{t-1}.$$

Derivative with respect to w_t.

$$\mu_t\varphi\left(L_t - (1-\alpha)\frac{Y_t}{w_t}(1-\frac{\kappa}{2}(\Pi_t-1)^2)\right) = \sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\hat{\psi}_{t,y^N}y_0^N l_{t,y^N}$$
$$-\frac{(1-\alpha)\varphi}{w_t}\left((\gamma_t-\gamma_{t-1})\Pi_t(\Pi_t-1) + \frac{\varepsilon-1}{\kappa}\gamma_t\right)Y_tM_t.$$

Derivative with respect to B_t.

$$\begin{aligned}\mu_t = \beta\mathbb{E}_t &\left[\mu_{t+1}\left(1-\delta+\zeta_{t+1}^{-1}(\tilde{r}_{t+1}^K+\delta)\left(1-\frac{\kappa}{2}(\Pi_{t+1}-1)^2\right)\right)\right] \\ &- \alpha\beta\mathbb{E}_t\left[\left((\gamma_{t+1}-\gamma_t)\Pi_{t+1}(\Pi_{t+1}-1)+\frac{\varepsilon-1}{\kappa}\gamma_{t+1}\right)\frac{Y_{t+1}}{K_t}M_{t+1}\right] \\ &+ \frac{\varepsilon-1}{\kappa}\beta\mathbb{E}_t\left[\gamma_{t+1}\frac{\zeta_{t+1}Y_{t+1}}{K_t}M_{t+1}\right] \\ &+ \beta(1-\tau_{SS}^K)\mathbb{E}_t\left[\Gamma_{t+1}\left(\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}}-1-\tilde{r}_{t+1}^K\right)\right].\end{aligned}$$

Derivative with respect to a_{t,y^N}.

$$\begin{aligned}\hat{\psi}_{t,y^N} = \beta\mathbb{E}_t &\left[(1+r_{t+1})\sum_{\tilde{y}^N\in\mathcal{Y}^N}\Pi_{t,y^N\tilde{y}^N}\hat{\psi}_{t+1,\tilde{y}^N}\right] \\ &+ \beta\mathbb{E}_t\left[\Gamma_{t+1}\left(r_{t+1}-(1-\tau_{SS}^K)\left(\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}}-1\right)\right)\right].\end{aligned} \tag{112}$$

Derivative with respect to T_t.

$$\sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\hat{\psi}_{t,y^N} = 0. \tag{113}$$

It can observed that FOCs in the truncated model are very similar to the ones of the full-fledged model in Section A.3.

D.3 Program in the economy without time-varying labor tax

We keep the same notation as in the no-capital tax case of Section D.2. The program is also very similar to the one of Section D.2, except that the capital tax is time-varying, and the labor tax is not. We have a new constraint:

$$\alpha\frac{w_t}{1-\tau_{SS}^L}L_t = (1-\alpha)K_{t-1}(\tilde{r}_t^K+\delta),$$

while the constraints on r_t (fund no-profit condition (9)) and on $\tilde{R}_t^{B,N}$ (no-arbitrage constraint (10)) are not binding anymore.

Derivative with respect to $\tilde{r}_t^K$.

$$\Gamma_t^L = \frac{\varepsilon-1}{\alpha(1-\alpha)\kappa}\gamma_t M_t.$$

Derivative with respect to Π_t.

$$0 = \mu_t \kappa \left(\Pi_t - 1\right) + \left(\gamma_t - \gamma_{t-1}\right)\left(2\Pi_t - 1\right) M_t. \tag{114}$$

Derivative with respect to r_t.

$$\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} \tilde{a}_{t,y^N} = - \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \tilde{\lambda}_{t,y^N} \xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}).$$

Derivative with respect to w_t.

$$\begin{aligned}\mu_t \varphi \left(L_t - (1-\alpha)\frac{Y_t}{w_t}\left(1 - \frac{\kappa}{2}\left(\Pi_t - 1\right)^2\right)\right) &= \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} y_0^N l_{t,y^N} \\ - \frac{(1-\alpha)\varphi}{w_t}\left(\left(\gamma_t - \gamma_{t-1}\right)\Pi_t\left(\Pi_t - 1\right) + \frac{\varepsilon - 1}{\kappa}\gamma_t\right) & Y_t M_t + \Gamma_t^L \frac{\alpha}{1 - \tau_{SS}^L}(1+\varphi)L_t.\end{aligned} \tag{115}$$

