Optimal Fiscal Policy With Heterogeneous Agents and Capital: Should We Increase or Decrease Public Debt and Capital Taxes?*

François Le Grand Xavier Ragot†

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Abstract

We analyze optimal fiscal policy in a heterogeneous-agent model with capital accumulation and aggregate shocks, where the government uses public debt, capital tax, and progressive labor tax to finance public spending. First, we provide conditions on utility functions and social welfare functions for the existence of a steady-state equilibrium with positive capital tax; we identify three conditions: a non-first-best condition, a Straub–Werning condition, and a Laffer condition. Second, we show theoretically and quantitatively that the optimal dynamics of public debt depend crucially on the persistence of the positive public spending shock, for a given net present value of the shock; the optimal public debt increases (resp. decreases) when the persistence of the shock is low (resp. high) because of a trade-off between consumption smoothing and the reduction of distortions. Third, labor tax progressivity increases after such a shock.

Keywords: Heterogeneous agents, optimal fiscal policy, public debt

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†Le Grand: Rennes School of Business and ETH Zurich; francois.le-grand@rennes-sb.com. Ragot: SciencesPo, OFCE, and CNRS; xavier.ragot@gmail.com.

1 Introduction

What is the optimal level of public debt? Should it increase or decrease when public spending is increasing? After a positive public spending shock, should the government temporarily increase capital tax or other distorting taxes, affecting the progressivity of the tax system? These old questions are likely to remain relevant in the coming years in many countries, as policymakers increasingly discuss additional public spending for climate change or military purposes. Such questions require considering both distorting and redistributive effects of tax changes, while accounting for general equilibrium effects. Heterogeneous-agent models in the tradition of the Bewley–Huggett–Imrohoroğlu–Aiyagari literature (Bewley, 1983; Imrohoroğlu, 1989; Huggett, 1993; Aiyagari, 1994; Krusell and Smith, 1998) are relevant tools for analyzing such questions because they generate a realistic amount of heterogeneity together with general and dynamic equilibrium effects. However, after seminal papers investigating optimal fiscal policy in these environments (Aiyagari, 1995; Aiyagari and McGrattan, 1998), the literature has mainly moved toward a positive analysis and little is known about the optimal dynamics of public debt and capital tax, because of the theoretical and computational difficulties of solving for optimal fiscal policy with aggregate shocks.

This paper analyzes optimal fiscal policy in heterogeneous-agent models, considering capital accumulation, progressive labor income taxation, capital tax, public debt, and aggregate shocks. The only frictions considered are incomplete markets for idiosyncratic risk, occasionally binding credit constraints (which appear to be the key friction), and the given set of fiscal instruments. In particular, the planner cannot use lump-sum taxes, which are known to possibly restore Ricardian equivalence in some environments (Bhandari et al., 2017). Considering capital accumulation allows characterizing the optimal dynamics of capital tax and discussing its relationship with the results of the vast Chamley–Judd literature on optimal capital taxation. Even though our analysis admittedly abstracts from other frictions, such as nominal rigidities or frictional labor markets, we identify new mechanisms that will also be present in more-general environments.[1]

Characterizing the optimal Ramsey allocation with full commitment in this environment provides two sets of results. We show that there exist steady-state equilibria where both optimal public debt and capital tax are positive, which is a necessary first step before

[1] Considering only price or wage stickiness would generate the same allocation as in our economy because the sole role of optimal monetary policy is price stability given the set of fiscal instruments that we consider (see LeGrand et al., 2022).

studying the effect of aggregate shocks. This was claimed by Aiyagari (1995) but with no formal proof of existence to the best of our knowledge, while Chien and Wen (2022a) claimed that such an equilibrium does not exist in some standard models. In addition, Straub and Werning (2020) showed that the steady state may not exist even in a complete-market environment. The existence of a steady state actually appears to be subtle. To make the algebra transparent, we provide a formal proof in an environment with deterministic income fluctuations. In the case of a utilitarian planner, we prove the existence of a steady-state equilibrium with optimal positive capital tax for a utility function without wealth effect on the labor supply (as in Aiyagari, 1995). We also prove it for the King-Plosser-Rebelo (KPR) utility function, as well as for different classes of utility functions separable in consumption and leisure (Stone–Geary, Fishburn, and CARA). Noticeably, the equilibrium does *not* exist for the standard CRRA separable utility function, which is consistent with the claim of Chien and Wen (2022a) and the numerical investigation of Auclert et al. (2022). When the planner is not utilitarian—implying that it can weight agents according to their idiosyncratic characteristics, as in Heathcote and Tsujiyama (2021) for instance—then we prove the equilibrium existence even in the standard CRRA utility case. In all cases, the existence of the steady-state equilibrium relies on three independent conditions: a non-first-best condition, a so-called Straub–Werning condition, and a standard Laffer condition. The Straub–Werning condition elaborates on Straub and Werning (2020) and states that the public spending must be low enough to ensure a stationary steady state and avoid the planner choosing to continuously decrease the capital stock (although they could levy enough resources at the steady state). In addition to these three conditions for the steady-state equilibrium, a fourth condition, a Blanchard–Kahn condition, ensures the equilibrium stability. All these conditions can be shown to be compatible with each other, in the sense that an equilibrium can actually exist. Also, we show that in addition to positive capital tax, the optimal fiscal system can exhibit (for some parameterization) a positive public debt that absorbs the excess savings. From this investigation, we deduce that heterogeneous-agent models are relevant tools for normative fiscal analysis around a well-defined steady state, even though it requires some conditions on the utility function or the social welfare function (SWF) of the planner.

The second set of results concerns the optimal dynamics of fiscal policy after a positive public spending shock. First, we find that for a given net present value (NPV) of public spending, public debt increases (resp. decreases) when the persistence of the shock is low (resp. high). Consequently, the persistence of the shock is a key driver of the optimal dynamics of public debt. We prove this result analytically in the simple model and then show that it holds in the quantitative model. We thus show that the key friction for our result is occasionally binding credit constraints and not aggregate or idiosyncratic risk. Furthermore, capital tax increases on impact, while labor tax barely moves. The intuition for these results is that, contrary to the complete-market case where agents initially hold some capital, the capital tax is not used to fully front-load the adjustment, because taxing capital reduces the ability of agents to self-insure when markets are incomplete. In addition, in this type of model, public debt converges back to its optimal steady-state value for any transitory public spending shock (when it exists). Consequently, when the persistence is high, a transitory increase in public debt would require a welfare-reducing highly persistent increase in taxes to finance public spending and to reduce public debt. Therefore, the optimal policy is to front-load the adjustment and to reduce public debt temporarily. When the persistence is low, the increase in public debt improves consumption smoothing and a small increase in taxes is enough to ensure public debt convergence.

The quantitative model features a realistic income risk, nonlinear labor tax as in Heathcote et al. (2017), and a general SWF. The results of the quantitative model are consistent with the ones of the theoretical model: Public debt increases when the persistence of the public spending shock is low, and decreases otherwise. The quantitative model generates additional results: labor-tax progressivity and capital tax both increase after a positive public spending shock, but the increase is smaller when the persistence of the shock is higher. Public debt also quantitatively exhibits much more persistent deviations than other variables. These dynamics generate a response of the market allocation that is relatively close to the first-best dynamics.

Computing Ramsey optimal fiscal policies with many instruments in the presence of aggregate shocks is difficult. We use a factorization method introduced by Marcet and Marimon (2019) and applied to heterogeneous-agent models by LeGrand and Ragot (2022a). This approach allows for occasionally binding credit constraints, which we show to be the key friction. Because we are interested in the dynamics of the fiscal system in the quantitative model, we first estimate an SWF consistent with the observed US tax system by solving the *inverse optimum taxation problem*, as in Heathcote and Tsujiyama (2021) among others. Once our model roughly reproduces at the steady state the US tax system and allocation, we then compute the optimal responses of capital tax, labor tax progressivity, and public debt after public spending shocks with different persistences, around a well-defined steady state.

This paper is related to the literature on optimal fiscal policy in heterogeneous-agent models.[2] As mentioned above, the existence of well-defined Ramsey equilibria is still an open question. Conesa et al. (2009) considered transitions with constant instruments. Chien and Wen (2022a) and Auclert et al. (2022) find that the Ramsey steady-state equilibrium does not exist for separable CRRA utility function. Dyrda and Pedroni (2022) solved quantitatively for optimal policy by considering the full path of the instruments and using a KPR utility function. Aiyagari (1995) and Açikgöz et al. (2018) analyze optimal public debt when there is no wealth effect for labor supply. Bassetto and Cui (2020) study an environment where public debt can relax the credit constraint of the producer. They find that steady-state capital taxes are positive, when public debt is constrained to be at the top of the Laffer curve. We prove that equilibria with positive capital tax and public debt can exist in standard incomplete market economies, depending on the utility function.[3]

Analyzing optimal fiscal policy in such an environment obviously relies heavily on results in complete-market economies for the idiosyncratic risk.[4] Compared to these environments, incomplete-market models allow consideration of optimal positive steady-state capital tax and redistribution.

More generally, recent literature reports the development of tools for solving for optimal policies with heterogeneous agents involving mostly monetary policy, for which the steady-state allocation is simpler to characterize (e.g., Bhandari et al., 2021; Acharya et al., 2022; LeGrand et al., 2022; Nuño and Thomas, 2022, among others). We use the truncation approach of LeGrand and Ragot (2022a), using the refinement of LeGrand and Ragot (2022c) to solve for the curse of dimensionality. This method allows one to easily simulate models with many instruments and aggregate shocks. Because it is relatively new, we summarize it below.

The rest of this paper is organized as follows. In Section 2, we present the general environment. In Section 3, we present simplifying assumptions and solve the tractable model, and in Section 4 we simulate the general model. In Section 5, we present some empirical evidence for heterogeneity in the persistence of public spending, and finally we conclude in Section 6.

[2]A large literature provides a positive analysis of fiscal policy in heterogeneous-agent models (e.g., Floden, 2001; Heathcote, 2005; Rohrs and Winter, 2017; Ferriere and Navarro, 2020, among many others).

[3]Albanesi and Armenter (2012) provide general sufficient conditions for the optimal steady-state capital tax to be 0 in many environments. These conditions are are not fulfilled in our setup for relevant cases, because the planner needs to use distorting labor tax to finance public spending when the capital tax is 0, preventing the economy to converge to the first-best allocation.

[4]For relevant contributions, see Barro (1979); Chari et al. (1994); Farhi (2010); Bassetto (2014); Chari et al. (2020); Straub and Werning (2020); Collard et al. (2023) among others.

2 The Environment

Time is discrete and indexed by $t = 0, 1, \ldots$, and the economy is populated by a continuum of agents distributed along a set I with measure ℓ. We follow Green (1994) and assume that the law of large numbers holds. The economy features production and a benevolent government that raises distorting taxes to finance an exogenous stream of public spending.

Risks. The aggregate shock solely affects public spending denoted by $(G_t)_{t\geq 0}$ and is therefore assimilated to a public spending shock. Furthermore, we assume that the whole path of public spending $(G_t)_{t\geq 0}$ becomes known to all agents in period 0. We will solve for the optimal adjustment of the economy after such a shock, also known as an MIT shock.[5]

Agents face an uninsurable productivity risk. Individual productivity levels, denoted by y, follow independent first-order Markov chains, whose state space is the finite set $\mathcal{Y}$ and whose transition matrix is denoted by Π. We assume that the Markov chain admits a stationary distribution that is denoted by the vector S_y, verifying $S_y = (S_y)^\top \Pi$.[6] In period t, when the productivity of agent i is y_t^i, they will earn a before-tax labor wage $\tilde{w}_t y_t^i l_t^i$, where l_t^i denotes their labor supply and $\tilde{w}_t$ the before-tax hourly wage. Their whole history of shocks up to t is denoted by $y^{i,t} := \{y_0^i, ..., y_t^i\}$.

Finally, it is assumed that agents enter the economy at date 0 with an endowment of wealth and productivity $(a_{-1}^i, y_0^i)_i$ drawn from a distribution Λ_0.

Production. The production sector is standard. The consumption–investment goods of the economy are produced by a profit-maximizing representative firm. At any date t, the firm production function combines labor L_t and capital K_{t-1}—which must be installed one period in advance—to produce Y_t units of the consumption goods. The production function is assumed to be of the Cobb–Douglas type featuring constant returns to scale and capital depreciation. The total factor productivity is normalized to one. Formally, the production is defined as

$$Y_t = F(K_{t-1}, L_t) = K_{t-1}^{\alpha} L_t^{1-\alpha} - \delta K_{t-1},$$

where $\alpha \in (0, 1)$ is the capital share and $\delta \in (0, 1)$ is the capital depreciation rate.

[5] It is known that one can derive a first-order approximation of the dynamics of the model in the presence of aggregate shocks, using the information obtained from MIT shocks (Boppart et al., 2018; Auclert et al., 2021).

[6] In the quantitative analysis of Section 4, the Markov chain can be shown to be irreducible and aperiodic, hence n^y exists and is unique.

The firm rents labor and capital at respective factor prices $\tilde{w}_t$ and $\tilde{r}_t$. The profit maximization conditions of the firm imply the following expressions for factor prices:

$$\tilde{w}_t = F_L(K_{t-1}, L_t) \text{ and } \tilde{r}_t = F_K(K_{t-1}, L_t). \tag{1}$$

Assets. In addition to capital, the economy also features public debt, whose size is denoted by B_t in period t. Public debt consists of one-period bonds issued by a benevolent government, which are assumed to be default-free. Because of our assumption of MIT shocks, there is no aggregate risk in this economy. Both capital and public debt are thus perfect substitutes, and no-arbitrage implies that they must pay the same after-tax return. Agents' savings are restricted to remain greater than an exogenous limit $-\underline{a} \leq 0$.

Period 0. We assume that the economy starts in period -1 with a given distribution of individual saving $(a^i_{-1})_i$, a given amount of public debt B_{-1} and a given amount of capital K_{-1}, verifying $K_{-1} + B_{-1} = \int_i a^i_{-1} \ell(di)$. The MIT shock is the amount of public spending $(G_t)_{t\geq 0}$, which is revealed at period -1 before households actually perform their portfolio choice at period -1. As a consequence, and as there is no aggregate risk, no arbitrage implies that the two assets must have the same after-tax return in all periods, including period 0. The before-tax real interest rate between period -1 and period 0 is denoted $\tilde{r}_0$, and the MIT shock affects the allocation from period 0 onward.

Government. A benevolent government has to finance the exogenous stream of public spending $(G_t)_{t\geq 0}$ by levying distorting taxes on capital and labor and issuing public debt. The tax on capital is linear with a rate $(\tau^K_t)_{t\geq 0}$, and the tax on labor income is assumed to be nonlinear and possibly time-varying. We denote by $T_t(\tilde{w}yl)$ the amount of labor tax paid at date t by an agent earning the labor income $\tilde{w}yl$. We follow Heathcote et al. (2017) (hereinafter HSV) and consider the following functional form:

$$T_t(\tilde{w}yl) := \tilde{w}yl - \kappa_t(\tilde{w}yl)^{1-\tau_t}, \tag{2}$$

where κ_t captures the level of labor taxation and τ_t the progressivity. Both parameters are assumed to be time-varying and will be the planner's instruments in the general model. When $\tau_t = 0$, labor tax is linear with rate $1 - \kappa_t$; oppositely, $\tau_t = 1$ corresponds to full income redistribution, where all agents earn the same post-tax income κ_t. Functional form (2) combined with the linear capital tax allows one to realistically reproduce the actual US system and its progressivity (see Heathcote et al., 2017 or Ferriere and Navarro, 2020).[7]

Using previous elements, the government budget constraint can thus be written as, $t \geq 0$

$$G_t + (1 + \tilde{r}_t)B_{t-1} = \int T_t(\tilde{w}_t y^i l_t^i)\ell(di) + \tau_t^K \tilde{r}_t(B_{t-1} + K_{t-1}) + B_t. \tag{3}$$

To simplify the government budget constraint, in the spirit of Chamley (1986) we introduce generalized post-tax factor prices, which are denoted without a tilde. We define the gross and net interest rates r_t and R_t, respectively, and the wage rate w_t as

$$w_t := \kappa_t(\tilde{w}_t)^{1-\tau_t}, \tag{4}$$

$$R_t := 1 + r_t = 1 + (1 - \tau_t^K)\tilde{r}_t. \tag{5}$$

The model can be expressed analytically using the pair of post-tax rates (R_t, w_t) rather than pre-tax ones $(\tilde{r}_t, \tilde{w}_t)$, which simplifies the algebra. The values of the fiscal instruments τ_t^K, κ_t, and τ_t can then be recovered from the allocation. Taking advantage of the homogeneity of the production function as in Chamley (1986), the governmental budget constraint (3) becomes

$$G_t + R_t B_{t-1} + (R_t - 1)K_{t-1} + w_t \int_i (y_t^i l_t^i)^{1-\tau_t}\ell(di) = F(K_{t-1}, L_t) + B_t. \tag{6}$$

Agents' program and resource constraints. At each date t, agents consume goods in quantity c_t, supply labor in quantity l_t, and save an amount a_t. They derive an instantaneous utility from consumption and labor supply denoted by $U(c_t, l_t)$; the utility function will be specified later. The discount factor is constant and denoted by $\beta \in (0, 1)$.

Using the post-tax rate definition (4), the post-tax labor income amounts to $\tilde{w}_t y_t^i l_t^i - T_t(\tilde{w}_t y_t^i l_t^i) = w_t(y_t^i l_t^i)^{1-\tau_t}$, while post-tax capital income is equal to $R_t a_{t-1}^i$. Formally, the program of agent i endowed with the given initial wealth a_{-1}^i can be expressed as

$$\max_{\{c_t^i, l_t^i, a_t^i\}_{t \geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t^i, l_t^i), \tag{7}$$

$$c_t^i + a_t^i = R_t a_{t-1}^i + w_t(y_t^i l_t^i)^{1-\tau_t}, \tag{8}$$

$$a_t^i \geq -\underline{a},\ c_t^i \geq 0,\ l_t^i \geq 0. \tag{9}$$

[7] The literature uses either the combination of a linear tax and a lump-sum transfer (e.g., Dyrda and Pedroni, 2022; Açikgöz et al., 2018) or the HSV structure. Heathcote and Tsujiyama (2021) showed that the HSV structure is quantitatively more relevant, and opting for the HSV tax structure enables us to discuss the dynamics of optimal tax progressivity following a public spending shock.

Note that because of our assumption of MIT shocks, the expectation operator in (10)—as well as in the rest—solely concerns idiosyncratic shocks. The constraint (8) is the budget constraint, and inequalities (9) are the credit constraint and the non-negativity constraints.

The solution of the previous program is a set of policy rules defined over the product space of productivity histories and initial asset holdings: $c_t : \mathcal{Y}^t \times [-\bar{a}; +\infty) \to \mathbb{R}^+$, $a_t : \mathcal{Y}^t \times [-\bar{a}; +\infty) \to [-\bar{a}; +\infty)$, and $l_t : \mathcal{Y}^t \times [-\bar{a}; +\infty) \to \mathbb{R}^+$. To lighten the notation, we will simply write c_t^i, a_t^i, and l_t^i (instead of $c_t(y_i^t, a_{-1}^i)$, $a_t(y_i^t, a_{-1}^i)$, and $l_t(y_i^t, a_{-1}^i)$) and use the same notation for all variables.[8]

Denoting by $\beta^t \nu_t^i \geq 0$ the Lagrange multiplier on the agent's credit constraint, the consumption Euler equation can be written as

$$U_c(c_t^i, l_t^i) = \beta \mathbb{E}_t \left[R_t U_c(c_{t+1}^i, l_{t+1}^i) \right] + \nu_t^i, \tag{10}$$

where U_c and U_l denote the derivatives of U with respect to consumption and labor, respectively.