Derivative with respect to T_t.

$$\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} = 0. \tag{116}$$

Derivative with respect to B_t: the public debt.

$$\begin{aligned}\mu_t = {} & \beta \mathbb{E}_t \left[\mu_{t+1}\left(1 - \delta + \zeta_{t+1}^{-1}(\tilde{r}_{t+1}^K + \delta)\left(1 - \frac{\kappa}{2}\left(\Pi_{t+1} - 1\right)^2\right)\right)\right] \\ & - \alpha\beta\mathbb{E}_t\left[\left(\left(\gamma_{t+1} - \gamma_t\right)\Pi_{t+1}\left(\Pi_{t+1} - 1\right) + \frac{\varepsilon - 1}{\kappa}\gamma_{t+1}\right)\frac{Y_{t+1}}{K_t} M_{t+1}\right] \\ & + \frac{\varepsilon - 1}{\kappa}\beta\mathbb{E}_t\left[\gamma_{t+1}\frac{\zeta_{t+1} Y_{t+1}}{K_t} M_{t+1}\right] - \beta(1-\alpha)\mathbb{E}_t\left[\Gamma_{t+1}^L(\tilde{r}_{t+1}^K + \delta)\right].\end{aligned} \tag{117}$$

Derivative with respect to a_t^i: the net saving of consumers.

$$\hat{\psi}_{t,y^N} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \sum_{\tilde{y}^N \in \mathcal{Y}^N} \Pi_{t,y^N \tilde{y}^N} \hat{\psi}_{t+1,\tilde{y}^N}\right]. \tag{118}$$

E Computing ξs and ωs using matrix representation

In this section, we provide closed-form formulas for preference multipliers ξs (Section E.1) and the Pareto weights ωs. We start with some notation:

- $\circ$ is the Hadamard product, $\otimes$ is the Kronecker product, $\times$ is the usual matrix product.

For any vector V, we denote by $diag\,(V)$ the diagonal matrix with V on the diagonal.

The matrix representation consists in stacking together the equations characterizing the steady-state, so as to provide a convenient matrix notation for solving the steady state. It also provides an efficient solution to compute the values for the coefficients (ξ_{y^N}) and (ω_{y^N}). The starting point is to observe that a history y^N can be seen as a N-length vector $\{y_{-N+1}, \ldots, y_0\}$ of elements of $\mathcal{Y}$. The number of histories is $N_{tot} = Y^N$. We can identify each history by an integer $k_{y^N} = 1, \ldots, N_{tot}$:

$$k_{y^N} = \sum_{k=0}^{N-1} N_{tot}^{-N+1-k}(y_k - 1) + 1, \tag{119}$$

which corresponds to an enumeration in base Y.

E.1 A closed-form formula for the ξs

Let $\mathbf{S}$ be the N_{tot}-vector of steady-state history sizes that is defined as $\mathbf{S} = (S_{k_{y^N}})_{k_{y^N}=1,\ldots,N_{tot}}$, by stacking history sizes for all histories using the enumeration given by (119). Similarly, let $\mathbf{a}$, $\mathbf{c}$, $\boldsymbol{\ell}$, $\boldsymbol{\nu}$, $\mathbf{U_c}$, $\mathbf{U_{cc}}$ be the N_{tot}-vectors of end-of-period wealth, consumption, labor supply, Lagrange multipliers, marginal utilities, and derivatives of the marginal utility, respectively. These vectors are known from the steady-state equilibrium of the Bewley model. Each element is defined as the truncation of the relevant variable computed using equation (49). We also define:

$$\mathbf{W} = w \begin{bmatrix} y_1 \\ \vdots \\ y_Y \end{bmatrix} \otimes 1_B, \; \mathbf{L} = \begin{bmatrix} y_1 \\ \vdots \\ y_Y \end{bmatrix} \otimes 1_B,$$