The first-order condition (FOC) on labor is

$$-U_l(c_t^i, l_t^i) = (1 - \tau_t) w_t (y_t^i l_t^i)^{-\tau_t} U_c(c_t^i, l_t^i), \tag{11}$$

and the clearing of financial and labor markets implies the following equalities:

$$A_t = K_t + B_t \text{ and } \int y_t^i l_t^i \ell(di) = L_t. \tag{12}$$

The clearing of the goods market reflects the fact that the sum of aggregate consumption, public spending, and new capital stock balances the output production and past capital:

$$\int_i c_t^i \ell(di) + G_t + K_t = K_{t-1} + F(K_{t-1}, L_t). \tag{13}$$

We can now formulate the standard equilibrium definition.

Definition 1 (Competitive equilibrium) *A competitive equilibrium is a collection of individual variables* $(c_t^i, l_t^i, a_t^i)_{t \geq 0, i \in \mathcal{I}}$*, aggregate quantities* $(K_t, L_t, Y_t)_{t \geq 0}$*, prices* $(\tilde{w}_t, \tilde{r}_t)_{t \geq 0}$*, fiscal policy* $(\tau_t^K, \kappa_t, \tau_t, B_t)_{t \geq 0}$*, and public spending* $(G_t)_{t \geq 0}$ *such that for an initial distribution of wealth and productivity* $(a_{-1}^i, y_0^i)_{i \in \mathcal{I}}$ *and for initial values of capital stock and public debt verifying* $K_{-1} + B_{-1} = \int_i a_{-1}^i \ell(di)$*, we have the following. i) Given prices, individual*

[8]Hence, the aggregation of the variable X_t in period t will be written as $\int_i X_t^i \ell(di)$ instead of the more involved explicit notation $\int_{a_{-1}} \sum_{y^t \in \mathcal{Y}^t} \theta_t(y^t) X(y^t, a_{-1}) d\Lambda_0(a_{-1}, y_0)$, where $\theta_t(y^t)$ is the probability of occurrence of history y^t in period t.

strategies $(c_t^i, l_t^i, a_t^i)_{t\geq 0, i\in\mathcal{I}}$ *solve the agent's optimization program in equations (7)–(9). iii) Financial, labor, and goods markets clear: for any* $t \geq 0$*, equations (12) and (13) hold. iii) The government budget is balanced: equation (3) holds for all* $t \geq 0$*. iv) The pre-tax factor prices* $(\tilde{w}_t, \tilde{r}_t)_{t\geq 0}$ *are consistent with the firm's program (1).*

A stationary equilibrium is a competitive equilibrium where all aggregate variables have converged toward constant values.

The Ramsey equilibrium. To consider general cases, we use a flexible form for the SWF embedding the standard utilitarian criterion. We assume that the planner considers a weighted sum of agents' utilities, where the agent's weight at date t depends on their current productivity; this weight is denoted by $\omega(y_t^i)$. The utilitarian case corresponds to $\omega(y) = 1$ for all y.[9] This specification is similar to the approach in Heathcote and Tsujiyama (2021), and it was used in an intertemporal setting by LeGrand et al. (2022), Dávila and Schaab (2022), and McKay and Wolf (2022) to deviate from the utilitarian case. Formally, the SWF that corresponds to the planner's aggregate welfare criterion can be expressed as

$$W_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) U(c_t^i, l_t^i) \ell(di) \right]. \tag{14}$$

The Ramsey program with full commitment consists in finding the fiscal policy that corresponds to the competitive equilibrium with the highest aggregate welfare for the SWF under consideration.

First-best outcome. In many cases studied below, the outcome of the Ramsey allocation will be compared to the first-best outcome. The latter is the solution of the program maximizing aggregate welfare subject to the resource condition, or formally

$$\max_{((c_{i,t}, l_{i,t})_{i\in I}, L_t, K_t)_{t\geq 0}} W_0 \tag{15}$$

$$\int_i c_t^i \ell(di) + G_t + K_t = K_{t-1} + F(K_{t-1}, L_t), \tag{16}$$

$$\int y_t^i l_t^i \ell(di) = L_t, \ K_{-1} \text{ given.}$$

[9] A more general specification would consider the weights as being a function of the whole history of idiosyncratic shocks for each agent: $\omega_t(y^{i,t})$. However, a generalization is not needed in the quantitative analysis, and so we follow the simpler formulation.

3 Analyzing Existence and Dynamics in a Simple Model

We now study optimal fiscal policy in a simple model to provide analytical results. The simple model is based on some simplifications (e.g., linear labor tax) but more importantly on the assumption of deterministic income fluctuations between two productivity levels, as introduced by Woodford (1990). The gain is analytical solutions—including a characterization of the Ramsey allocation—but also the proof that positive capital tax and public debt are the result of credit constraints and not of incomplete insurance markets.

First, we list the simplifying assumptions introduced in the environment of Section 2.

Assumption A 1. *The labor tax is linear: in (2) we set* $\tau_t = 0$ *and denote* $\tau_t^L := 1 - \kappa_t$ *such that* $T_t(\tilde{w}yl) := \tau_t^L \tilde{w}yl$.

2. *The credit constraint is set to zero:* $\underline{a} = 0$.

3. *There are only two productivity levels, i.e.,* 0 *and* 1. *There is initially a unit mass of agents in each state, and the transition matrix is anti-diagonal:* $\Pi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

To summarize, the planner has three instruments, i.e., a linear capital tax, a linear labor tax, and public debt, and agents face deterministic productivity shock subject to non-negativity of savings. For simplicity, we refer to the two types of agents according to their current employment status: "employed" (subscript e, when productivity is 1) and "unemployed" (subscript u, when productivity is 0). Consistent with this two-agent economy, we also renormalize the total population size to 2. We discuss the equilibrium existence and its characterization depending on the specification of the utility function.

The remainder of this section is organized as follows. In Section 3.1, we study existence conditions. We start by providing a detailed analysis of equilibrium existence and properties in the case of a Greenwood–Hercowitz–Huffman (GHH) utility function and a utilitarian planner. We then consider the KPR utility function. We extend our analysis to the case of separable utility functions while maintaining the assumption of a utilitarian planner. We present conditions for an equilibrium to exist in the general case, and we show that they can hold in the cases of CARA, Fishburn, and Stone–Geary utility functions, while they never do for CRRA utility functions. We conclude this subsection by showing that equilibrium exists in the CRRA case when we depart from the utilitarian criterion and consider productivity-contingent weights in the SWF. In Section 3.2, we analyze the dynamics of public debt in the simple model.

3.1 Existence Conditions for the Steady-State Equilibrium

GHH Utility Function and Utilitarian Planner

We start by considering a utility function without wealth effect on the labor supply, as in the initial contribution by Aiyagari (1995) but also in Diamond (1998) to obtain analytical results. The instantaneous utility function U is of the GHH type with the log specification:

$$U(c,l) := u\left(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}\right) \text{ and } u(c) := \log(c), \tag{17}$$

where $\varphi > 0$ is the Frisch elasticity of labor supply, and $\chi > 0$ scales labor disutility. The utilitarian planner sets the two weights to 1.

Structure of the economy. In any non-trivial equilibrium, employed agents cannot be credit-constrained at any date, otherwise unemployed agents would consume zero, as they do not earn any labor income. Thus there are only two possible steady-state equilibria: one in which unemployed agents are not credit-constrained, and one in which they are. Thus, the Ramsey program can be written as follows:

$$\max_{(c_{e,t},c_{u,t},a_{e,t},a_{u,t},l_{e,t}B_t,K_t,R_t,w_t)} \sum_{t=0}^{\infty} \beta^t \left(u\Big(c_{e,t} - \chi^{-1}\frac{l_{e,t}^{1+1/\varphi}}{1+1/\varphi}\Big) + u(c_{u,t}) \right) \tag{18}$$

$$\text{s.t. } u'\Big(c_{e,t} - \chi^{-1}\frac{l_{e,t}^{1+1/\varphi}}{1+1/\varphi}\Big) = \beta R_{t+1} u'(c_{u,t+1}), \tag{19}$$

$$u'(c_{u,t}) \geq \beta R_{t+1} u'\Big(c_{e,t+1} - \chi^{-1}\frac{l_{e,t+1}^{1+1/\varphi}}{1+1/\varphi}\Big), \tag{20}$$

$$\text{with equality if } a_{u,t} > 0,$$

$$c_{e,t} + a_{e,t} = R_t a_{u,t-1} + w_t l_{e,t}, \tag{21}$$

$$c_{u,t} + a_{u,t} = R_t a_{e,t-1}, \tag{22}$$

$$l_{e,t} = (\chi w_t)^{\varphi}, \tag{23}$$

$$F(K_{t-1}, l_{e,t}) + B_t = G_t + R_t B_{t-1} + (R_t - 1)K_{t-1} + w_t l_{e,t}, \tag{24}$$

$$B_t + K_t = a_{e,t} + a_{u,t}, \tag{25}$$

$$a_{e,t}, a_{u,t} \geq 0, \tag{26}$$

corresponding to maximizing the aggregate welfare criterion (18) subject to constraints (19)–(23) guaranteeing the optimality of individual choices (Euler equations, budget constraints, and labor FOC, respectively), to the governmental budget constraint (24), to the financial market clearing condition (25), and to the credit constraints (26).

First-best allocation and possible decentralization. We solve program (15) to derive the first-best allocation; see Appendix A.1 for the computations. As is standard in this type of problem, the first-best outcome can be attained if public spending is not too high. In this case, public debt is negative (the state thus holds some capital) and the government finances public spending out of interest payments on the capital stock. This is stated formally in the next proposition, whose proof can be found in Appendix A.2, together with the value of the steady-state first-best level of output Y_{FB}.[10]

Proposition 1 *Define*

$$\overline{g}_1 := \frac{1-\beta}{\beta}\frac{\alpha}{1/\beta+\delta-1} - \frac{1-\beta}{1+\beta}\frac{1-\alpha}{\varphi+1}. \tag{27}$$

If the public spending verifies $G \leq \overline{g}_1 Y_{FB}$, then the steady-state Ramsey allocation is the first-best steady-state allocation characterized by zero taxes and perfect consumption smoothing.

Steady-state allocation with binding credit constraints. We now assume that $G > \overline{g}_1 Y_{FB}$ and characterize the equilibrium where the credit constraint binds for unemployed agents ($a_{u,t} = 0$ for all t). We then provide the conditions for the existence of this equilibrium. Before deriving FOCs, two important remarks are in order. First, even in this simple framework, we must check that the Karush–Kuhn–Tucker conditions apply to our problem, and that the FOCs actually characterize an optimum. Because of the nonlinearity of the constraints (19)–(24), the standard Slater (1950) condition does not apply in our problem. Therefore, we must check another constraint qualification; this is done in Appendix A.3, where we verify that the linear independence constraint qualification holds. Second, we verify that the second-order conditions are also fulfilled; this is done in Appendix A.4, where we prove that the FOCs indeed characterize a maximum.

The FOCs are understood more easily when one uses the factorization approach of

[10]This non first-best condition is the condition identified in more general settings by Albanesi and Armenter (2012), for optimal steady-state capital tax not to be 0.

LeGrand and Ragot (2022a).[11] Denoting by $\lambda_{e,t}$ the discounted Lagrange multiplier on the constraint (19), the objective of the planner can be rewritten as

$$\max_{(a_{e,t},w_t,R_t,B_t)_t} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left(u\left(c_{e,t} - \chi^{-1}\frac{(\chi w_t)^{1+\varphi}}{1+1/\varphi}\right) - \lambda_{e,t} u'\left(c_{e,t} - \chi^{-1}\frac{(\chi w_t)^{1+\varphi}}{1+1/\varphi}\right) \right.$$
$$+ u(c_{u,t}) + \lambda_{e,t-1} R_t u'(c_{u,t}))$$
$$- \mu_t \Big(G_t + (R_t - 1)a_{e,t-1} + B_{t-1} + \chi^{\varphi} {w_t}^{1+\varphi} - F(a_{e,t-1} - B_{t-1}, (\chi w_t)^{\varphi}) - B_t \Big),$$

where we have used (23) and (25) and consumption choices are actually functions of instruments: $c_{e,t} = \chi^{\varphi} {w_t}^{1+\varphi} - a_{e,t}$ and $c_{u,t} = R_t a_{e,t-1}$. The FOCs can be interpreted easily after introducing the following new variables:

$$\hat{\psi}_t^e = \mu_t - u'\left(c_{e,t} - \chi^{-1}\frac{l_{e,t}^{1+1/\varphi}}{1+1/\varphi}\right) + \lambda_{c,t} u''\left(c_{e,t} - \chi^{-1}\frac{l_{e,t}^{1+1/\varphi}}{1+1/\varphi}\right), \tag{28}$$

$$\hat{\psi}_t^u = \mu_t - u'(c_{u,t}) - \lambda_{c,t-1} R_t u''(c_{u,t}), \tag{29}$$

where $\hat{\psi}_t^e$ is interpreted as the gain for the planner of transferring one unit of resources from agent e to its own budget constraint. We will call it the marginal value of public funds financed by agent e. Indeed, the marginal value of one unit of goods for the planner is μ_t, while the social cost for agent e is the sum of their private cost of a reduction in consumption (the second term, $-u'(\cdot)$) and the effect on saving incentives (the third term $\lambda_t u''(\cdot)$). Similarly, $\hat{\psi}_t^u$ is the gain for the planner of transferring one unit of resources from agent u to its own budget. The planner cannot set $\hat{\psi}_t^e$ or $\hat{\psi}_t^u$ to zero because it does not have access to productivity-contingent lump-sum tax. We show in Appendix A.5 and A.6 that the first-order steady-state conditions of the planner are

$$1 + F_K = \frac{1}{\beta}, \tag{30}$$

$$\hat{\psi}^e = \beta R \hat{\psi}^u, \tag{31}$$

$$\hat{\psi}^e = \varphi\mu\left(\frac{1}{1-\tau^L} - 1\right), \tag{32}$$

$$\hat{\psi}^u a_e = \lambda_e u'(c_u). \tag{33}$$

Equation (30) is the modified golden rule, already discussed in Aiyagari (1995), which states

[11] This method is based on the factorization of the Lagrangian introduced by Marcet and Marimon (2019). Two other solution methods can easily be implemented in this simple case. The first is the primal approach, where prices are substituted using the FOCs of households (e.g., as in Bhandari et al., 2021). Second, the Lagrangian approach keeps the prices as instruments but introduces Lagrange multipliers on Euler equations. All methods provide the same FOCs, as we show in Section C.2.

that the marginal product of capital is the same as in the first-best equilibrium because of the Euler equation of the planner for the choice of public debt. This equation implies that the capital-to-labor ratio is the same as in the first best: $K/L = K_{FB}/L_{FB}$. Equation (31) states that the planner intertemporally smooths out the marginal value of public funds using the post-tax interest rate. Equation (32) states that trade-off faced by the planner in raising resources from employed agent e is the distortion in labor supply, which depends on the Frisch elasticity of the labor supply, φ. Equation (33) states that taxing capital to obtain resources from unemployed agent u generates a cost in the distortion of saving incentives, measured by $\lambda_e u'(c_u)$. Finally, one can check that $\hat{\psi}^e, \hat{\psi}^u > 0$, reflecting the fact that the planner wants to tax both agents for financing public spending. The first result is presented below.

Proposition 2 *If the steady-state Ramsey equilibrium has a positive capital tax, $\tau_K > 0$, then capital and labor taxes are such that the post-tax rate and wage satisfy*

$$\underbrace{1 - \beta R}_{\text{Smoothing wedge}} = \underbrace{\frac{F_L - w}{w}}_{\text{Labor wedge}} \underbrace{\varphi\,(1 + \beta)}_{\text{Net distribution gain}}, \tag{34}$$

or equivalently

$$(1 - \beta)\tau^K = \frac{\tau^L}{1 - \tau^L}\varphi(1 + \beta). \tag{35}$$

The term on the left-hand side of equation (34) is positive because of the capital tax, as $1 - \beta R = \tau^K(1 - \beta)$. It captures the low equilibrium post-tax interest rate faced by agents. Perfect consumption smoothing would imply $\beta R = 1$, delivering $c_e = c_u$. For this reason, we call this difference "smoothing wedge".[12] The planner trade-off implies that the smoothing wedge is proportional to the labor wedge, which is the first term on the right-hand side. In the first-best allocation, the remuneration of labor would be equal to its marginal productivity ($F_L = w$) and thus the labor wedge reflects a positive labor tax.

The third term, called the net distribution gain (NDG), weights the labor wedge by its distribution cost. It is equal to the Frisch elasticity φ, which measures the effect of the labor wedge on labor supply, multiplied by $1+\beta$, as the additional resources benefit to both employed and unemployed agents (though savings). We discuss the NDG further below when considering other utility functions. Finally, using equation (30), the relationship

[12]The denominations are related to the ones used by Chari et al. (2007). We call the low interest rate "smoothing wedge" and not "investment wedge" as they do, because the planner uses public debt to reach the optimal pre-tax interest rate. Thus, capital tax affects consumption smoothing, but the marginal productivity of capital is equal to its first-best value because the modified golden rule holds.

(34) can also be written as in equation (35). It shows that in equilibrium, the capital tax increases with the labor tax: both distortions increase together with the financial requirements that the planner has to finance.[13] In particular, the capital tax is positive whenever the labor tax is.

The next proposition characterizes the existence of this equilibrium.

Proposition 3 *There exist two thresholds $\overline{g}_{La}$ and $\overline{g}_{SW}$, defined in equations (89) and (93) of Appendix A.7, such that when $\overline{g}_1 Y_{FB} < G \leq \min(\overline{g}_{SW}, \overline{g}_{La}) \times Y_{FB}$, there exists a steady-state equilibrium with binding credit constraint for unemployed agents where both taxes τ^L and τ^K are positive.*

The proposition is proved in Appendices A.7 and A.9. In addition to the non-first-best condition, $\overline{g}_1 Y_{FB} < G$, the existence of the steady-state equilibrium is subject to two additional conditions, reflected in two thresholds on the public spending. The first threshold $\overline{g}_{SW}$ ensures that the Lagrange multiplier μ is positive. This independent constraint has been discussed recently by Straub and Werning (2020), justifying the SW subscript and the denomination of Straub-Werning condition. If $G > \overline{g}_{SW} Y_{FB}$, then no (stationary) steady-state equilibrium exists, and a non-stationary equilibrium may exist, as studied in Appendix A.10. The threshold $\overline{g}_{La}$ corresponds to a more traditional Laffer condition. When G is higher than this last threshold, not enough resources can be raised with the distorting taxes to finance it. We prove in Appendix A.7 that the constraints $\overline{g}_1 Y_{FB} < G \leq \min(\overline{g}_{SW}, \overline{g}_{La}) \times Y_{FB}$ are compatible for some G and some parameter values. However, stating which of $\overline{g}_{SW}$ or $\overline{g}_{La}$ is greater is not possible in general, as both cases are possible depending on parameter specification.[14]

When is optimal public debt positive? We now show that in addition to a positive capital tax, this model can generate a positive amount of public debt. This is stated in the following result, proved in Appendix A.11.

Result 1 *There exists a threshold $\overline{g}_{pos}$ defined in equation (114) of Appendix A.11, such that steady-state public debt is positive, $B \geq 0$, iff $\overline{g}_1 \leq 0$ and $G \leq \overline{g}_{pos} Y_{FB}$.*

[13] One can check that τ^K/τ^L increases with the discount factor β and the Frisch elasticity.

[14] When β increases from below towards 1, equation (35) would imply that the capital tax would increase without limit relative to the labor tax. However, the equilibrium does not exist in this case. More precisely, we find that the Straub-Werning threshold decreases and $\overline{g}_{SW} < \overline{g}_1$, implying that there is no steady-state equilibrium for any G (see equations (92) and (93) in Appendix).

The joint positivity of public debt and capital tax is not obvious: why would the planner provide more public debt to the market (more liquidity in the sense of Woodford, 1990) and then tax the return on public debt with a positive capital tax? In an equilibrium with positive public debt, the equilibrium savings of employed agents are higher than the optimal capital stock, and the extra savings are absorbed by the public debt. From this allocation, decreasing public debt would inefficiently increase the capital stock, and would further require an increase in the capital tax to reduce savings, which would hinder consumption smoothing. Thus, public debt enables the planner to absorb the excess of savings and reconcile the high savings of private agents with the optimal capital stock without affecting consumption smoothing.

KPR utility function

We consider the KPR utility function (King et al., 1988), for which we use the following standard functional form:

$$U(c,l) = \frac{\left(c^{\gamma}(1-l)^{1-\gamma}\right)^{1-\sigma}}{1-\sigma}, \sigma > 0, \sigma \neq 1, 0 < \gamma < 1,$$

and $U(c,l) = \gamma \log(c) + (1-\gamma)\log(1-l)$ if $\sigma = 1$. In this case, the IES is $\frac{1}{1-\gamma+\gamma\sigma}$.

We provide the main results in the following proposition.

Proposition 4 *If an interior steady-state with $\tau^K > 0$ exists, then:*

1. *The equilibrium allocation satisfies:*

$$\underbrace{1-\beta R}_{\text{Smoothing wedge}} = \underbrace{\frac{F_L - w}{w}}_{\text{Labor wedge}} \underbrace{(1-\gamma)(\sigma-1)l_e.}_{\text{Net distribution gain}} \tag{36}$$

2. *The Straub–Werning condition always holds.*

The proof can be found in Appendix B. Equation (36) provides the equilibrium relationship between wedges, when the equilibrium with positive capital tax exists. The first two terms are interpreted as smoothing and labor wedges, as in equation (34) of the GHH case. The NDG has now a different expression which can be summarized by preference parameters and the equilibrium labor supply of employed agents, l_e, which affects the saving of employed agents and thus the next-period utility of unemployed agents. The equation (36) of Item 1 can also be written as:

$$(1-\beta)\tau^K = \frac{\tau^L}{1-\tau^L}(1-\gamma)(\sigma-1)l_e,$$

which means that a steady-state equilibrium with positive capital and labor taxes implies an IES below 1 (i.e., $\sigma > 1$). In Appendix E.1, we provide a numerical example of such a steady-state equilibrium.[15] When the IES is exactly one ($\sigma = 1$), the steady-state equilibrium with binding credit constraint does not exists as $\tau^K = 0$. Equilibria with an IES greater than one ($\sigma < 1$) may exist for some parametrizations but they will feature negative labor taxes.

Item 2 stipulates that the Straub-Werning condition is always verified with KPR utility function. The existence of a steady-state equilibrium is thus only subject to a non-first-best condition and a Laffer condition. We provide the full characterization of the allocation in Proposition 9 of Appendix B.

Separable utility function and utilitarian Planner

We now focus on the separable utility function of the form $U(c,l) := u(c) - v(l)$, with $u' > 0, u'' < 0, v' \geq 0, v'' \geq 0$. First, we provide general results before focusing on specific separable utility functions. Define as $\epsilon^u(c) := -\frac{cu''(c)}{u'(c)} > 0$ the relative risk aversion coefficient for consumption level c. In this environment, it is more convenient to think about $\epsilon^u(c)$ as the inverse of the intertemporal elasticity of substitution. Similarly, $\epsilon^v(l) := \frac{lv''(l)}{u'(l)} > 0$ is the curvature of the disutility of labor at l. We now provide the main result.

Proposition 5 *An interior steady-state solution* (c_e, c_u, l_e) *with* $\tau^K > 0$ *(if it exists) must satisfy the following conditions.*

1. *Equilibrium allocation*

$$\underbrace{1-\beta R}_{\textit{Smoothing wedge}} = \underbrace{\frac{F_L - w}{w}}_{\textit{Labor wedge}} \underbrace{\frac{\epsilon^u(c_u) - \epsilon^u(c_e)}{\epsilon^u(c_e) + \epsilon^v(l)}}_{\textit{Net distribution gain}}. \tag{37}$$

2. *The Straub–Werning condition is*

$$\frac{\epsilon^u(c_u) - \epsilon^u(c_e)}{u'(c_e)\left(1-\epsilon^u(c_e)\right) - u'(c_u)\left(1-\epsilon^u(c_u)\right)} > 0.$$

[15]The case with an IES below 1 is the one considered by Dyrda and Pedroni (2022) and thus does not raise any existence concern in our setup.

The proof can be found in Appendix C, where we also provide in Proposition 10 a full characterization of the equilibrium allocation and its existence conditions. Considering condition (37), the main difference with the previous cases (equations (34) and (36)) is the expression of the NDG. In the separable utility case, the effect of capital tax on consumption smoothing is captured by the shape of the utility function u, appearing at the numerator of the NDG. The NDG now captures the difference between the local concavity of the utility function for unemployed and employed agents; the higher the difference, the higher the gain of transferring resources from employed to unemployed agents. As there is a wealth effect on labor supply, the elasticity of the function v appears in the numerator if the NDG.

The steady-state capital tax can thus be positive under two conditions. First, the labor tax is positive—and so is the labor wedge—and the NDG is positive. The latter case holds for instance if $u(\cdot)$ exhibits decreasing relative risk aversion (DRRA).[16] Second and oppositely, the labor tax and the NDG are both negative; the planner chooses to subsidize labor. A negative NDG can be the consequence of an increasing relative risk aversion (IRRA) utility function. Below, we derive implications of these findings with different utility functions.

CRRA utility function. We start with the standard CRRA utility function: $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ if $\sigma \neq 1$ or $u(c) = \log c$ if $\sigma = 1$. That case features $\epsilon^u(c_u) = \epsilon^u(c_e) = \sigma$ and the NDG is null. This implies $\beta R = 1$ and a zero capital tax in any steady-state Ramsey allocation. This result shows that a steady-state Ramsey equilibrium with positive capital tax cannot exist in this case, which is consistent with the KPR utility case when $\sigma = 1$ (which is then separable with a constant IES) and the claims of Chen et al. (2020); Auclert et al. (2022); Chien and Wen (2022b) – the latter provided a general proof considering the CRRA case.[17]

Fishburn utility function. A simple DRRA utility function is the one proposed in Fishburn (1977), which is isoelastic below a threshold and linear after it. More formally,

[16] In the absence of risk, it would be more precise to write that u exhibits increasing intertemporal elasticity of substitution. However, we stick here to the more standard denomination.

[17] In Result 3 of Appendix B.2, we derive a relationship between wedges extending (37) to non-separable utility functions, thereby nesting the KPR and separable utility functions. It makes it clear that the separable constant IES utility has the peculiar feature of imposing a zero capital tax and hence ruling out the existence of an equilibrium with binding credit constraint.

$v(l) = \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}$, and

$$u(c) = \begin{cases} \underline{c}\log\left(\frac{c}{\underline{c}}\right) & \text{if } 0 < c \leq \underline{c}, \\ c - \underline{c} & \text{if } \underline{c} \leq c, \end{cases}$$

where $\varphi > 0$ is the Frisch elasticity of labor supply, $\chi > 0$ scales labor disutility, and $\underline{c} > 0$ is a threshold. The function u is continuously differentiable on $\mathbb{R}_+^*$, with $u'(c) = \frac{\underline{c}}{c}$ if $0 < c \leq \underline{c}$, and $u'(c) = 1$ if $\underline{c} \leq c$. This utility function was used by Challe and Ragot (2016) and LeGrand and Ragot (2018) because it generates tractable models.

Assuming $c_e > \underline{c} > c_u$, which must be checked in equilibrium, we have $\epsilon^u(c_e) = 1$, $\epsilon^u(c_u) = 0$, and $\epsilon^v(l) = 1/\varphi$ that can be plugged into (37) to obtain a relationship between capital and labor taxes. In Appendix E.2, we detail the system characterizing the Ramsey allocation and its existence conditions. We also provide a numerical example satisfying all equilibrium conditions.

Stone–Geary utility function. Another example of a simple DRRA utility function is the Stone–Geary one, which can be written in the separable case as $U(c,l) = \frac{(c-\underline{c})^{1-\sigma}-1}{1-\sigma} - \chi^{-1}\frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$, where when $\sigma = 1$, the first term should be substituted by $\log(c - \underline{c})$. The term $\underline{c}$ is a minimum consumption level, and $\sigma, \chi, \varphi > 0$ are positive parameters whose interpretation has already been discussed. In this case, $\epsilon^u(c) = -c\frac{u''(c)}{u'(c)} = \frac{\sigma c}{c-\underline{c}}$ is decreasing in $c > \underline{c}$, while $\epsilon^v(l) = \frac{1}{\varphi}$ is constant. Again, these expressions can be plugged into (37) to obtain the wedge relationship. As in the Fishburn case, in Appendix E.3 we provide the characterization of the Ramsey allocation and a numerical example of an equilibrium with positive taxes and binding credit constraints.

CARA utility function. Finally, the CARA case corresponds to the utility functions $u(c) = -\frac{1}{\gamma}e^{-\gamma c}$ and $v(l) = \frac{1}{\chi\varphi}e^{\varphi l}$, where $\gamma, \varphi > 0$. We then have $\epsilon^u(c) = \gamma c$ and $\epsilon^v(l) = \varphi l$, which are both increasing. Similar to the two previous cases, in Appendix E.4 we provide the characterization of the equilibrium allocation and a numerical example of an equilibrium with binding credit constraints. Because the utility function is IRRA, the equilibrium allocation features positive capital tax, labor subsidy, and negative NDG.

Non-utilitarian planner

Stationary Ramsey equilibria with positive capital tax can also exist because of deviations from the utilitarian SWF. In this subsection, we focus on the CRRA utility function, for which no steady-state Ramsey equilibrium with positive capital tax exists with a utilitarian SWF. We relax the assumption of a utilitarian SWF and assume that the planner considers the following objective:

$$\sum_{t=0}^{\infty} \beta^t \left(u(c_{e,t}) - v(l_{e,t}) + \omega u(c_{u,t}) \right),$$

where ω is the weight for unemployed agents, $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ if $\sigma \neq 1$ or $u(c) = \log c$ if $\sigma = 1$, and $v(l) = \chi^{-1} \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$. The following proposition summarizes our main result.

Proposition 6 *An interior solution (c_e, c_u, l_e) (if it exists) must satisfy the following sets of conditions.*

1. *The allocation satisfies:*

$$\underbrace{\omega - \beta R}_{\text{Smoothing wedge}} = \underbrace{\frac{F_L - w}{w}}_{\text{Labor wedge}} \underbrace{\frac{\omega(1 - \epsilon^u(c_e)) - (1 - \epsilon^u(c_u))}{\epsilon^u(c_e) + \epsilon^v(l_e)}}_{\text{Net distribution gain}}. \tag{38}$$

2. *The Straub–Werning condition is $\omega < 1$.*

The proof can be found in Appendix C.3, where we also provide in Proposition 11 a formal characterization of the allocation and equilibrium existence conditions. Proposition 6 provides a wedge equation (38) in the case of states that a non-utilitarian planner. This equation falls back on the separable case when the weight of unemployed is set to one. The proposition also states that a Ramsey equilibrium with positive capital tax can exist if $\omega < 1$. The intuition is that in this simple setup, the utilitarian planner chooses a zero capital tax. A positive capital tax, which is actually paid by agents in the unemployment state, thus requires lowering the weight of those agents in the planner's objective.

In Appendix E.5, we provide a numerical example that illustrates the existence of an equilibrium with positive capital tax with CRRA utility function and a non-utilitarian planner.

3.2 Dynamic Analysis of Public Debt in Simple Models

We now study the dynamics of public debt after a public spending shock to prove the second main result of the paper (after the one on equilibrium existence), which is the relationship between the dynamics of public debt and the persistence of the aggregate shock. Because the goal with the simple model is to provide clear analytical characterization, we focus here on the GHH utility function and discuss other ones in the last paragraph of this section.

In addition, to simplify the algebra, we focus on the case with full capital depreciation: $\delta = 1$.

Time consistency. In the GHH case with log period utility (17), the program of the planner is time-consistent, although capital is fixed at the period 0 and capital taxes are chosen at period 0 (which is not the case for other utility functions, as discussed below and in LeGrand and Ragot, 2022b). Indeed, in this case, and when credit constraints bind, the saving of employed agents does not depend on the post-tax real interest rate, but only on the post-tax real wage (see equation 65in Appendix A.5). [18].

Linearization. We denote with a hat the relative deviation to the steady-state value: $\widehat{x}_t = \frac{x_t - x}{x}$ for generic variable x_t with steady-state value x. The public spending shock is assumed to be defined as follows:

$$\widehat{G}_t = \begin{cases} \widehat{G}_0 & \text{if } t = 0, \\ \rho_G \widehat{G}_{t-1} & \text{if } t > 0, \end{cases} \tag{39}$$

with $\widehat{G}_0$ small enough for a first-order approximation of the dynamics to be relevant, and $\rho_G \in (-1, 1)$. The shock only happens at date $t = 0$ and then persists with parameter ρ_G, as is consistent with our assumption of an MIT shock.

Characterization of the system stability. Our first result is to characterize the stability of the dynamic system characterizing the Ramsey allocation, using the FOCs of the planner. Interestingly, the dynamic of the Ramsey allocation can be summarized by the capital as a unique state variable and the public spending shock. Our first result characterizes the dynamic of the capital stock.

[18] More precisely, the Lagrange multipliers of the Euler equations in the previous period do not affect the current period allocation. See Appendix A.6.

Result 2 *The optimal dynamic of the capital stock is given by the following system:*

$$\widehat{K}_t = \rho_K \widehat{K}_{t-1} + \sigma_K \widehat{G}_t, \tag{40}$$

where $\rho_K > 0, \sigma_K < 0$, ρ_K *do not depend on* ρ_G *and* $\frac{\partial \sigma_K}{\partial \rho_G} > 0$.

See Appendix D.1 for the expressions of the coefficients and computations. Thus at impact, an increase in public spending diminishes capital, and the higher the persistence of the public spending shock, the stronger the effect.

The dynamic system (40) is stable when the auto-regressive coefficient ρ_K is smaller than one in absolute value. In our setup, this is equivalent to verifying Blanchard–Kahn conditions. The result regarding system stability is summarized in the following proposition.

Proposition 7 *The system (40) is stable, i.e.,* $|\rho_K| < 1$, *iff*

$$\alpha \leq \frac{1}{1 + (1-\beta)(1+\varphi)}. \tag{41}$$

The dynamic system is stable under condition (41), which imposes an upper bound on α. Note that this upper bound is always strictly smaller than one and hence can be binding. This condition on α always holds when public debt is positive, i.e., when $\overline{g}_1 < 0$. This fourth condition, called Blanchard-Kahn, guarantees the dynamic stability of the equilibrium with positive capital tax, while the three previous ones (non-first-best, Straub–Werning and Laffer ones) ensure the existence of the steady-state equilibrium.

By induction, we can derive from (39) and (40) the closed-form expression of the optimal capital impulse response function:

$$\widehat{K}_t = \sigma_K \widehat{G}_0 \frac{\rho_K^{t+1} - \rho_G^{t+1}}{\rho_K - \rho_G}, \tag{42}$$

which allows us to characterize completely the capital path following a public spending shock. At impact and after a positive shock ($\widehat{G}_0 > 0$), the relative variation of capital is always negative by a quantity $\sigma_K \widehat{G}_0 < 0$. Then, the profile of the capital variation is hump-shaped: it starts decreasing further, before increasing and reverting back to zero (see Appendix D.2 for further characterization of the dynamics of the capital stock)

Role of the persistence of the public spending shock ρ_G on public debt. From the expression of capital (42), it is possible to derive the explicit expression for the optimal dynamics of public debt:

$$\widehat{B}_t = \widehat{G}_0(\Theta^K \rho_K^t - \Theta^G \rho_G^t), \tag{43}$$

where the coefficients Θ^K, Θ^G are functions of the parameters of the model but not of $\widehat{G}_0$ and are provided in equations (186) and (187) of Appendix D.2. These parameters can be either positive or negative. As a consequence, on impact, the change in public debt, $\widehat{B}_0 = \widehat{G}_0(\Theta^K - \Theta^G)$, after a positive public spending shock ($\widehat{G}_0 > 0$) can be either positive or negative, because the sign $\Theta^K - \Theta^G$ is ambiguous. We can characterize the effect of the persistence of the shock on the initial change of public debt, considering two cases. First, we analyze the effect of ρ_G with fixed $\widehat{G}_0$ to understand the mechanisms. Our second experiment focuses on studying the effect of ρ_G while keeping the public spending NPV unchanged. More formally, we keep unchanged the following quantity denoted by $N\hat{P}V_0$:

$$N\hat{P}V_0 = \sum_{t=0}^{\infty} \frac{\hat{G}_t}{R^t} = \widehat{G}_0 \sum_{t=0}^{\infty} \left(\frac{\rho_G}{R}\right)^t = \widehat{G}_0 \frac{R}{R - \rho_G}.$$

Keeping the NPV unchanged while changing ρ_G implies setting the initial size of the shock to $\widehat{G}_0(\rho_G) = N\hat{P}V_0 \frac{R-\rho_G}{R}$. This is summarized in the following proposition.

Proposition 8 *Assume that the steady-state public debt is positive: $B > 0$. Denoting by $\widehat{B}_0$ the public debt variation on impact, we have*

$$\left.\frac{\partial \widehat{B}_0}{\partial \rho_G}\right|_{\widehat{G}_0} < 0.$$

Moreover, if we further assume $\widehat{B}_0 > 0$, we also have

$$\left.\frac{\partial \widehat{B}_0}{\partial \rho_G}\right|_{N\hat{P}V_0} < 0.$$

See Appendix D.2 for the proof. The intuition that the dynamics of the debt depend on the persistence of the shock is the following. After a positive public spending shock, the capital is always falling, but to implement consumption smoothing, the planner does not want to decrease private saving (which is used by unemployed agents to consume). Consequently, when the persistence of the shock is low, the planner increases public debt to provide a store of value to private agents. Then, a small increase in future taxes allows one to reduce the public debt. When the persistence is high, this strategy is very costly in terms of welfare, because the fall of the capital stock is persistent, and the planner would have to increase taxes to reduce public debt in periods when capital and output are low. Consequently, the planner does not increase public debt to avoid having to raise taxes in the future to stabilize the public debt.

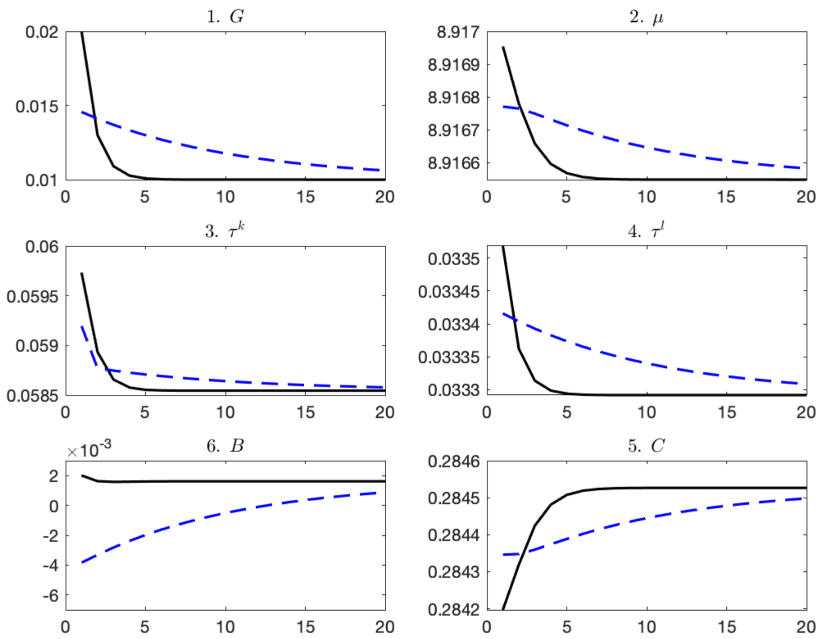

Figure 1: Examples of the dynamics of fiscal variables for a shock with the same net present value and persistences $\rho_G = 0.3$ (black line) and $\rho_G = 0.9$ (blue dashed line) for the parameters $\alpha = 0.3, \beta = 0.7, \varphi = 0.3, \delta = 1, G = 0.01, \chi = 1$.Variables are plotted in level, 20 periods after the MIT shock. Optimal steady-state capital tax is 5.85%, labor tax is 3.33%, public debt-to-GDP is 0.16%.