where 1_B is a vector of 1 of length Y^{N-1}. We define $\mathbb{P}$ as the diagonal matrix having 1 on the diagonal at y^N if and only if the history y^N is not credit constrained (i.e., $\nu_{y^N} = 0$), and 0 otherwise. We similarly define $\mathbb{P}^c = \mathbf{I} - \mathbb{P}$, where $\mathbf{I}$ is the $(N_{tot} \times N_{tot})$-identity matrix. Noting Π as the transition matrix across histories, we obtain the following steady-state relationships:

$$\mathbf{S} = \Pi \mathbf{S}, \tag{120}$$

$$\mathbf{S} \circ \mathbf{c} + \mathbf{S} \circ \mathbf{a} = (1+r)\Pi \left(\mathbf{S} \circ \mathbf{a}\right) + \left(\mathbf{S} \circ \mathbf{W} \circ \boldsymbol{\ell}\right), \tag{121}$$

$$\mathbb{P}^c \mathbf{a} = -\mathbb{P}^c \bar{a} 1_{N_{tot} \times 1}, \tag{122}$$

where (120) is the dynamics of history-sizes, (121) is history budget constraints and (122) history credit-constraints.

Euler equations (52) imply that we are looking for a vector $\boldsymbol{\xi}$ such that:

$$\mathbf{S} \circ \boldsymbol{\xi} \circ u'\left(\mathbf{c}\right) = \beta(1+r)\Pi^\top \left(\mathbf{S} \circ \boldsymbol{\xi} \circ u'\left(\mathbf{c}\right)\right) + \boldsymbol{\nu},$$

where the matrix $\Pi^\top$ (the transpose of Π) is used to make expectations about next period histories. We deduce from this equation that the vector $\boldsymbol{\xi}$ is defined by the following closed-form formula:

$$\boldsymbol{\xi} = \left[\mathbb{P}\left(diag\left(u'\left(\mathbf{c}\right)\right) - \beta(1+r)\Pi^{\top} \times diag\left(u'\left(\mathbf{c}\right)\right)\right) + \mathbb{P}^{c}\right]^{-1} \boldsymbol{\nu}. \tag{123}$$

E.2 Computing the Pareto weights ω

The first-order conditions (104)–(107) of the planner at the steady-state can respectively be written as follows using matrix notation:

$$\begin{aligned}
\mathbb{P}\hat{\psi} &= \beta\mathbb{P}\Pi^{\top}\hat{\psi}\left(1+r\right), \\
\mathbb{P}^{c}\lambda^{p} &= 0, \\
(\mathbf{y}\circ\mathbf{l})' \times \left(\mathbf{S}\circ\hat{\psi}\right) &= \mu L\left(1+\varphi\frac{w-F_L}{w}\right), \\
\mathbf{1}_{YL}^{\intercal} \times \left(\mathbf{S}\circ\hat{\psi}\circ\tilde{a}\right) &= \mathbf{1}_{YL}^{\intercal} \times \left(\mathbf{S}\circ\tilde{\lambda}\circ\xi_h\circ\mathbf{U}_c\right), \\
1 &= \beta(F_K+1), \\
\mathbf{S}\circ\hat{\psi} &= 0.
\end{aligned}$$

We can deduce after some algebra that the Pareto weights ω must satisfy the following two relationships for the planner's first-order conditions to hold:

$$\mathbf{H}^{\mathbf{1}}\omega = 0 \text{ and } \mathbf{H}^{2}\omega = 0,$$

where $\mathbf{H}^1 = \tilde{\mathbf{H}}^1\mathbf{D}(\mathbf{S}\circ\xi\circ\mathbf{U}_c)$, $\mathbf{H}^2 = \tilde{\mathbf{H}}^2\mathbf{D}\left(\mathbf{S}\circ\xi\circ\mathbf{U}_c\right)$ and:

$$\begin{aligned}
\tilde{\mathbf{H}}^1 &\equiv 1_{YL}^{\intercal} \times \left[D\left(\tilde{a}\right)\mathbf{N} + \mathbf{D}\left(\xi_h\circ\mathbf{U}_c\right)\Pi\mathbf{J}\right] - A\mathbf{Q}, \\
\mathbf{Q} &\equiv \left(\frac{\mathbf{y}\circ\mathbf{l}}{L\left(1+\varphi\frac{w-F_L}{w}\right)}\right)' \times \mathbf{N}, \\
\mathbf{N} &\equiv \mathbf{Id}_{\mathbf{YN}} - \mathbf{B}\left(\mathbb{P}^c + \mathbb{P}\mathbf{MB}\right)^{-1}\mathbb{P}\mathbf{M}, \\
\mathbf{J} &\equiv -\left(\mathbb{P}^c + \mathbb{P}\mathbf{MB}\right)^{-1}\mathbb{P}\mathbf{M},
\end{aligned}$$