We check with a simple numerical example that the result of Proposition 8 still holds when we consider a non-marginal variation in the persistence. Figure 1 plots the illustrative dynamics of the economy and of the instruments of the planner for two shocks with the same NPV but different persistences, the initial size of the shock $\hat{G}_0$ being adjusted. The parameters are $\alpha = 0.3, \beta = 0.7, \varphi = 0.3, \delta = 1, G = 0.01, \chi = 1$, and one can check that $\bar{g}_1 Y_{FB} < G$, $G \leq \bar{g}_{SW} Y_{FB}$, and $G < \bar{g}_{La} Y_{FB}$. This economy has an equilibrium capital tax of 6%, a labor tax of 3%, and a (small) positive public debt. The low-persistence economy with $\rho_G = 0.2$ corresponds to the black solid line, while the high-persistence economy with $\rho_G = 0.9$ corresponds to the blue dashed line.

Panel 1 plots the increase in public spending. For the increase to be the same in NPV, it increases by 1% on impact in the case of low persistence and by 0.44% in the case of high persistence. Panel 2 plots the increase in μ, the social value of public liquidity (i.e., the Lagrange multiplier on the government budget constraint). When the persistence is low, the increase is higher on impact but much less persistent compared to the high-persistence case. Panel 3 plots the capital tax, and panel 4 the labor tax. When the persistence is low, both capital and labor taxes increase more on impact but are much less persistent. Capital tax increases by one order of magnitude more than the labor tax on impact, to front-load the adjustment, because period-0 capital taxes are not distorting (see Farhi, 2010 for a discussion of a similar result with complete insurance markets). However, to avoid reducing the resources of credit-constrained agents, the planner does not fully front-load the adjustment and the labor tax is used on the whole transition. Labor taxes are barely increasing in both economies. Consequently, there is a long-lasting increase in both capital and labor taxes when the persistence is high. Therefore, any further increase in taxes would be very costly. This creates a strong incentive not to increase public debt, to avoid a higher interest repayment and hence higher taxes. As can be seen in panel 5, public debt increases in the low-persistence economy whereas it decreases in the high-persistence economy. Finally, panel 6 plots aggregate consumption, which falls in both cases, much more so when the persistence is low, but it returns much faster to its steady-state value.

Other utility functions. One can also check that the result of Proposition 8 and the public debt pattern of Figure 1 do not depend on the specification of the utility function. In particular, it also holds in the CRRA case with non-utilitarian planner of Section 3.1. However, rather than focusing on the simple two-agent model, we focus directly on a model that allows one to consider a realistic calibration (and relaxes Assumption A).

4 The General Model

4.1 Description and First-order Conditions of Ramsey Planner

We now solve for the Ramsey allocation for the general model of Section 2. We thus dispose of the simplifying assumptions of Section 3: the period utility function is separable in labor and consumption ($U(c,l) = u(c) - v(l)$), the productivity process features an arbitrary number of levels and a general transition matrix, the labor tax has an HSV functional form $T_t(\tilde{w}yl) := \tilde{w}yl - \kappa_t(\tilde{w}yl)^{1-\tau_t}$, and Pareto weights depend on current productivity level $(\omega(y_t^i))_i$.

The Ramsey program involves choosing the fiscal instruments $(\tau_t^K, \kappa_t, \tau_t, B_t)_{t\geq 0}$ (as a function of the realization of the aggregate shock and of the initial distribution of the state variables of agents) that correspond to the competitive equilibrium with the highest aggregate welfare. Formally, the Ramsey program can be written as follows:

$$\max_{\left(r_t,w_t,B_t,K_t,L_t,(a_t^i,c_t^i,l_t^i,\nu_t^i)_i\right)_{t\geq 0}} \sum_{t=0}^{\infty}\beta^t \int_i \omega(y_t^i)(u(c_t^i)-v(l_t^i))\ell(di), \tag{44}$$

$$G_t + R_t B_{t-1} + (R_t-1)K_{t-1} + w_t \int_i (y_t^i l_t^i)^{1-\tau_t}\ell(di) = F(K_{t-1},L_t) + B_t \tag{45}$$

$$\text{for all } i \in \mathcal{I}\text{: } a_t^i + c_t^i = R_t a_{t-1}^i + w_t (y_t^i l_t^i)^{1-\tau_t}, \tag{46}$$

$$a_t^i \geq -\bar{a},\ \nu_t^i(a_t^i+\bar{a}) = 0,\ \nu_t^i \geq 0, \tag{47}$$

$$u'(c_t^i) = \beta \mathbb{E}_t R_{t+1} u'(c_{t+1}^i) + \nu_t^i, \tag{48}$$

$$v'(l_t^i) = (1-\tau_t) w_t y_t^i (y_t^i l_t^i)^{-\tau_t} u'(c_t^i), \tag{49}$$

$$K_t + B_t = \int_i a_t^i \ell(di),\ L_t = \int_i y_t^i l_t^i \ell(di). \tag{50}$$

The constraints guarantee that the governmental budget is balanced in (45) and that the planner actually selects a competitive equilibrium characterized by individual budget constraints (46), individual Euler equations (48) and (49), individual credit and positivity constraints (47), market clearing conditions (50), and factor price definitions (1), (4), and (5).[19]

We denote as $\beta^t \lambda_{c,t}^i$ the Lagrange multiplier on the period-t Euler equation of agent i, equation (48). When the credit constraint of agent i is binding, we have $a_t^i = -\bar{a}$ and $\lambda_{c,t}^i = 0$ because the Euler equation is not a constraint. When the credit constraint does not bind, the equilibrium can feature either $\lambda_{c,t}^i > 0$ or $\lambda_{c,t}^i < 0$ depending on whether the agents save too much or too little as seen from the planner's perspective. Similarly, we denote by $\beta^t \lambda_{l,t}^i$ the Lagrange multiplier on the labor supply (49), and by $\beta^t \mu_t$ the Lagrange multiplier on the government budget constraint (45).

To save place, we derive the FOCs of the planner in Appendix F. Note that we follow the literature and assume that the solution is interior and that the FOCs of the planner are sufficient to characterize the optimal allocation. We provide some quantitative checks below.

To simplify the interpretation of the FOCs of the Ramsey program, we introduce the

[19] We solve this program using a factorization approach as presented by LeGrand and Ragot (2022a). We show that this method can be used with occasionally binding credit constraints, taking limits of smooth penalty functions. In addition, the signs of the Lagrange multipliers on Euler equations are analyzed.

marginal social valuation of liquidity for agent i, defined as

$$\psi_t^i := \omega_t^i u'(c_t^i) - \left(\lambda_{c,t}^i - (1+r_t)\lambda_{c,t-1}^i\right) u''(c_t^i) \\ - \lambda_{l,t}^i (1-\tau_t) w_t (y_t^i)^{1-\tau_t} (l_t^i)^{-\tau_t} u''(c_t^i). \tag{51}$$

This complex expression has a simple interpretation. It is the value for the planner of transferring one unit of resources to agent i (if it could). First, the extra unit is valued by the marginal utility weighted with the proper factor $\omega_t^i u'(c_t^i)$. Second, this extra unit of resources also affects the savings incentives, both from period $t-1$ to t (term in $\lambda_{c,t-1}^i$) and from period t to $t+1$ (term in $\lambda_{c,t}^i$). Finally, this unit also modifies the labor supply incentives (term in $\lambda_{l,t}^i$). These last two effects are weighted by the variation in marginal utility of consumption, $u''(c_t^i)$.

From equation (51), we also define the marginal value of public funds financed by agent i:

$$\hat{\psi}_t^i := \mu_t - \psi_t^i, \tag{52}$$

which is the generalization of $\hat{\psi}_t^e$ and $\hat{\psi}_t^u$ of equations (28) and (29). With this notation, the FOCs of the planner are easily interpreted. First, for an unconstrained agent i, the planner implements a public-funds smoothing condition:

$$\hat{\psi}_t^i = \beta \mathbb{E}_t[R_{t+1} \hat{\psi}_{t+1}^i], \tag{53}$$

where the expectation is taken with respect to the idiosyncratic risk. Equation (53) is a generalized version of the Euler equation (10) (and is actually the same equation when all Lagrange multipliers are zero and all weights are set to 1), in which the planner internalizes in the definition of $\hat{\psi}_t^i$ the general equilibrium externalities when setting individual savings. For credit-constrained agents, we have $\lambda_t^i = 0$, and the Euler equation is not a constraint.

Here we present FOCs related to the fiscal tools.[20] The FOC with respect to public debt can be written as

$$\mu_t = \beta(1 + \tilde{r}_{t+1})\mu_{t+1} \tag{54}$$

without an expectation operator because of the MIT shock assumption. Equation (54) shows that the planner aims at smoothing the shadow cost of the government budget constraint through time. This yields the modified golden rule at the steady state.

The other FOC with respect to the post-tax interest rate captures the effect of a change

[20] See equation (190) in Appendix F.1 for the FOC with respect to labor supply.

in capital tax:

$$\underbrace{\int_j \hat{\psi}_t^j a_{t-1}^j \ell(dj)}_{\text{Net distributive gain}} = \underbrace{\int_j \lambda_{c,t-1}^j u'(c_t^j)\ell(dj)}_{\text{Cost on savings incentives}}. \tag{55}$$

The change in capital tax generates benefits for the government through the taxation of heterogeneous households. Because the capital tax is levied on agents' asset holdings, the benefits are proportional to their beginning-of-period wealth, which is the net distributive effect (which is the term at the left-hand side (LHS)). These benefits are set equal to the costs, which operate through the savings incentives. From the planner's perspective, these costs depend on the Lagrange multiplier $\lambda_{c,t-1}^j$ on the Euler equation of each agent (term at the right-hand side (RHS)).

The FOC on post-tax wages captures the effect of a change in the linear labor tax schedule:

$$\underbrace{\int_j (y_t^j l_t^j)^{1-\tau_t} \hat{\psi}_t^j \ell(dj)}_{\text{Net distributive gain}} = \underbrace{\int_j \lambda_{l,t}^j (y_t^j)^{1-\tau_t} (l_t^j)^{-\tau_t} (1-\tau_t) u'(c_t^j)\ell(dj)}_{\text{Cost on labor supply incentives}}. \tag{56}$$

As in FOC (55), the benefit of setting the labor tax level consists of public-funds transfers weighted by the tax base, which is here the labor supply, equal to $(y_t^j l_t^j)^{1-\tau_t}$ for each agent j (LHS). The cost is related to the modification of labor supply incentives that are affected by labor tax (RHS).

The FOC for the progressivity coefficient τ_t has a similar interpretation:

$$0 = \underbrace{\int_j \frac{\partial}{\partial \tau_t} \left((y_t^j l_t^j)^{1-\tau_t} \right) \hat{\psi}_t^j (dj)}_{\text{Net distributive gain}} + \underbrace{\int_j \lambda_{l,t}^j \frac{1}{l_t^j} \frac{\partial}{\partial \tau_t} \left((1-\tau_t) y_t^i (y_t^i l_t^i)^{-\tau_t} \right) u'(c_t^j)\ell(dj)}_{\text{Cost on labor supply incentives}}. \tag{57}$$

Setting the labor tax progressivity is very similar to setting the labor tax level. Indeed, on the one hand, benefits are public-funds transfers weighted by the tax base, which is the term $\frac{\partial}{\partial \tau_t}\left((y_t^j l_t^j)^{1-\tau_t}\right)$. On the other hand, the costs are related to the modification of labor supply incentives. However, even though setting the average tax level or the progressivity (coefficient τ_t) has similar effects, they are two independent instruments because they affect the distribution of agents differently.

Consistency with the analytical model. In Appendix F.2, we check that the general approach of this section and the analytical one of Section 3 provide comparable results.

We proceed in two steps. In Section F.2.1, we verify that the FOCs of the two approaches are identical, even though the Ramsey problems are formulated differently. Then in Section F.2.2, we check that the solution of the analytical approach is quantitatively equal to the limit of the solution of the general approach when the transition matrix converges to the anti-diagonal matrix of Assumption A (see Figure 6 in appendix).

Quantitative strategy. We now show that the intuitions about the dynamics of public debt derived in the simple environment of Section 3 are valid when considering a realistic calibration of the general setup. Because we are interested in the dynamics of the public debt and not in the optimality of the overall tax system, we use the following strategy. First, we calibrate standard parameters to obtain a realistic steady-state allocation in light of the parameters of actual US fiscal policy. Second, following the literature on the inverse optimum taxation problem (Bourguignon and Amadeo, 2015; Chang et al., 2018; Heathcote and Tsujiyama, 2021), we estimate an empirically motivated SWF such that the actual US fiscal policy (before the financial crisis) is optimal for the planner at the steady state. The gain with this methodology is being able to observe the dynamics of the tax system considering a quantitatively realistic initial allocation. Starting from this allocation, we implement period-0 shocks (with different persistences) on public spending to observe the responses of fiscal instruments. As these shocks are transitory, we can check that the value of the fiscal tools return to their initial values, which are the optimal ones in the long run.

Numerical tools. Solving for the dynamics of optimal policies with many tools is a difficult task in heterogeneous-agents models. Here, we follow the methodology of LeGrand and Ragot (2022a). The formal algebra is provided in Appendix G.

The main elements of the method can be summarized as follows. In heterogeneous-agent models, agents differ according to their idiosyncratic history. An agent i has a period-t history $\{y_{i,0}, \ldots, y_{i,t}\}$. Let $h = (\tilde{y}_{-N+1}, \ldots, \tilde{y}_{-1}, \tilde{y}_0)$ be a given history of length N. In period t, an agent i is said to have *truncated history* h if the history of this agent for the last N periods is equal to h: $(y_{i,t-n+1}, \ldots, y_{i,t}) = (\tilde{y}_{-N+1}, \ldots, \tilde{y}_{-1}, \tilde{y}_0)$. The truncation method aggregates agents with the same truncated history and then expresses the model in terms of these groups of agents. This generates the so-called *truncated model*, which features a finite state space. In the truncated model, the agents' aggregation assumes full risk-sharing within each truncated history and thus "forgets" the heterogeneity in histories prior to the aggregation period (i.e., more than N periods ago). We capture this within-truncated-history via additional parameters denoted by "ξs" and that are truncated-history specific. This construction yields a finite state-space representation that is exogenous to agents' choices and thereby allows one to compute optimal policies.[21]

The previous truncation method is simple to implement, but it has the drawback of considering many histories, some of which are very unlikely to be experienced by agents. By the law of large numbers, these histories concern a very small number of agents. LeGrand and Ragot (2022c) proposed considering different truncation lengths for different histories; for clarity, we call this method *refined* truncation and the former one *uniform* truncation. Histories that are more likely to be experienced (i.e., larger ones) can be "refined", meaning that they can be substituted by a set of histories with higher truncation lengths. For instance, the truncated history (y_1, y_1) $(N = 2)$ can be refined into $\{(y, y_1, y_1) : y \in \mathcal{Y}\}$, where the group of agents who have been in productivity y_1 for two consecutive periods is split into $Card(\mathcal{Y})$ truncated histories. A benefit of this construction is that the number of histories is a *linear* function of the maximum truncation length, instead of an exponential function. A difficulty of the construction is that the set of refined histories must form a well-defined partition of the set of idiosyncratic histories in each period. The construction of the refinements is detailed in LeGrand and Ragot (2022c). In what follows, we provide results and check that they are robust to an increase in the truncation length.

4.2 Calibration

Preferences. The period is a quarter. The utility function is separable in labor $U(c, l) = u(c) - v(l)$, with

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \text{ and } v(l) = \frac{1}{\chi} \frac{l^{1+\frac{1}{\phi}}}{1 + \frac{1}{\phi}}.$$

We set the inverse of intertemporal elasticity of substitution to $\sigma = 2$, which is a standard value in the literature. For the disutility of labor, we set a Frisch elasticity for labor supply of $\phi = 0.5$, which is the value recommended by Chetty et al. (2011) for the intensive margin in heterogeneous-agent models. The scaling parameter is set to $\chi = 0.05$, which implies normalizing the aggregate labor supply to 1/3. The discount factor is set to $\beta = 0.99$.

[21] Considering wealth bins is not possible because the savings function and thus the transitions across wealth bins are endogenous to the planner's policy. This would imply a fixed point that would be very hard to solve. LeGrand and Ragot (2022a) showed that the truncated allocation converges to the true one when the truncation length increases. The question is then quantitative, and LeGrand and Ragot (2022b) showed that a tractable truncation length provides accurate results. The presentation of the method is related to LeGrand et al. (2022, Section 4.4)

Idiosyncratic risk. We focus on a standard AR(1) process: $\log y_t = \rho_y \log y_{t-1} + \varepsilon_t^y$, where $\varepsilon_t^y \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_y^2)$. Following the strategy of Castaeneda et al. (2003), we choose the parameters (ρ_y, σ_y) to target three key moments.[22] The first target is the variance of the logarithm of consumption, which enables us to capture consumption inequality; Heathcote and Tsujiyama (2021) reported a value of $\text{Var}(\log c) = 0.23$. We also target the log-variance of wages to match income inequality, which Heathcote and Tsujiyama (2021) found to be $\text{Var}(\log w) = 0.47$. The third target is the debt-to-GDP ratio, which allows us to replicate a realistic financial market equilibrium; we target a value of $B/Y = 61.5\%$, which is the mean ratio over the period (Dyrda and Pedroni, 2022). Calibrating these three moments yields $\rho_y = 0.993$ and $\sigma_y = 0.082$; these parameters are close to those from a direct estimation of the productivity process on PSID data, which corresponds to $\rho_y = 0.9923$ and $\sigma_y = 0.0983$ (see Boppart et al., 2018 and Krueger et al., 2018). The data targets and their model counterparts are reported in Table 1.

	Data	Model
Variance of log consumption $\text{Var}(\log c)$	0.23	0.20
Variance of log income $\text{Var}(\log y)$	0.47	0.48
Debt-to-GDP ratio B/Y	61.5%	61.4%

Table 1: Model calibration: targets and model counterparts.

This simple representation does a good job in matching the three targeted moments. Furthermore, we can check that this calibration generates a reasonable wealth distribution, even though we do not calibrate it explicitly.[23] Indeed, the calibrated model implies a Gini coefficient of wealth equal to 0.66, which is close—albeit below—its empirical counterpart of 0.77. It is known that additional model features must be introduced to match the high wealth inequality in the US, such as heterogeneous discount rates (Krusell and Smith, 1998), entrepreneurship (Quadrini, 1999), or stochastic financial returns, which are not considered here.

[22] More precisely, we minimize the quadratic difference between the model-generated moments and their empirical counterpart, following the simulated method of moments (SMM). In the present environment, we see this procedure as a "sophisticated" calibration rather than an actual SMM because we weight the three moments equally.

[23] For the problem under consideration, we consider that matching the dispersion of consumption may be more important than the distribution of wealth, which motivates the exclusion of this moment from our calibration strategy. It is possible to match the wealth distribution more closely, but at the cost of a worse match of the moments of the consumption distribution.

Finally, we discretize the productivity process using the Rouwenhorst (1995) procedure with seven idiosyncratic states.

Technology. The production function is Cobb–Douglas: $F(K, L) = K^\alpha L^{1-\alpha} - \delta K$. The capital share is set to $\alpha = 36\%$ and the depreciation rate to $\delta = 2.5\%$, as in Krueger et al. (2018) among others.

Taxes and government budget constraint. The capital tax is taken from Trabandt and Uhlig (2011), who used the methodology of Mendoza et al. (1994) on public finance data prior to 2008. Their estimation for the US in 2007 (before the financial crisis) yields a capital tax (including both personal and corporate taxes) of $\tau^K = 36\%$. For labor, we consider the HSV functional form of equation (2). The progressivity of the labor tax is taken from Heathcote et al. (2017), who reported an estimate of $\tau = 0.181$. We choose κ to match a public-spending-to-GDP ratio of 19%, as in Heathcote and Tsujiyama (2021).

Table 2 summarizes the model parameters.

Parameter	Description	Value
	Preference and technology	
β	Discount factor	0.99
α	Capital share	0.36
δ	Depreciation rate	0.025
$\bar{a}$	Credit limit	0
χ	Scaling param. labor supply	0.05
φ	Frisch elasticity labor supply	0.5
	Shock process	
ρ_y	Autocorrelation idio. income	0.993
σ_y	Standard dev. idio. income	0.082
	Tax system	
τ^K	Capital tax	36%
κ	Scaling of labor tax	0.75
τ	Progressivity of tax	0.181

Table 2: Parameter values in the baseline calibration. See text for descriptions and targets.

4.3 Truncation and Estimating Pareto Weights

The model resolution relies on the truncation method that is explained in Section 4.1. To investigate the optimal dynamics of the instruments after a shock, we start by deriving an exact truncated representation of the steady-state model, then we follow the dynamics of the truncated representation using perturbation methods. In Appendix G, we provide a detailed account of the computational implementation, which is of independent interest because solving such Ramsey problems is not straightforward.

The refined truncation length is set to $N = 20$, which is shown to provide an accurate representation of the dynamics in the robustness check of Appendix I. We consider 350 relevant histories. We have to estimate the weights of the SWF such that the FOCs of the planner at the steady state are consistent with the actual US tax system (as described in Section 4.2). However, the problem is generally under-identified because we have only three constraints (one for the capital tax and two for the labor tax) but seven different weights (one per productivity level). Following Heathcote and Tsujiyama (2021), we introduce productivity weights that depend on the productivity level and define a parametric quadratic representation of weights as follows:

$$\log \omega_y := \theta_1 \log y + \theta_2 (\log y)^2 .$$

As explained in Appendix G, matching capital and labor tax yields $\theta_1 = 0.93$ and $\theta_2 = 0.33$. In an environment without savings, Heathcote and Tsujiyama (2021) estimated the relationship $\log \omega_y = \theta \log y$ and found a positive value $\theta = 0.517$. The quantitative difference comes mostly from the additional instruments that we use.[24]

4.4 Model Dynamics

We now simulate the optimal dynamics of the four fiscal tools $(\tau_t^K, B_t, \kappa_t, \tau_t)$ after a public spending shock occurring in period $t = 0$. The dynamics of the shock are the same as in equation (39) of the analytical section.[25] After an initial shock in period 0, public spending

[24]We cannot strictly reproduce the specification of Heathcote and Tsujiyama (2021) within our framework because we need two parameters to match the planner's FOCs because we have more instruments. The correlation between the estimated value $\log \omega_y$ and $\log y$ is 0.61 in our model, which is consistent with the value in Heathcote and Tsujiyama (2021).

[25]As discussed above, the period-0 problem is time inconsistent in this economy. To avoid the effect of a re-optimization shock in period 0, we report the result following the *timeless* perspective: The Lagrange multipliers of all constraints entering the FOCs of the planner are initialized to their steady-state values. As a consequence, the dynamics below represent the sole effect of a public spending shock. Conversely, a pure re-optimization shock would be a period-0 shock where Lagrange multipliers entering the FOCs of

reverts back to equilibrium at a rate ρ_G.

Dynamics of the instruments as a function persistence. We simulate the model for two values of the persistence of public spending shocks while keeping the NPV constant. The higher value is $\rho_G = 0.97$, which is the value used by Farhi (2010) on US data. The lower value is $\rho_G = 0.7$, which corresponds to some specific transitory increase in public spending in the US, such as episodes of military build-ups. The initial size of the shock is adjusted for the NPV of public spending to be the same in the two economies. Results are plotted in Figure 2, which reports public spending shock G and Lagrange multiplier μ in proportional deviations, labor tax level κ, labor tax progressivity τ, and capital tax τ^k in level deviations, and finally public debt B in proportional deviations. The high-persistence economy is plotted with blue dashed lines, while the low-persistent one is plotted with black solid lines.

Panel 1 represents the dynamics of public spending, G, which increases by 1% of GDP when $\rho_G = 0.7$ (black solid line) and by 0.12% of GDP when $\rho_G = 0.97$ (blue dashed line), for the NPV to remain the same. Panel 2 plots the value of the Lagrange multiplier (in proportional deviation), which represents the marginal value of additional public resources. Panels 3–5 report the labor tax level, labor tax progressivity, and capital tax (in level deviations). In our tax schedule, an increase in κ (panel 3) corresponds to a decrease in the labor tax (as agents receive more labor income). An increase in τ (panel 4) is an increase in the progressivity of the labor tax.

First, after a public spending shock, capital tax increases (panel 5), and the planner reduces labor tax (panel 3) but increases progressivity (panel 4) to levy some resources. Note that the change in the capital tax is one order of magnitude higher than the change in the labor tax. In addition, the higher the persistence, the smaller the change in these variables. However, whereas the public spending path is very different for the two persistence levels (panel 6), the change in taxes is less so. Consequently, public debt increases when the persistence is low, which facilitates financing the sharp increase in public spending in the first periods, whereas public debt decreases when the persistence is high because of a front-loading of the cost of the new public spending.

To summarize, in both cases (high and low persistence), the planner implements a significant increase in capital taxes for a few quarters. Labor tax moves much less, with

the planner are set to 0 in period 0, without any public spending shock, to observe how time-inconsistency makes the planner deviate from steady state before converging back to it. This exercise is performed in LeGrand and Ragot (2022b).

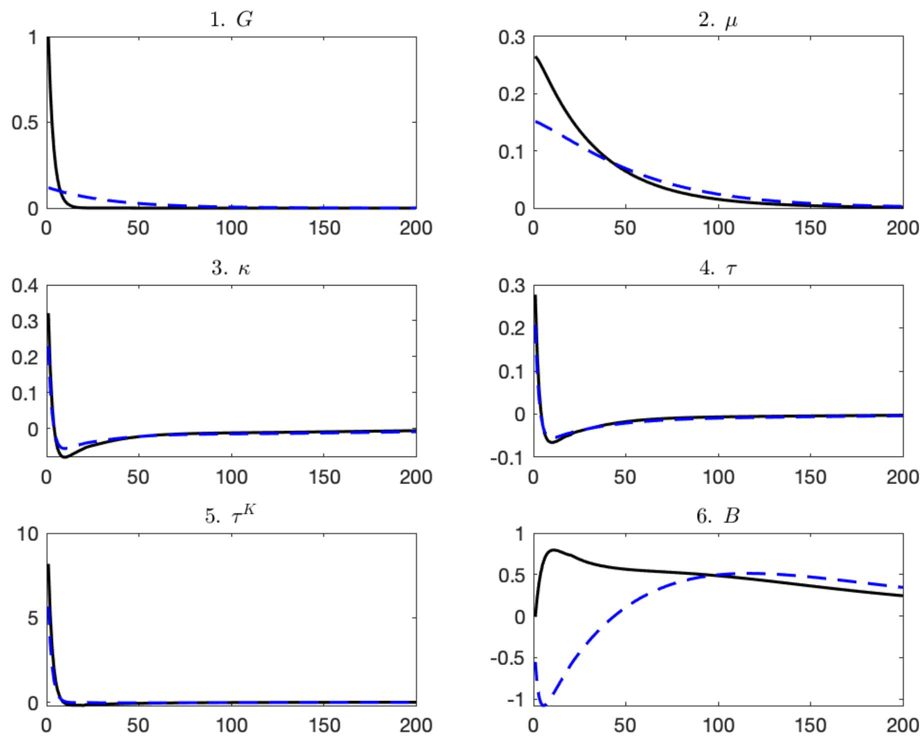

Figure 2: Dynamics of selected variables for two shocks with different persistence but the same NPV. G—public spending; μ—value of public resources; κ—level of labor tax; τ—progressivity of labor tax; τ^k—capital tax; B—public debt. The black solid lines correspond to persistence $\rho_G = 0.7$, and the blue dashed lines correspond to persistence $\rho_G = 0.97$. G is in percent of GDP, B is in proportional deviations, and other variables are in level deviations.

a small decrease in the overall level and a small increase in progressivity. The increase in capital tax and progressivity is smaller when the persistence is higher (for the same NPV). Public debt exhibits much more persistent deviations than do other variables. It can either decrease or increase depending on the persistence, consistent with the analytical results of Section 3.2.

Allocation and comparison with the first-best outcome. Figure 3 plots the dynamics of output Y, capital K, labor L, and consumption C, all in proportional deviations, for the two levels of persistence. Panel A reports results for the low-persistence case ($\rho_G = 0.7$), and panel B reports results for the high-persistence case ($\rho_G = 0.97$). In both figures, the dotted line is the first-best allocation (in proportional deviation) for the same shock. The first-best allocation is defined as the complete market economy where the planner maximizes aggregate welfare solely subject to the resource constraint (thereby assuming that it has access to lump-sum taxes, as in the standard real business-cycle model). Pareto weights do not affect the dynamics in this first-best case, but only the intra-period allocation.

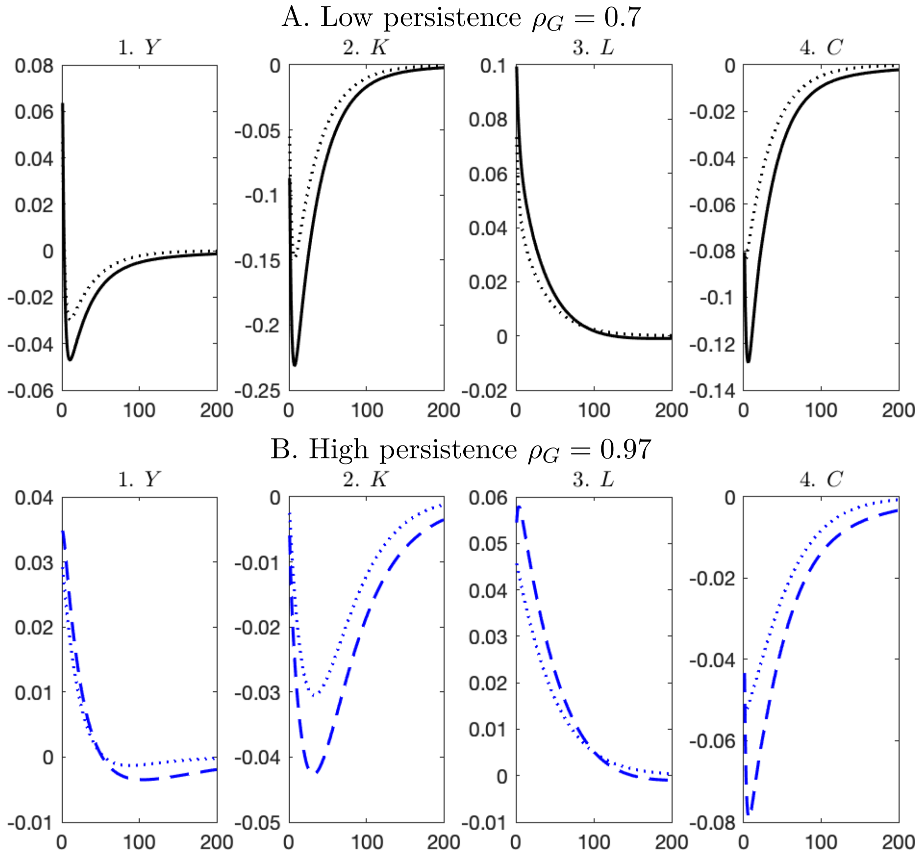

Figure 3: Optimal output Y, capital K, labor L, and consumption C for low and high persistence values, in proportional deviations. The dotted line is the first-best allocation for both persistence values.

Consumption and the capital stock fall in all cases, but much more so when the persistence is low. Total labor supply increases on impact (because of a negative wealth effect for households), as does output. One can observe that the volatility and the persistence of the market economy for both low and high persistence values are higher than in the first-best economy, although the dynamics of variables are qualitatively similar.[26]

[26]It is also possible to compute the dynamics of the allocation with complete markets (representative-agent case) but with distorting taxes. It is known (from Chari et al., 1994, Chari and Kehoe, 1999, and Farhi, 2010, among others) that the optimal steady-state outcome features (i) a null capital tax, (ii) a government holding the whole capital stock (public debt thus being negative), and (iii) a labor set to finance the share of public spending, which is not financed by interest payment on the capital stock. After

Optimal path of public debt and persistence of the shock. In Figure 4, we plot the optimal debt dynamics for four persistence levels of the public spending shock, with a normalization of the initial shock G_0 to generate the same NPV of public spending. We report only the path of public debt, because the paths of other instruments are similar to those presented in Figure 2.

We observe in Figure 4 that the deviation of public debt decreases with the persistence. When the persistence is very small ($\rho_G = 0.01$), public debt increases on impact and then decreases monotonically. For higher persistence ($\rho_G = 0.7$), the path of public debt has an inverted U shape which then becomes J-shaped for higher persistence ($\rho_G = 0.97$). In other words, persistent public spending shock should be financed by taxes and not by public debt.

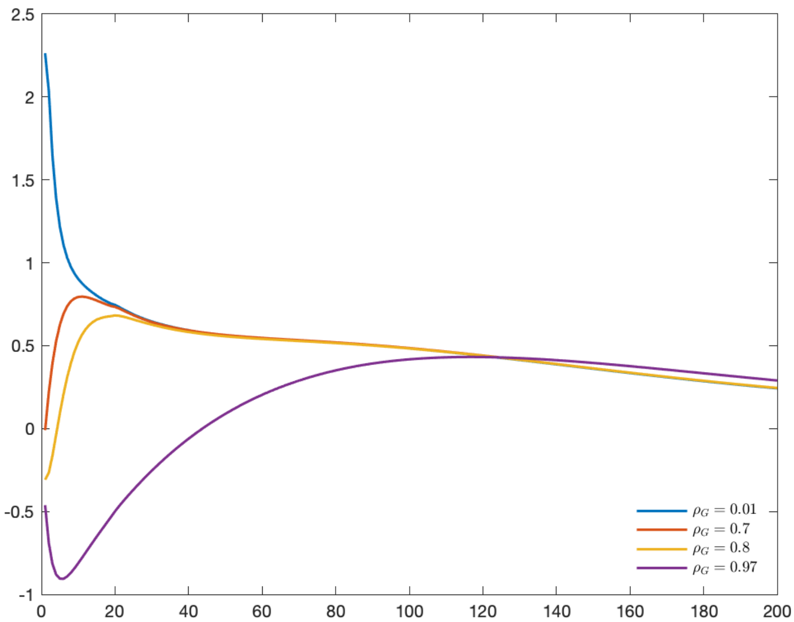

Figure 4: Comparison of optimal public debt dynamics for different persistence values of the public shock (same NPV of public spending), in proportional deviation from steady-state value of public debt.

Robustness of the truncation method. Finally, to check the accuracy of the truncation method, we compute the simulation outcomes for a truncation length of $N = 25$ instead of $N = 20$ in the benchmark case. As shown in Appendix I, the outcomes are

a public spending shock, public debt follows the capital stock. Because this outcome is very different from the incomplete-market economy (where steady-state public debt is positive), we do not report the simulation of this economy.

undistinguishable for both the instruments and the allocation.

5 The Dynamics of US Public Spending Shock

This section documents the fiscal policy following historical public spending shocks of varying persistence in the United States, in order to assess the relevance of results of the previous sections. Identifying the perceived persistence of public spending shocks is a challenging task. The empirical identification of shocks often relies on the assumption of a common dynamic structure for all shocks.[27] For this reason, we consider four events: World War I, World War II, the Korean War and the Vietnam War. Using the data from Ramey and Zubairy (2018), we construct the quarterly time series of public spending, normalized by potential output for each event.[28] To identify the perceived persistence of the shock, we estimate an autoregressive process for each event only after the peak in public spending, before spending starts to fall continuously. Figure 5 shows the deviation of public spending after the peak of each event, as a percentage of potential output. We also plot the path of the estimated process, which roughly matches the data. The estimated persistences show considerable heterogeneity in their magnitudes. For example, the persistence for World War I is 0.59, while it is 0.78 for the Korean War, and 0.94 for the Vietnam War. Table 3 lists the events in increasing order of the persistence.

Table 3 shows the events (first column), the estimated persistence (second column), the key dates (third column), and the share of total spending financed by direct taxes. This last statistic is taken from Goldin (1980) and Ohanian (1997). The tax share is estimated as the deviation of taxes from the pre-tax level. Joines (1981) estimates that the tax increase is mainly due to an increase in capital taxes, which is consistent with the results of the model.

The financing of the Vietnam War is not reported in Goldin (1980) because it is particularly difficult to assess. During the Vietnam War, the ratio of public debt to GDP fell from 45% in 1965 to 32% in 1975. The marginal tax rate on labor rose from 22% in 1965 to 25% in 1969, and the marginal tax rate on capital rose from 49% in 1965 to 54%

[27] The literature estimates a process for the deviation of public spending from a long-run trend, conditional on a well-identified public spending shock. The estimation typically assumes a constant process for public spending, often using a local projection method (see Ramey and Zubairy, 2018 for a recent analysis). Here, we analyze the impact of differences in persistence by relying on event studies in the spirit of Ohanian (1997).

[28] As in Ramey and Zubairy (2018), we normalize macro variables by real potential GDP, based on a 6th degree polynomial fit from 1889:1–2015:4, omitting the Great Depression and World War II. We then remove a linear trend from the public spending data.

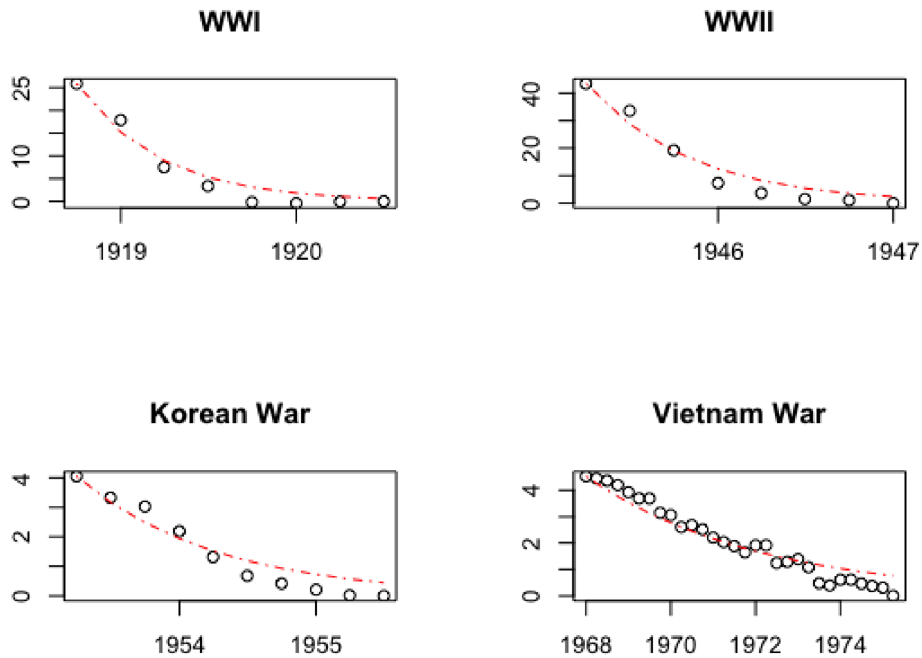

Figure 5: Public spending, deviation from potential output in percent, for six major military events. The black circles are data, the red dashed line is the estimated process, modeled as an AR(1).

in 1969 (Joines, 1981). However, this decline in public debt occurred during a period of high inflation, following the President Johnson's Great Society programs and during the period of oil price shocks. As argued in Bordo and Orphanides (2013), high inflation was largely the result of an inadequate response by the Fed to negative supply shocks during this period.