as well as:

$$\begin{aligned}
\mathbf{M} &\equiv \mathbf{Id}_{\mathbf{YN}} - \beta\left(1+r\right)\Pi^{S} + \beta\frac{r-F_K}{L\left(1+\varphi\frac{w-F_L}{w}\right)}\mathbf{S}\left(\mathbf{y}\circ\mathbf{l}\right)', \\
\mathbf{\Pi}^{S} &\equiv \mathbf{DS}\Pi^{\top}D^{-1}\mathbf{S}, \\
\mathbf{B} &\equiv \mathbf{D}\left(\xi_h\circ\mathbf{U}_{cc}\right)\left(\left(1+r\right)\Pi - \mathbf{Id}_{YL}\right), \\
\tilde{\mathbf{H}}^2 &\equiv \mathbf{1}_{YL}^{\intercal} \times \mathbf{N} - \mathbf{Q}.
\end{aligned}$$

The Pareto weights are finally given as a solution of the following minimization problem:

$$\min_{\omega} \left\| \omega - \frac{1}{N_{tot}} \mathbf{1}_{N_{tot}\times 1} \right\|^2,$$
$$\text{s.t. } \mathbf{H^1}\omega = 0, \ \mathbf{H}^2\omega = 0, \text{ and } \sum_{y^N} \omega_{y^N} = 1.$$

The solution linear-quadratic problem can be found by simple linear algebra ($\mathbf{H}^1$ and $\mathbf{H}^2$ are known), introducing Lagrange multipliers on each constraint.

F Comparison of the truncation method with other solution methods

We simulate the same model with three different solution methods. We assume full price flexibility and that there is no public spending need ($G = 0$). Taxes are thus set to 0. Table 4 presents the calibrated model parameters. The labor process is the same as in the simulations of Section 5.1. The three simulation methods solve for transitory dynamics around the steady-state allocation, which is the same for the three solution methods.

Parameter	Description	Value
	Preference and technology	
β	Discount factor	0.99
α	Capital share	0.36
δ	Depreciation rate	0.025
$\bar{a}$	Credit limit	0.0
χ	Scaling param. labor supply	0.068
φ	Frisch elasticity labor supply	0.5
	Shock process	
ρ_z	Autocorrelation TFP	0.95
σ_z	Standard deviation TFP shock	0.31%
ρ_y	Autocorrelation idio. income	0.99
σ_y	Standard dev. idio. income	14.0%

Table 4: Parameter values in the baseline calibration. See text for descriptions and targets.

Table 5 presents the first- and second-order moments of model simulations with three methods: (i) the histogram method developed by Rios-Rull (2001), Reiter (2009), and Young (2010) (column "Reiter"), (ii) the method of Boppart et al. (2018) and Auclert et al. (2019) (column "BKM"), (iii) the truncation method with $N = 5$ (column "Trunc"). We report the simulation outcomes for several variables. We report the mean (Mean) and, to allow for comparison, the standard deviation normalized by the mean of the relevant variable (rows Std). For example, the variable "Std" for the variable GDP should be understood as $std(GDP)/mean(GDP)$. We also report two autocorrelations. The three solution methods for the incomplete market economy yield very close outcomes. Note that we have chosen to use the same truncation length ($N = 5$) as in the main paper. Increasing the truncation length would improve the accuracy, but at the cost of a higher computing time. This would be manageable in this comparison exercise, but less so when solving for the optimal Ramsey program. Finally, the incomplete-market economy generates very

		Reiter	BKM	Trunc
GDP Y	Mean	1.397	1.397	1.397
	Std	0.017	0.017	0.016
Capital K	Mean	18.34	18.34	18.34
	Std	0.015	0.015	0.015
Aggregate consumption C	Mean	0.939	0.939	0.939
	Std	0.014	0.014	0.013
Aggregate labor$L$	Mean	0.328	0.328	0.328
	Std	0.006	0.006	0.005
$corr(C, Y)$		0.954	0.958	0.951
$corr(Y, Y_{-1})$		0.973	0.974	0.971

Table 5: Method Comparison

different allocations from the complete-market one. In this last case, the equilibrium interest rate is $1 + r = 1/\beta$, the capital stock 11.64 (instead of 18.34), and the GDP is 1.135 (instead of 1.397). This shows that the results of Table 5 are not driven by the fact that heterogeneity has a little role to play in the IM economy.

G Further comparison results

We present here additional comparison elements with respect to the literature. This extends the comparison of Section 6. We start with presenting how we solve for the time-0 problem that we use in this comparison.

G.1 Solving for the time-0 and timeless problems without perturbation

When the initial distribution differs from the steady-state one (as it is the case in Section 6) we cannot rely on perturbation technique. The resolution implies then the computation of a transition path under perfect foresight from initial conditions to the long-run steady state. This computation relies on the resolution of a non-linear system, for which we impose initial conditions and that the steady state is reached after 150 periods of time. We check that our results are not too sensitive to this convergence within 150 periods. From a technical point of view, we rely on the non-linear "`simul`" command of Dynare for this computation.

For the time-0 problem, the initial values of the Lagrange multipliers for Euler equations and for the Phillips curve are set to 0. This reflects the fact that the planner is not constrained by its past commitments.

For a timeless model, the initial values of Lagrange multipliers (for Euler equations and the Phillips curve) are not set to zero but to their steady-state values.

G.2 Additional comparison elements

We extend the discussion of Section 6 and report additional results regarding the comparison with the literature in the case of the time-0 problem. More precisely, we conduct a comparative statics on several model parameters by measuring the change in inflation on impact. We focus on three parameters: (i) the cost of price adjustment κ, (ii) the borrowing limit $\bar{a}$, (iii) a time-varying idiosyncratic risk, as in Acharya et al. (2020).

We model the time-varying idiosyncratic productivity of the last point as follows. We denote by $\bar{y}$ the mean productivity across agents type, which is constant by construction of the productivity process. We recall that the aggregate risk is z_t (and aggregate TFP is $Z_t = \exp(z_t)$). The time-varying productivity in state i and date t is defined as:

$$y_{i,t} = y_i - \phi^{TV} \times z_t \times (y_i - \bar{y}), \tag{124}$$

where y_i is the constant productivity level in state i and ϕ^{TV} a scalar. A positive ϕ^{TV} generates a countercyclical time-varying mean-preserving spread in idiosyncratic productivity. When aggregate productivity falls (z_t is negative), productivity levels below the mean $\bar{y}$ fall, while productivity levels above the mean increase. With specification (124), it can be checked that the proportional change in the standard deviation of idiosyncratic risk is $-\phi^{TV} z_t$.

Table 6 presents our comparative static results. We consider a negative productivity shock, corresponding to a 1% fall in TFP on impact. The parameter values are in columns, while the economy is in rows. The last column reports the change in inflation on impact, in percent.

Economy	κ	$\bar{a}$	ϕ^{TV}	$\Delta\pi$ (%)
(0)	30	5	0	1.19
(i)	60	5	0	0.79
(ii)	30	2	0	0.78
(iii)	30	5	3	1.25

Table 6: Comparative statics about the inflation on impact.

Economy (0) is the benchmark economy, presented in the main text. The optimal inflation increases by 1.19% on impact, as can be seen in Figure 3. Economy (i) has a higher price adjustment cost κ. The same productivity shock generates a smaller increase in inflation, equal to 0.79%. As can be expected, the higher the cost of adjusting prices, the lower the optimal inflation volatility. Economy (ii) has a stricter borrowing limit – agents can borrow less. Inflation volatility is reduced to 0.78%, which reflects the fact that the planner has smaller incentives to use inflation for redistribution purposes. Indeed, because of a smaller outstanding amount of nominal debt in the economy, inflation generates less redistribution on impact through the Fisher effect discussed by Nuño and Thomas (2020). This result is consistent with the fact that inflation volatility increases when the borrowing limit approaches the natural borrowing limit as in Bhandari et al. (2020). Economy (iii) is the benchmark economy but with a counter-cyclical income risk (as in equation (124)). Consistently with Acharya et al. (2020), we find that this increases inflation volatility, which reaches 1.25% here.