Overall, without assuming that fiscal policy was fully optimally set during these periods, these historical fiscal episodes are broadly consistent with the model.

6 Conclusion

We have investigated the optimal fiscal policy after a public spending shock in a heterogeneous-agent model. Our first contribution was to clarify the conditions for relevant equilibria to exist; the key friction for equilibrium existence is an occasionally binding credit constraint, which provides a rationale for both positive capital tax and public debt. Our second contribution was to show that the dynamics of public debt and taxes depend crucially on the persistence of the public spending shock; public debt is pro-cyclical for low persistence but countercyclical for high persistence. In the general model, we found that both capital

Event	Quarterly Persistence	Dates			Share financed by direct taxes
		Beg.	Peak	End	
World War I	59%	1914:Q3	1918:Q4	1920:Q3	24%
World War II	66%	1939:Q3	1944:Q1	1947:Q1	41%
The Korean War	78%	1950:Q3	1953:Q3	1957:Q1	100%
The Vietnam War	94%	1965:Q1	1968:Q1	1975:Q2	n.a.

Table 3: Estimated persistence of public spending in percent for the six events (in ascending order) and change in public debt divided by the net present value of public spending.

and labor taxes increase when persistence is high and decrease otherwise. We considered a quantitative model whereby the actual US tax system is implemented at the steady state thanks to an inverse optimal taxation approach.

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Appendix

A Properties of the Simple Model, with GHH Utility Function and Utilitarian Planner

A.1 First-best Steady-State Allocation in the Simple Model

We derive the first-best allocation of the simple model. Considering the utilitarian SWF, the Lagrangian associated to the program is:

$$\mathcal{L}^{FB} = \sum_{t=0}^{\infty} \beta^t \left[\log(c_t^u) + \log\left(c_t^e - \chi^{-1} \frac{l_{e,t}^{1+1/\varphi}}{1+1/\varphi} \right) \right] + \sum_{t=1}^{\infty} \beta^t \mu_t \left(K_{t-1} + K_{t-1}^{\alpha} l_{e,t}^{1-\alpha} - \delta K_{t-1} - c_t^e - c_t^u - G_t - K_t \right),$$

together with non-negativity constraints $c_t^e, c_t^u, l_{e,t} \geq 0$, which are not binding. In that case, it is straightforward to check that the linear independence constraint qualification (LICQ) holds and the optimization yields a maximum (see Section A.3 below for a lengthier discussion). Denoting by $L_{FB} = l_e$ the steady-state labor supply, the FOCs imply the following equations at the steady state:

$$\frac{K_{FB}}{L_{FB}} = \Big(\frac{\alpha}{\frac{1}{\beta} + \delta - 1} \Big)^{\frac{1}{1-\alpha}}, \tag{58}$$

$$L_{FB} = (\chi(1-\alpha))^{\varphi} \Big(\frac{\alpha}{\frac{1}{\beta} + \delta - 1} \Big)^{\frac{\alpha}{1-\alpha}\varphi}, \; K_{FB} = (\chi(1-\alpha))^{\varphi} \Big(\frac{\alpha}{\frac{1}{\beta} + \delta - 1} \Big)^{\frac{1+\alpha\varphi}{1-\alpha}}, \tag{59}$$

$$Y_{FB} = (\chi(1-\alpha))^{\varphi} \Big(\frac{\alpha}{\frac{1}{\beta} + \delta - 1} \Big)^{\frac{\alpha(1+\varphi)}{1-\alpha}}, \tag{60}$$

as well as $c^u = c^e - \chi^{-1} \frac{L_{FB}^{1+1/\varphi}}{1+1/\varphi}$. With the resource constraint (16), we can compute consumption levels.

A.2 Proof of Proposition 1

The first-best equilibrium is characterized by optimal consumption smoothing and no inefficient distortion. We now analyze the necessary and sufficient conditions for which the first-best allocation can be decentralized. Using the Euler equations (19) and (20) with equality, one finds:

$$\beta R_{FB} = 1, \tag{61}$$

Distorting taxes are also null: $\tau^K = \tau^L = 0$, while the government budget constraint (24) implies that the public debt verifies: $B_{FB} = -\frac{\beta}{1-\beta}G < 0$. To implement the first-best allocation, we further need to check that no agent is credit-constrained.

Factor prices definitions (1) with (61) and $L_{FB} = l_e = (\chi w_{FB})^\varphi$ yield the same capital-to-labor ratio K_{FB}/L_{FB} as in (58) the same output as in (58), and the same labor supply and capital as in (59). The wage is equal to:

$$w_{FB} = (1-\alpha)\Big(\frac{\alpha}{\frac{1}{\beta}+\delta-1}\Big)^{\frac{\alpha}{1-\alpha}}. \tag{62}$$

Furthermore, since agents are unconstrained, Euler equations imply $c_{u,FB} = c_{e,FB} - \frac{1}{\chi}\frac{l_{e,FB}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$, or after substituting by budget constraints: $R_{FB}a_{u,FB} - a_{e,FB} + \frac{w(\chi w)^\varphi}{\varphi+1} = R_{FB}a_{e,FB} - a_{u,FB}$. Using (61) and the financial market clearing condition stating that $a_{e,FB} + a_{u,FB} = B_{FB} + K_{FB}$ implies:

$$2\frac{1-\beta}{\beta}\frac{a_{u,FB}}{Y_{FB}} = \overline{g}_1 - \frac{G}{Y_{FB}}, \tag{63}$$

with $\overline{g}_1$ defined in (27). The credit constraint $a_{u,FB} \geq 0$ and equation (63) imply the first-best condition $\frac{G}{Y_{FB}} \leq \overline{g}_1$, which concludes the proof of Proposition 1.

A.3 Constraint Qualification

In our problem, even though the objective function is concave, the equality constraints are not linear and the standard Slater (1950) conditions do not apply. However, we can check that the linear independence constraint qualification (LICQ) holds in our problem. This constraint qualification requires the gradients of equality constraints to be linearly independent at the optimum (or equivalently that the gradient is locally surjective). At any date t, two constraints matter for the instruments of date t. These are the constraints at dates t and $t+1$. We can check that their gradient can be written as:

$$\begin{pmatrix} 1 & \varphi(\chi w_t)^\varphi \frac{\tilde{w}_t}{w_t} - (\varphi+1)(\chi w_t)^\varphi & -\frac{\beta}{1+\beta}\frac{w_{t-1}(\chi w_{t-1})^\varphi}{1+\varphi} \\ -\tilde{r}_{t+1} - 1 & \frac{\beta}{1+\beta}(\chi w_t)^\varphi \tilde{r}_{t+1} - (R_{t+1}-1)\frac{\beta}{1+\beta}\frac{w_t(\chi w_t)^\varphi}{1+\varphi} & 0 \end{pmatrix}, \tag{64}$$

which forms a matrix of rank 2. Indeed, looking at the first and third columns of the matrix in (64) makes it clear that a sufficient condition is $(1+\tilde{r}_{t+1})w_{t-1} \neq 0$. This condition must hold at the optimum, since: (i) equation (1) implies $\tilde{r}_{t+1} \geq 0$, and (ii) $w_{t-1} > 0$.

A.4 Second-Order Conditions

In the program (18)–(26), we use (24) to substitute for the expression of R_t. We can further use financial market constraint (26) to express B_t as a function of K_t and w_t. The planner's program can be equivalently rewritten as a function of K_t and $W_t = w_t(\chi w_t)^{\varphi}$:

$$\max_{(K_t,w_t)_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg(\log(W_t) \\ + \log\left(K_{t-1} + F(K_{t-1}, \chi^{\frac{\varphi}{1+\varphi}} W_t^{\frac{\varphi}{1+\varphi}}) - \frac{1+\varphi+\varphi\beta}{(1+\beta)(1+\varphi)} W_t - K_t - G_t \right)\Bigg).$$

The function $(W_t, K_{t-1}) \mapsto F(K_{t-1}, \chi^{\frac{\varphi}{1+\varphi}} W_t^{\frac{\varphi}{1+\varphi}})$ is concave as the composition of concave and increasing functions. We thus deduce that the mapping defined by $(W_t, K_{t-1}, K_t) \mapsto \log(W_t) + \log\left(K_{t-1} + F(K_{t-1}, \chi^{\frac{\varphi}{1+\varphi}} W_t^{\frac{\varphi}{1+\varphi}}) - \frac{1+\varphi+\varphi\beta}{(1+\beta)(1+\varphi)} W_t - K_t - G_t\right)$ is concave. Any interior optimum characterized by the FOCs must thus be a maximum.

A.5 FOCs Derivation with Binding Credit Constraints

We first derive the FOCs using a direct method and, we then check that the factorization method yields the same FOCs (see Section A.6). Using individual budget constraints and log utility, Euler equations (19) becomes:

$$a_{e,t} = \frac{\beta}{1+\beta} \frac{w_t(\chi w_t)^{\varphi}}{1+\varphi} \geq 0. \tag{65}$$

The Ramsey program can then be written as:

$$\max_{\{B_t,w_t,R_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(\frac{1}{1+\beta} \frac{w_t(\chi w_t)^{\varphi}}{\varphi+1} \right) + \log(R_t \frac{\beta}{1+\beta} \frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi}) \right), \tag{66}$$

$$w_{t+1}(\chi w_{t+1})^{\varphi} > \beta^2 R_{t+1} R_t w_t (\chi w_t)^{\varphi}, \tag{67}$$

$$G + B_{t-1} + (R_t - 1)\frac{\beta}{1+\beta} \frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi} + w_t(\chi w_t)^{\varphi} = B_t \\ + F(\frac{\beta}{1+\beta} \frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi} - B_{t-1}, (\chi w_t)^{\varphi}). \tag{68}$$

Note that the Euler inequality for unemployed agents (67) is equivalent at the steady state to $\beta R < 1$, which will always hold in equilibrium (see below).

Defining by convention w_{-1} as $\frac{\beta}{1+\beta}\frac{w_{-1}(\chi w_{-1})^{\varphi}}{1+\varphi} = a_{-1}$ and by $\beta^t \mu_t$ the Lagrange multiplier on (68), the FOCs associated to the program (66)–(68) can be written as (for $t \geq 0$):

$$0 = (1+\beta)(\varphi+1)\frac{1}{w_t} + \beta(\chi w_t)^{\varphi}\frac{\beta}{1+\beta}\mu_{t+1}(F_{K,t+1} - R_{t+1} + 1) \tag{69}$$

$$+ \chi\mu_t(\chi w_t)^{\varphi-1}\left(\varphi F_{L,t} - (\varphi+1)w_t\right),$$

$$\mu_t = \beta(1 + F_{K,t+1})\mu_{t+1}, \tag{70}$$

$$1 = R_t\mu_t\frac{\beta}{1+\beta}\frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi}. \tag{71}$$

We can take advantage of FOCs (70) and (71) to simplify FOC (69) as follows:

$$\mu_t w_t(\chi w_t)^{\varphi}\left(1 - (1+\beta)\varphi\frac{\tau_t^L}{1-\tau_t^L}\right) = (1+\varphi)(1+\beta), \tag{72}$$

which is a time-t equation only. The only dynamic FOC is the forward-looking equation (70). We will check that the system is well-defined and does not raise convergence issues.

A.6 Deriving First-Order Conditions with Factorization

We check here that the FOCs do not depend on the resolution method. Denoting $\lambda_{e,t}$ the discounted Lagrange multiplier on the constraint (19), and using the definitions of $\hat{\psi}_t^e$ and $\hat{\psi}_t^u$ of equations (28) and (29), the FOCs of the planner are:

$$\hat{\psi}_t^e = \beta R_{t+1}\hat{\psi}_{t+1}^u, \tag{73}$$

$$\hat{\psi}_t^e = -\varphi\mu_t\left(1 - \frac{F_{L,t}}{w_t}\right), \tag{74}$$

$$\hat{\psi}_t^u a_{e,t-1} = \lambda_{c,t-1}u'(c_{u,t}), \tag{75}$$

$$\mu_t = \beta\mu_{t+1}(1 + F_{K,t+1}), \tag{76}$$

which fall back on the FOCs (30)–(33) at the steady state.

We check that the FOCs (73)–(76) derived with the Lagrangian method exactly simplify to the FOCs (69)–(71) of Section A.5. The extra equation is related to the Lagrange multiplier $\lambda_{c,t}$. Note that the FOC (76) is identical to (70). Denoting $C = c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}$, we obtain that the FOC (74) becomes:

$$\hat{\psi}_t^u C_{u,t} = \frac{\lambda_{c,t-1}}{a_{e,t-1}}. \tag{77}$$

Remark. It seems that the previous relationship induces time-inconsistency. However, this is not the case. Indeed, from (29), we have:

$$\begin{aligned}\hat{\psi}_t^u &= \mu_t - u'(c_{u,t}) - \lambda_{c,t-1} R_t u''(c_{u,t}),\\ &= \mu_t - \frac{1}{C_{u,t}} + \lambda_{c,t-1} \frac{R_t}{C_{u,t}^2},\end{aligned}$$

which comes from $C_{u,t} = c_{u,t} - \chi^{-1} \frac{l_{u,t}^{1+1/\varphi}}{1+1/\varphi} = c_{u,t}$ and $u = \log$. Multiplying by $C_{u,t}$ the former equality and using $R_t = \frac{C_{u,t}}{a_{e,t-1}}$, we obtain

and

$$\begin{aligned}\hat{\psi}_t^u C_{u,t} &= \mu_t C_{u,t} - 1 + \lambda_{c,t-1} \frac{R_t}{C_{u,t}},\\ &= \mu_t C_{u,t} - 1 + \frac{\lambda_{c,t-1}}{a_{e,t-1}},\end{aligned}$$

which with (77) implies $\mu_t C_{u,t} - 1 = 0$, for which there is not time-incosistency issue.

FOCs (73), (74), and (77) thus become after substituting the expressions of $\hat{\psi}_t^e$ and $\hat{\psi}_t^u$:

$$\mu_t - \frac{1}{C_{e,t}} - \frac{\lambda_{c,t}}{C_{e,t}^2} = \beta R_{t+1} \left(\mu_{t+1} - \frac{1}{C_{u,t+1}} + R_{t+1} \lambda_{c,t} \frac{1}{C_{u,t+1}^2} \right), \tag{78}$$

$$\mu_t - \frac{1}{C_{e,t}} - \frac{\lambda_{c,t}}{C_{e,t}^2} = \varphi \mu_t \left(\frac{F_{L,t}}{w_t} - 1 \right), \tag{79}$$

$$\left(\mu_t - \frac{1}{C_{u,t}} + R_t \lambda_{c,t-1} \frac{1}{C_{u,t}^2} \right) a_{e,t-1} = \frac{\lambda_{c,t-1}}{C_{u,t}}. \tag{80}$$

Using $a_{e,t-1} = \frac{C_{u,t}}{R_t}$ (i.e., the budget constraint of agents u), FOC (80) becomes:

$$\mu_t C_{u,t} = 1, \tag{81}$$

with the expression of $C_{u,t} = R_t \frac{\beta}{1+\beta} \frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi}$ (coming from (65)) is identical to FOC (71). Using the Euler equation $C_{u,t+1} = \beta R_{t+1} C_{e,t}$ to substitute for R_{t+1} and (81) to substitute for μ_{t+1}, equation (78) becomes after some simplification:

$$\frac{\lambda_{c,t}}{C_{e,t}} = \frac{\beta}{1+\beta} (\mu_t C_{e,t} - 1). \tag{82}$$

Finally, we turn to FOC (79), which, combined with (28) and (82), becomes: $(1+\beta)C_{e,t}\mu_t\left(\frac{1}{1+\beta}-\varphi\frac{\tau_t^L}{1-\tau_t^L}\right)=1$. Using the budget constraint $C_{e,t}=\frac{w_t(\chi w_t)^\varphi}{(1+\beta)(1+\varphi)}$, the previous equation falls back on FOC (72) (and hence FOC (69)). This completes the proof that the Focs are the same.

A.7 Proof of Proposition 2

Note that because of FOC (71), $\mu=0$ or $R=0$ is not possible at the steady state. FOCs (69)–(71) and governmental budget constraint (68) become at the steady state, where we denote variables without subscripts:

$$\frac{1}{1+\beta}\mu w(\chi w)^\varphi=\varphi+1+\mu(\chi w)^\varphi\varphi(F_L-w), \tag{83}$$

$$1=\beta(1+F_K) \tag{84}$$

$$1=R\mu\frac{\beta}{1+\beta}\frac{w(\chi w)^\varphi}{1+\varphi} \tag{85}$$

$$F\left(\frac{\beta}{1+\beta}\frac{w(\chi w)^\varphi}{1+\varphi}-B,(\chi w)^\varphi\right)=G+(R-1)\frac{\beta}{1+\beta}\frac{w(\chi w)^\varphi}{1+\varphi}+w(\chi w)^\varphi. \tag{86}$$

Using (85) and $w=(1-\tau^L)F_L$, equation (83) becomes (34). Further using $w=(1-\tau^L)F_L$, and $R-1=(1-\tau^K)F_K=(1-\tau^K)(\beta^{-1}-1)$ yields: $\tau^K=\varphi\frac{1+\beta}{1-\beta}\frac{\tau^L}{1-\tau^L}$ (equation (35)). This concludes the proof of Proposition 2.

A.8 Proof of Proposition 3

The Laffer threshold. After several manipulations and using (35) and (84), as well as the properties of F, the governmental budget constraint (86) implies that τ^L is a solution of $\mathcal{T}(\tau^L)=0$, where:

$$\mathcal{T}:\tau\in(-\infty,1)\mapsto\tau-\frac{1}{1-\alpha}\frac{\frac{G}{Y_{FB}}(1-\tau)^{-\varphi}-\overline{g}_1}{1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}}. \tag{87}$$

The mapping $\tau\mapsto\mathcal{T}(\tau)$ is akin to a Laffer curve. Indeed, we can check that $\mathcal{T}$ is continuously differentiable, strictly concave, with a unique maximum over $(-\infty,1)$. In consequence, the function $\mathcal{T}$ admits either zero, one, or two solutions. The number of solutions depends on the level of public spending G in (87). When public spending is too high, there is no level of labor tax that makes this public spending sustainable: $T(\tau)<0$ for all $\tau\in(-\infty,1)$. When the public spending is sustainable, T typically admits two roots. The smaller root corresponds to a low tax and a high labor supply, while the larger root corresponds to a high tax and a low labor supply. There is a third case that is the limit between sustainability and no sustainability. In this situation, there is a unique tax rate that enables public spending to be financed.

The limit case of the Laffer curve happens when the extremum point of the Laffer curve is the only root of the function. It can be checked that this corresponds to the tax level $\overline{\tau}^L_{La}$ that verifies $\mathcal{T}(\overline{\tau}^L_{La}) = \mathcal{T}'(\overline{\tau}^L_{La}) = 0$, or equivalently to:

$$\overline{\tau}^L_{La} = \frac{1}{1+\varphi} - \frac{1}{1-\alpha}\frac{\varphi}{1+\varphi}\frac{\overline{g}_1}{1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}}. \tag{88}$$

This corresponds to a ratio of public spending $\frac{G}{Y_{FB}}$, defined as:

$$\overline{g}_{La} := \frac{1-\alpha}{\varphi}\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}\right)(1-\overline{\tau}^L_{La})^{1+\varphi}, \tag{89}$$

where $\overline{\tau}^L_{La}$ is defined in (88) and which can be shown to be always well-defined since $\overline{g}_1 \geq -\frac{1-\beta}{1+\beta}\frac{1-\alpha}{\varphi+1}$. So, any public spending such that $\frac{G}{Y_{FB}} > \overline{g}_{La}$ is not sustainable and cannot be financed by any tax system.

Oppositely, when $\frac{G}{Y_{FB}} < \overline{g}_{La}$, two different tax levels enable the government to finance public spending, and the planner will always opt for the lowest tax rate. Indeed, taxes have an unambiguously negative impact on consumption levels, since: $c_e = \frac{1}{1+\beta}(1-\tau^L)^{\varphi+1}\frac{w_{FB}(\chi w_{FB})^{\varphi}}{1+\varphi}$ and $c_u = (1-(1-\beta)\tau^K)c_e$. So larger taxes decrease consumption and hence individual welfare.

As a conclusion, let us prove that $\overline{g}_{La} \geq \overline{g}_1$ and more precisely the following lemma.

Lemma 1 *We have $\overline{g}_{La} \geq \overline{g}_1$. The equality only holds if $\frac{\varphi}{1-\alpha}\frac{\overline{g}_1}{1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}} = 1$. Otherwise, the inequality is strict.*

Proof. Note that by construction, $\overline{g}_{La} \geq 0$. The result thus holds if $\overline{g}_1 < 0$. We assume that $\overline{g}_1 \geq 0$. Using the definitions of $\overline{g}_{La}$ and $\overline{g}_1$, we have:

$$\frac{\overline{g}_{La} - \overline{g}_1}{\kappa} = \left(\frac{\varphi}{1+\varphi}\right)^{\varphi}\frac{1-\alpha}{1+\varphi}\left(1+\frac{\overline{g}_1}{\kappa}\right)^{1+\varphi} - \frac{\overline{g}_1}{\kappa},$$

with $\kappa = (1-\alpha)(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}) > 0$. The sign of $\overline{g}_{La} - \overline{g}_1$ can be determined by focusing on the function $s : x \in \mathbb{R}_+ \mapsto \left(\frac{\varphi}{1+\varphi}\right)^{\varphi}\frac{1}{1+\varphi}(1+x)^{1+\varphi} - x$, which is well-defined and continuously differentiable on $\mathbb{R}_+$. We have $s'(x) \geq 0$ iff $\left(\frac{\varphi}{1+\varphi}\right)^{\varphi}(1+x)^{\varphi} \geq 1$ or $x \geq \varphi^{-1}$. The function s thus admits a minimum for $x = \varphi^{-1}$, whose value is:

$s(\varphi^{-1}) = \left(\frac{\varphi}{1+\varphi}\right)^{\varphi} \frac{1}{1+\varphi}(\frac{1+\varphi}{\varphi})^{1+\varphi} - \frac{1}{\varphi} = 0$. We deduce that $s(x) \geq 0$ and the equality holds iff $x = \varphi^{-1}$, which concludes the proof. ■

Regarding the allocation, we have:

$$c_e = \frac{1}{1+\beta}\frac{w(\chi w)^{\varphi}}{1+\varphi}, \tag{90}$$

$$c_u = \frac{1-(1-\beta)\tau^K}{1+\beta}\frac{w(\chi w)^{\varphi}}{1+\varphi}. \tag{91}$$

The Straub-Werning threshold. The relationship (35) does not provide any upper bound on the capital tax, which diverges when τ^L becomes close to 100%. However, the post-tax interest rate sets an implicit bound on the capital tax. Indeed, the post-tax interest rate must remain positive – otherwise unemployed agents would face negative consumption. The positivity of the post-tax rate is equivalent to the positivity of the Lagrange multiplier μ through FOC (85). Equation (91) implies that the capital tax must remain below a threshold

$$\overline{\tau}^K_{SW} := \frac{1}{1-\beta} \tag{92}$$

– where SW stands for Straub-Werning (see the discussion below). This tax threshold implies an upper bound on the labor tax $\overline{\tau}^L_{SW}$ (through equation (35)) and also an upper bound on the level of public spending: $G < \overline{g}_{SW} Y_{FB}$, where:

$$\overline{g}_{SW} := \overline{g}_1 + (1-\alpha)\left(1 + \frac{1-\beta}{1+\beta}\frac{1}{1+\varphi} + \frac{\varphi}{1+\varphi}\right)(1-\overline{\tau}^K_{SW})^{\varphi}. \tag{93}$$

This concludes the proof of Proposition 3.

A.9 The $\tau^K = 0$-Equilibrium

We prove here that the steady-state equilibrium featuring $\tau^K = 0$ is always dominated by the equilibrium featuring binding credit constraint and $\tau^K > 0$. We write with the 0-subscript the allocation where $\tau^K = 0$, and with no subscript the allocation where $\tau^K > 0$. The proof is split into three parts: (i) the characterization of the $\tau^K = 0$-equilibrium (Section A.9.1); (ii) when the $\tau_k > 0$-equilibrium exists, i.e., when the Straub-Werning condition holds (Section A.9.2); and (iii) when the $\tau_k > 0$-equilibrium does not exist, i.e., when the Straub-Werning condition does not hold (Section A.9.3).

A.9.1 Characterization of the $\tau^K = 0$-Equilibrium

With the same steps as in Section A.2, we have:

$$w_0 = (1-\tau^L)w_{FB},\ K_0 = (1-\tau^L)^{\varphi}K_{FB},\ Y_0 = (1-\tau^L)^{\varphi}Y_{FB}. \tag{94}$$

Governmental budget constraint becomes: $B_0 = -\frac{\beta}{1-\beta}G + \frac{\beta}{1-\beta}\tau_0^L(1-\tau_0^L)^{\varphi}w_{FB}(\chi w_{FB})^{\varphi}$. Perfect risk sharing (i.e., $c_{u,0} = c_{e,0} - \frac{1}{\chi}\frac{l_{e,0}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$) and financial market clearing (i.e., $A_0 = K_0 + B_0$) imply after some manipulation:

$$2\frac{a_{u,0}}{Y_0} = \frac{\beta}{1-\beta}(\overline{g}_1 - g_{FB}(1-\tau_0^L)^{-\varphi}) + \left(\frac{1}{1-\beta} + \frac{1}{1+\beta}\frac{1}{\varphi+1}\right)\beta\tau_0^L(1-\alpha), \tag{95}$$

$$2\frac{a_{e,0}}{Y} = 2\frac{a_{u,0}}{Y} + 2\frac{\beta}{1+\beta}\frac{1-\alpha}{\varphi+1}(1-\tau_0^L), \tag{96}$$

meaning that $a_{e,0} \geq a_{u,0}$ for all values of $\tau_0^L \leq 1$. We compute the consumption level $c_{u,0}$ from individual budget constraint:

$$2\frac{c_{u,0}}{Y_{FB}} = (1-\tau_0^L)^{\varphi}\overline{g}_1 - \frac{G}{Y_{FB}} + \frac{2}{1+\beta}\frac{1-\alpha}{\varphi+1}(1-\tau_0^L)^{\varphi} + \frac{\varphi}{\varphi+1}(1-\alpha)\tau_0^L(1-\tau_0^L)^{\varphi}. \tag{97}$$

Computing the derivative of $2\frac{c_{u,0}}{Y_{FB}}$ with respect to the labor tax τ_0^L yields: $\frac{1}{\varphi(1-\tau_0^L)^{\varphi-1}}\frac{\partial}{\partial\tau_0^L}2\frac{c_{u,0}}{Y_{FB}} = -\frac{(1-\beta)\alpha}{1+\beta(\delta-1)} - (1-\alpha)\tau_0^L < 0$, whenever $\tau_0^L \geq 0$. We deduce from the last inequality that $c_{u,0}$ is decreasing with τ_0^L (and hence aggregate welfare since $c_{u,0} = c_{e,0} - \frac{1}{\chi}\frac{l_0^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$). Since $a_{e,0} \geq a_{u,0}$ for all values of τ_0^L, the value of τ_0^L is chosen as small as possible for credit constraints not to bind and hence such that $a_{u,0} = 0$. From (95), τ_0^L is the solution of:

$$\tau_0^L = \frac{1}{1+\frac{1-\beta}{1+\beta}\frac{1}{\varphi+1}}\frac{g_{FB}(1-\tau_0^L)^{-\varphi} - \overline{g}_1}{1-\alpha}, \tag{98}$$

which is a Laffer-like curve, as (87), admitting 0, 1 or 2 solutions. Finally, regarding allocation, we have:

$$c_{u,0} = c_{e,0} - \chi^{-1}\frac{l_0^{1+1/\varphi}}{1+1/\varphi} = \frac{1}{1+\beta}\frac{w_0(\chi w_0)^{\varphi}}{1+\varphi}. \tag{99}$$

A.9.2 Case where the $\tau_k > 0$-Equilibrium Exists

The fact that the planner implements $a_{u,0} = 0$ in the equilibrium with full risk-sharing implies that the objective of the planner is actually the same as in the case with binding credit constraints. As a consequence, the allocation with $\tau^K = 0$ and $\tau^K > 0$ can be written as the outcome of the same program, with the constraint $\tau^K \geq 0$. Formally:

$$\max_{\{B_t, w_t, R_t\}} \sum_{t=0}^{\infty} \beta^t \left((1+\beta) \log \left(\frac{1}{1+\beta} \frac{w_t (\chi w_t)^{\varphi}}{\varphi + 1} \right) + \log(\beta R_t) \right) \tag{100}$$

$$\begin{aligned} G + B_{t-1} + (R_t - 1) \frac{\beta}{1+\beta} \frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi} + w_t (\chi w_t)^{\varphi} &= B_t \\ + F(\frac{\beta}{1+\beta} \frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi} - B_{t-1}, (\chi w_t)^{\varphi}), & \end{aligned} \tag{101}$$

with $R_t \geq 1 + \tilde{r}_t$, where $\tilde{r}_t = F_{K,t}$ is exogenous. We now show that the previous program has the desired properties.

We start with the case $\tau^K = 0$. Denoting by $\beta^t \mu_t$ the Lagrange multiplier associated to the constraint (101), the maximization with respect to B_t yields: $\mu_t = \beta(1 + F_{K,t+1})\mu_{t+1}$, or at the steady state: $\beta(1 + F_K) = 1$. The constraint (101) implies then at the steady state, using (58)–(59), that the labor tax, denoted $\hat{\tau}_0^l$ verifies:

$$(1-\alpha) \left(1 + \frac{1-\beta}{1+\beta} \frac{1}{1+\varphi} \right) \hat{\tau}_0^l = \frac{g_{FB}}{(1-\hat{\tau}_0^l)^{\varphi}} - \overline{g}_1, \tag{102}$$

which is the equation as (98) for τ_0^L. Since the planner will also choose the lowest solution for (102), we deduce that $\hat{\tau}_0^l = \tau_0^L$. Consumption levels then mechanically verify equation (99), which proves that the steady-state equilibrium with $\tau^K = 0$ is a steady-state solution of the program (100)–(101) where we impose $\tau_t^K = 0$ at all dates.

We now turn to the unconstrained case ($\tau^K \neq 0$). In that case, the FOCs of the program (100)–(101), with respect to B_t, R_t, and w_t, respectively, are:

$$\mu_t = \mu_{t+1} \beta (1 + F_{K,t}), \tag{103}$$

$$1 = R_t \mu_t \frac{\beta}{1+\beta} \frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi}, \tag{104}$$

$$\begin{aligned} \frac{(1+\beta)(1+\varphi)}{w_t} &= \frac{\mu_t}{w_t} \left((\varphi + 1) w_t (\chi w_t)^{\varphi} - \varphi F_{L,t} (\chi w_t)^{\varphi} \right) \\ &+ \frac{\beta \mu_{t+1}}{w_t} (R_{t+1} - 1 - F_{K,t+1}) \frac{\beta}{1+\beta} w_t (\chi w_t)^{\varphi}, \end{aligned} \tag{105}$$

which are identical to the FOCs (69)–(71) of the unconstrained case.

We therefore deduce that the allocation with $\tau^K = 0$ is the solution of a constrained program and is hence dominated by the allocation $\tau_k \neq 0$ – whenever the latter exists.[29]

[29]Note that the argument could not be applied right away from the initial program formulation because

A.9.3 Case where the $\tau_k > 0$-Equilibrium Does Not Exist

We now show that an equilibrium with $\tau^K = 0$ does not exist even when the equilibrium where $\tau^K > 0$ does not exist. Assume now that the solution of (87) does not verify the Straub-Werning condition. We will show that in that case the $\tau_k = 0$-equilibrium does not exist either. To do so, we focus on the limit case when the Straub-Werning condition does not hold, implying that the solution, denoted τ_m^L, to (87) is:

$$\tau_m^L = \frac{1}{1 + (1+\beta)\varphi}. \tag{106}$$

The argument easily extends to any value $\tau^L \geq \tau_m^L$ (see explanation after equation (108)). Equation (87) implies that it corresponds to a public spending $g_{FB,0}$ verifying:

$$g_{FB,0}(1-\tau_m^L)^{-\varphi} = (1-\alpha)\left(1 + \frac{1-\beta}{1+\beta}\frac{1}{1+\varphi} + \frac{\varphi}{1+\varphi}\right)\tau_m^L + \overline{g}_1. \tag{107}$$

To show that the $\tau_k = 0$-equilibrium does not exist, we show that there is no solution to (98), and more precisely that, for all τ_0^L:

$$\tau_0^L < \frac{g_{FB,0}(1-\tau_0^L)^{-\varphi} - \overline{g}_1}{(1-\alpha)\left(1 + \frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)}. \tag{108}$$

The argument we develop would easily extend to any solution τ^L to (87), such that $\tau^L \geq \tau_m^L$. Indeed, these cases would imply public spending levels higher than $g_{FB,0}$. The equilibrium non-existence would then be implied by inequality (108).

To show inequality (108), notice that $\tau_0 \in (-\infty, 1) \mapsto g_{FB,0}(1-\tau_0^L)^{-\varphi} - \overline{g}_1 - (1-\alpha)\left(1 + \frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)\tau_0^L$ is convex admits a global minimum denoted $\tau_{0,\min}^L$, defined as:

$$1 - \tau_{0,\min}^L = \left(\frac{\varphi g_{FB,0}}{(1-\alpha)\left(1 + \frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)}\right)^{\frac{1}{\varphi+1}} \tag{109}$$

To prove inequality (108), using (109), we only need to show that $\Delta > 0$, where

$$\Delta = (\varphi^{-1}+1)\left(\frac{\varphi g_{FB,0}}{(1-\alpha)\left(1 + \frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)}\right)^{\frac{1}{\varphi+1}} - \frac{\overline{g}_1}{(1-\alpha)\left(1 + \frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)} - 1. \tag{110}$$

with $\tau_k \neq 0$, the constraint $a_{u,t} = 0$ was binding – which is not present anymore with the modified program (100)–(101).

The definition (107) of $g_{FB,0}$ implies using (106) that (110) becomes:

$$\Delta = \frac{(1+\beta)(\varphi+1)}{1+(1+\beta)\varphi}\left(\frac{2(1+(1+\beta)\varphi)}{(1+\beta)((1+\beta)(1+\varphi)+1-\beta)}+\frac{\frac{1+(1+\beta)\varphi}{1+\beta}\overline{g}_1}{(1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)}\right)^{\frac{1}{\varphi+1}} \tag{111}$$
$$-\frac{\overline{g}_1}{(1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)}-1,$$

which can be seen as a function of $\tilde{g}_1 = \frac{\overline{g}_1}{(1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)}$, defined on $(-\frac{2}{(1+\beta)(1+\varphi)+1-\beta}, \infty)$. This function is concave, admits a unique maximum, $\frac{(1+\beta)\varphi}{(1+\beta)(1+\varphi)+1-\beta} > 0$, in $\tilde{g}_1^* = \frac{-2\varphi(1+\beta)}{(1+(1+\beta)\varphi)((1+\beta)(1+\varphi)+1-\beta)}$. Thus, there exist two (mathematical) bounds denoted $\tilde{g}_1^{\inf} < \tilde{g}_1^* < \tilde{g}_1^{\sup}$, such that $\Delta(\tilde{g}_1) > 0$ iff $\tilde{g}_1 \in (\tilde{g}_1^{\inf}, \tilde{g}_1^{\sup})$. The rest of the proof consists in finding two economical bounds on $\tilde{g}_1$, denoted by $\tilde{g}_1^{\min}$ and $\tilde{g}_1^{\max}$ and to prove that $\Delta(\tilde{g}_1^{\min}) > 0$ and $\Delta(\tilde{g}_1^{\max}) > 0$. We can then deduce from the properties of the function Δ that $\Delta(\tilde{g}_1) > 0$ for all economically acceptable $\tilde{g}_1$, which concludes the proof.

Lower bound on $\tilde{g}_1$. The definition (27) of $\overline{g}_1 = \frac{1-\beta}{\beta}\frac{\alpha}{1/\beta+\delta-1} - \frac{1-\beta}{1+\beta}\frac{1-\alpha}{\varphi+1}$ readily implies: $\frac{\overline{g}_1}{(1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)} \geq -\frac{1-\beta}{(1+\beta)(1+\varphi)+1-\beta} = \tilde{g}_1^{\min}$, or from (111): $\Delta(\tilde{g}_1^{\min}) \geq \frac{(1+\beta)(1+\varphi)}{(1+\beta)(1+\varphi)+1-\beta}\Big((1+\frac{1}{1+(1+\beta)\varphi})^{\frac{\varphi}{\varphi+1}} - 1\Big) > 0$, where the second inequality comes from $\beta \in (0,1)$ and $\varphi > 0$.

Upper bound on $\tilde{g}_1$. The upper bound on $\tilde{g}_1$ is less straightforward. Equation (107) – seen as an equation in τ_m^L for a given $g_{FB,0}$ – admits one or two roots (since by construction the no-root case is excluded). To guarantee that the smallest solution is chosen, the derivative of the $\tau \mapsto (1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}\right)\tau + \overline{g}_1 - g_{FB,0}(1-\tau)^{-\varphi}$ must be positive in τ_m^L (the function being concave, it has to intercept 0 before it reaches its maximum). Or equivalently: $\varphi g_{FB,0}(1-\tau_m^L)^{-\varphi-1} \leq (1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}\right)$. Using (107), we obtain that this condition is equivalent to:

$$\frac{\overline{g}_1}{(1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)} \leq \frac{2\beta}{(1+\beta)(1+\varphi)+1-\beta} = \tilde{g}_1^{\max}.$$

From (111), we obtain, after some manipulations:

$$\frac{\Delta(\tilde{g}_1^{\max})}{\tau_m^L} \geq (1+\beta)(1+\varphi)\left(\left(1+\frac{\varphi(1+\beta)}{1+(1+\varphi(1+\beta))}\right)^{\frac{1}{\varphi+1}}-1\right) - \beta\frac{\varphi(1+\beta)}{(1+(1+\varphi(1+\beta)))},$$

whose left-handside can be seen as a function of $\frac{\varphi(1+\beta)}{1+(1+\varphi(1+\beta))}$ (that lies in $(0,1)$). We denote: $\tilde{\Delta} : x \in (0,1) \mapsto (1+\beta)(\varphi+1)\left((1+x)^{\frac{1}{\varphi+1}}-1\right) - \beta x$. Using a second-order

Taylor development, we have for $x \in (0,1)$: $\frac{\tilde{\Delta}(x)}{x} \geq 1 - \frac{\varphi}{\varphi+1}\frac{1+\beta}{2}x > 0$, where the second inequality comes from $x < 1$, $\beta < 1$, and $\varphi > 0$.

A.10 A non-interior steady-state equilibrium

Here we investigate the case when (87) admits a solution that does not verify the Straub-Werning condition. FOC (70) holds and FOCs (69) and (71) can also be written as:

$$\left(1 - (1+\varphi(1+\beta))\tau_t^L\right)(1-\tau_t^L)^{\varphi}\mu_t\tilde{w}_t(\chi\tilde{w}_t)^{\varphi} = (1+\beta)(1+\varphi), \tag{112}$$

$$(1 + (1-\tau_t^K)F_{K,t})\mu_t(1-\tau_{t-1}^L)^{\varphi+1}\tilde{w}_{t-1}(\chi\tilde{w}_{t-1})^{\varphi} = \frac{(1+\beta)(1+\varphi)}{\beta}. \tag{113}$$

Equation (112) implies that for all t: $\tau_t^L \leq \frac{1}{1+\varphi(1+\beta)}$ and $\tau^L = \lim_{t\to\infty}\tau_t^L \leq \frac{1}{1+\varphi(1+\beta)}$. From (112), there are possibly non-interior steady states, featuring $\lim_t \mu_t = \infty$ or $\lim_t \tilde{w}_t = \infty$.

First case: $\lim w_t = w^* < \infty$**.**

- The case $w^* = 0$ is not possible. Otherwise there are no resources to pay G.
- Assume that $\lim \mu_t = \infty$, then equation (112) implies $\lim_t \tau_t^L = (1+\varphi(1+\beta))^{-1}$. Equation (113) then yields $\lim_t (1 + (1-\tau_t^K)F_{K,t}) = \lim_t R_t = 0$.

Second case: $\lim_t w_t = \infty$**.** We thus have $\lim_t \tilde{w}_t = \infty$. Using factor price definitions: $\chi\tilde{w}_t = \left(\frac{\chi(1-\alpha)}{(1-\tau_t^L)^{\alpha\varphi}}\right)^{\frac{1}{1+\varphi\alpha}} K_{t-1}^{\frac{\alpha}{1+\varphi\alpha}}$ yields $\lim_t K_t = \infty$ and $\lim_t \frac{K_{t-1}}{(\chi w_t)^{\varphi}} = \infty$. We deduce $\lim_t F_{K,t} = -\delta$, as well as $\lim_t \mu_t = \infty$, $\lim_t \tau_t^L = (1+\varphi(1+\beta))^{-1}$, and $\lim_t R_t = 0$.

These two non-stationary equilibria feature $\lim_t \mu_t = \infty$ and $\lim_t R_t = 0$.

A.11 Characterization of Positive Public Debt

We prove here Result 1. The financial market clearing condition implies using (65) and the definition of w: $B = (\chi w)^{\varphi}\left(\frac{\beta}{1+\beta}\frac{1-\tau^L}{1+\varphi}F_L - \frac{K}{L}\right)$, which is positive iff: $\frac{\beta}{1+\beta}\frac{1-\tau^L}{1+\varphi} > \frac{1}{F_L}\frac{K}{L}$. Using the definitions of F and $\overline{g}_1$, we can simplify $\frac{1}{F_L}\frac{K}{L}$ and obtain that $B > 0$ iff: $\tau^L < -\frac{1+\varphi}{1-\alpha}\frac{1+\beta}{1-\beta}\overline{g}_1$. Using (87), we get the equivalent condition $g_{FB}(1-\tau^L)^{-\varphi} < \overline{g}_{\text{pos}}$, with:

$$\overline{g}_{\text{pos}} = \frac{1+\beta}{1-\beta}(1+2\varphi)(-\overline{g}_1). \tag{114}$$

B Proof of Proposition 4

We consider the utility: $U(c,l) = \frac{1}{1-\sigma} c^{\gamma(1-\sigma)}(1-l)^{(1-\gamma)(1-\sigma)}$. We prove a result that extends the results of Proposition 4 and provides a full characterization of the equilibrium.

Proposition 9 *In the case of a KPR utility function, an interior steady-state solution* (c_e, c_u, l_e) *with* $\tau^K > 0$ *(if it exists) must satisfy the following conditions.*

1. *Equilibrium allocation definition:*

$$\begin{cases} c_e - \frac{U(c_e,l_e)}{U(c_u,0)} c_u & = & (w_{FB}\gamma(1-l_e) - (1-\gamma)c_e)(\sigma-1)l_e, \\ \frac{\gamma - l_e}{1-l_e} U(c_e,l_e) + \beta\gamma U(c_u,0) & = & 0, \\ G + c_u + c_e & = & y_{FB} l_e. \end{cases} \tag{115}$$

2. *The no first-best condition is*

$$0 \leq \frac{K_{FB}}{L_{FB}} l_{FB} - \frac{\beta}{1-\beta} G \tag{116}$$
$$+ \frac{\beta}{1+\beta} w_{FB} \left(\frac{\gamma}{1-\gamma} \left(1 - l_{FB} - (1-l_{FB})^{\frac{\sigma}{1+\gamma(\sigma-1)}} \right) - l_{FB} \right),$$

where l_{FB} *is the unique root of* $l \in \mathbb{R}_+ \mapsto \frac{\gamma}{1-\gamma} w_{FB} \left(1 - l + (1-l)^{\frac{\sigma}{1+\gamma(\sigma-1)}} \right) + G - y_{FB} l$.

3. *The Straub–Werning condition always holds.*

We first characterize the first best, and then the equilibrium with binding credit constraints, for which we discuss existence conditions.

B.1 The First Best

The first-best corresponds to the allocation chosen by the planner that maximizes aggregate welfare, subject to the sole resource constraint of the economy. The FOCs are:

$$U_c(c_{e,t}, l_{e,t}) = U_c(c_{u,t}, 0), \tag{117}$$
$$-U_l(c_{e,t}, l_{e,t}) = w_{FB} U_c(c_{e,t}, l_{e,t}), \tag{118}$$
$$\mu_t = \beta \mu_{t+1}(1 + F_{K,t}). \tag{119}$$

We thus have at the steady-state: $K/L = K_{FB}/L_{FB}$ and w_{FB} are given by (58) and (62). Using the expression of derivatives, we deduce: $c_{u,FB} = \frac{\gamma}{1-\gamma} w_{FB}(1-l_{e,FB})^{\frac{\sigma}{1+\gamma(\sigma-1)}}$ and

$c_{e,FB} = \frac{\gamma}{1-\gamma} w_{FB}(1 - l_{e,FB})$ with $\frac{\sigma}{1+\gamma(\sigma-1)} > 0$, where using the resource constraint $l_{e,FB}$ is the solution of:

$$\frac{\gamma}{1-\gamma} w_{FB} \left(1 - l_{e,FB} + (1 - l_{e,FB})^{\frac{\sigma}{1+\gamma(\sigma-1)}}\right) = y_{FB} l_{e,FB} - G,$$

that always exists whenever $y_{FB} \geq G$. Combining the resource constraints and the budget constraints, we compute saving choices $a_{e,FB}$ and $a_{u,FB}$. The condition $a_{u,FB} \geq 0$ is equivalent to condition (116), which is an implicit condition on G.

B.2 Case with Binding Credit Constraints

The Ramsey program can then be written as

$$\max_{(c_{e,t}, l_{e,t}, c_{u,t}, a_e, R_t, w_t, B_t)_{t \geq 0}} \sum_{t=0}^{\infty} \beta^t \left(U(c_{e,t}, l_{e,t}) + U(c_{u,t}, 0) \right), \tag{120}$$

$$\text{s.t. } G_t + B_{t-1} + (R_t - 1) a_{e,t-1} + w_t l_{e,t} = F(a_{e,t-1} - B_{t-1}, l_{e,t}) + B_t \tag{121}$$

$$U_c(c_{e,t}, l_{e,t}) = \beta R_{t+1} U_c(c_{u,t+1}, 0), \tag{122}$$

$$-U_l(c_{e,t}, l_{e,t}) = w_t U_c(c_{e,t}, l_{e,t}), \tag{123}$$

where (120) is the Ramsey objective with KPR utility and two-agents types, (121) is the governmental budget constraint, (122) is the Euler equation, and (123) the labor FOC. Observe that the latter FOC can also be written as: $c_{e,t} = \frac{\gamma}{1-\gamma} w_t (1 - l_{e,t})$. Combined with the budget constraint of employed agents, we obtain: $\frac{1-l_{e,t}}{1-\gamma} = 1 - \frac{a_{e,t}}{w_t}$. The constraints $c_{e,t} \geq 0$ and $a_{e,t} \geq 0$ imply $0 \leq \frac{1-l_{e,t}}{1-\gamma} \leq 1$ or:

$$1 \geq l_{e,t} \geq \gamma. \tag{124}$$

The FOCs of the problem (120)–(123) with respect to B_t, $l_{e,t}$, $a_{e,t}$, w_t and R_t can respectively be written as:

$$
\begin{aligned}
\mu_t &= \beta\mu_{t+1}(1+F_{K,t+1}), &(125)\\
0 &= \mu_t(F_{L,t}-w_t)-\lambda_{c,t}(w_tU_{cc,t}+U_{cl,t}) &(126)\\
&\quad +\lambda_{l,t}\left(2w_tU_{cl}(c_{e,t},l_{e,t})+U_{ll}(c_{e,t},l_{e,t})+w_t^2U_{cc}(c_{e,t},l_{e,t})\right),\\
0 &= \mu_t-\beta\mu_{t+1}R_{t+1}-\lambda_{c,t}\left(-U_{cc}(c_{e,t},l_{e,t})-\beta R_{t+1}^2U_{cc}(c_{u,t+1},0)\right) &(127)\\
&\quad +\lambda_{l,t}\left(-U_{cl}(c_{e,t},l_{e,t})-w_tU_{cc}(c_{e,t},l_{e,t})\right),\\
0 &= U_c(c_{e,t},l_{e,t})-\mu_t-\lambda_{c,t}U_{cc}(c_{e,t},l_{e,t}) &(128)\\
&\quad +\frac{\lambda_{l,t}}{l_{e,t}}\left(U_c(c_{e,t},l_{e,t})+l_{e,t}w_tU_{cc}(c_{e,t},l_{e,t})+l_{e,t}U_{cl}(c_{e,t},l_{e,t})\right),\\
0 &= U_c(c_{u,t},0)-\mu_t+\frac{\lambda_{c,t-1}}{a_{e,t-1}}\left(U_c(c_{u,t},0)+R_ta_{e,t-1}U_{cc}(c_{u,t},0)\right). &(129)
\end{aligned}
$$

By difference of (128) and of (129) (shifted by one period and multiplied bu βR_{t+1}) and using FOC (127), we obtain:

$$
\frac{\lambda_{l,t}}{l_{e,t}}=\frac{\lambda_{c,t}}{a_{e,t}} \tag{130}
$$

Plugging this into (126)–(129), we obtain the Ramsey allocation is characterized at the steady state by the following equations:

$$
1=\beta(1+F_K), \tag{131}
$$

$$
\mu\left(\frac{w_{FB}}{w}-1\right)=-\frac{\lambda_cU_c(c_e,l_e)}{a_e}(c_e\frac{U_{cc}(c_e,l_e)}{U_c(c_e,l_e)}-c_e\frac{U_{cl}(c_e,l_e)}{U_l(c_e,l_e)}+l_e\frac{U_{cl}(c_e,l_e)}{U_c(c_e,l_e)}-l_e\frac{U_{ll}(c_e,l_e)}{U_l(c_e,l_e)}), \tag{132}
$$

$$
\mu(1-\beta R)=\frac{\lambda_cU_c(c_e,l_e)}{a_e}\left(c_e\frac{U_{cc}(c_e,l_e)}{U_c(c_e,l_e)}-c_u\frac{U_{cc}(c_u,0)}{U_c(c_u,0)}+l_e\frac{U_{cl}(c_e,l_e)}{U_c(c_e,l_e)}\right), \tag{133}
$$

$$
\mu=U_c(c_e,l_e)+\frac{\lambda_cU_c(c_e,l_e)}{a_e}\left(1+c_e\frac{U_{cc}(c_e,l_e)}{U_c(c_e,l_e)}+l_e\frac{U_{cl}(c_e,l_e)}{U_c(c_e,l_e)}\right). \tag{134}
$$

Equation (131) is the usual modified Golden rule, combining equations (132) and (133) yields a general wedge equation, and equation (134) characterizes the Straub-Werning condition (i.e., $\mu>0$). In the general case, the steady-state Ramsey allocation (c_e,l_e,c_u) and prices (R,w) are characterized by equations (131)–(133), as well as FOCs (122) and (123), the resource constraint $c_e+c_u+G=y_{FB}l_e$ and the budget constraint $c_e+c_u/R=wl_e$.

It can be observed that the equilibrium will never exist if for every allocation (c_e,l_e,c_u), we have $\tau^K = 0$ or $1 - \beta R = 0$. From equation (133), we deduce that this only corresponds to the separable CRRA utility function (of the type $\frac{c^{1-\sigma}}{1-\sigma} + v(l)$). We formalize in the following result.

Result 3 *The capital tax is null for every allocation* (c_e, l_e, c_u) *iff the utility function is separable in consumption and labor and features a constant IES for consumption.*

In other words, the constant CRRA utility function has a particular status: there is no parametrization for which an equilibrium with binding credit constraint can exist.

We now specify the allocation characterization in the case of the KPR utility function. Computing the different partial derivatives of the KPR utility functions implies that (132)–(134) imply:

$$1 - \beta R = (F_L/w - 1)(1-\gamma)(\sigma - 1)l_e, \tag{135}$$

$$\frac{U_c(c_e, l_e)}{\mu} = 1 + (F_L/w - 1)(\gamma - l_e)(\sigma - 1), \tag{136}$$

where (135) is the wedge equation (36) of Proposition 4.

Using the properties of the KPR utility function and (122) and (123) to express prices in the aggregate budget constraint $c_e + \frac{1}{R}c_u = wl_e$ and in the wedge equation (135), we obtain:

$$\frac{\gamma - l_e}{1 - l_e} U(c_e, l_e) + \beta\gamma U(c_u, 0) = 0,$$

$$1 - \frac{U(c_e, l_e)}{U(c_u, 0)} \frac{c_u}{c_e} = \left(w_{FB}\gamma \frac{1 - l_e}{c_e} - (1 - \gamma) \right) (\sigma - 1)l_e.$$

These two equations, together with the resource constraint ($c_e + c_u + G = y_{FB}l_e$) correspond to the system (115) characterizing the allocation in Proposition 9.

Finally, proceeding to similar substitutions to express prices R and w, equation (136) can also be written as:

$$\frac{U_c(c_e, l_e)}{\mu} = 1 - \left(1 - \frac{U(c_e, l_e)}{U(c_u, 0)} \frac{c_u}{c_e}\right) \frac{1 - \gamma/l_e}{1 - \gamma}. \tag{137}$$

Note that $\frac{1-\gamma/l_e}{1-\gamma} \in (0, 1]$ since $l_e \in [\gamma, 1)$ (see inequality (124)). We deduce that equation (137) can also be written as $\frac{U_c(c_e,l_e)}{\mu} = 1 - (1 - \frac{U(c_e,l_e)}{U(c_u,0)} \frac{c_u}{c_e})\tilde{l}_e$ with $\tilde{l}_e \in (0, 1]$. We deduce that $\frac{U_c(c_e,l_e)}{\mu} \geq \min(1, \frac{U(c_e,l_e)}{U(c_u,0)} \frac{c_u}{c_e}) > 0$. Hence, $\mu > 0$ and the Straub-Werning condition always holds. This concludes the proof.

C Proof of Proposition 5

We define

$$y_{FB} := \alpha^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{\beta} - 1 + \delta\,(1-\alpha)\right)\left(\frac{1}{\beta} - 1 + \delta\right)^{\frac{1}{\alpha-1}} > 0, \tag{138}$$

$$w_{FB} := (1-\alpha)(K/L)^{\alpha} (= F_L(K,L)). \tag{139}$$

The quantity y_{FB} corresponds to the GDP per unit of labor supply (in efficient units) in the first-best steady-state equilibrium. Actually, this quantity depends solely on the capital-to-labor ratio, which is the same in all equilibria because the modified golden rule always holds; therefore, it does not depend on whether credit constraints bind. Similarly, w_{FB} is the pre-tax wage, which is identical in all equilibria. We state a result that extends Proposition 5 and characterizes equilibrium in the case of the a separable utility function.

Proposition 10 *In the case of a separable utility function, an interior steady-state solution (c_e, c_u, l_e) with $\tau^K > 0$ (if it exists) must satisfy the following conditions.*

1. *Equilibrium allocation definition:*

$$\begin{cases} \frac{\epsilon^u(c_u)-\epsilon^u(c_e)}{\epsilon^u(c_e)+\epsilon^v(l)} \frac{w_{FB}u'(c_e)-v'(l_e)}{v'(l_e)} &= \frac{u'(c_u)-u'(c_e)}{u'(c_u)}, \\ \beta u'(c_u)c_u + u'(c_e)c_e &= v'(l_e)l_e, \\ G + c_u + c_e &= y_{FB}l_e. \end{cases} \tag{140}$$

2. *The no first-best condition is*

$$\frac{1+\beta}{1-\beta}G \leq \left(\frac{1+\beta}{\beta}\frac{\alpha}{\frac{1}{\beta}+\delta-1} - (1-\alpha)\right)\left(\frac{\alpha}{\frac{1}{\beta}+\delta-1}\right)^{\frac{\alpha}{1-\alpha}} l_{FB}, \tag{141}$$

where l_{FB} is the unique root of $l \in \mathbb{R}_+ \mapsto 2u'^{-1}(w_{FB}^{-1}v'(l)) + G - y_{FB}l$.

3. *The Straub–Werning condition is*

$$\frac{\epsilon^u(c_u) - \epsilon^u(c_e)}{u'(c_e)\,(1-\epsilon^u(c_e)) - u'(c_u)\,(1-\epsilon^u(c_u))} > 0.$$

The first item of the proposition states that the steady-state allocation (c_e, c_u, l_e) can be computed as the solution of a system of three equations (140); the three conditions of Proposition 3 are still present. The second and third items correspond to the no first-best condition and the Straub–Werning condition, respectively. The Laffer condition appears implicitly in system (140), particularly in the third equation. As in the previous section, we start with the first best and then discuss the case with binding credit constraints, using three different methods to show their equivalence.

C.1 First Best

The first-best allocation is the solution of $\max_{(c_{e,t},c_{u,t},l_{e,t},K_t)_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t (u(c_{e,t}) - v(l_{e,t}) + u(c_{u,t}))$ subject to the resource constraint (16). This yields the following FOCs: $c_{e,t} = c_{u,t}$, $v'(l_{e,t}) = F_L(K_{t-1}, l_{e,t})u'(c_{e,t})$, and $\mu_t = \beta(1 + F_K)\mu_{t+1}$. We deduce that at the steady state: $c_{FB} := c_e = c_u$, and $\frac{K}{L} = \alpha^{\frac{1}{1-\alpha}} \left(\frac{1}{\beta} - 1 + \delta\right)^{\frac{1}{\alpha-1}}$. Using y_{FB} and w_{FB} defined in (138) and (139), we obtain: $2c_{FB} + G = y_{FB}l_{FB}$ and $v'(l_{FB}) = w_{FB}u'(c_{FB})$. We deduce that c_{FB} is the root of $c \mapsto 2c + G - y_{FB}v'^{-1}(w_{FB}u'(c))$. With u is strictly concave, v strictly convex, $u'(0) = \infty = v'(\infty)$, and $v'(0) = 0$, c_{FB} exists and is unique.

If the first-best is decentralized, budget constraints and market clearing imply: $2a_{u,FB} = \frac{K}{L}l_{FB} - \frac{\beta}{1-\beta}G - \frac{\beta}{1+\beta}w_{FB}l_{FB}$. The condition $a_{u,FB} \geq 0$ implies then condition (141).

C.2 Binding Credit Constraints

Since unemployed agents are credit-constrained, the planner's program is:

$$\max_{(c_{e,t},l_{e,t},c_{u,t},a_e,R_t,w_t,B_t)_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t \left(u(c_{e,t}) - v(l_{e,t}) + \omega u(c_{u,t})\right), \tag{142}$$

$$\text{s.t. } G_t + B_{t-1} + (R_t - 1)a_{e,t-1} + w_t l_{e,t} = F(a_{e,t-1} - B_{t-1}, l_{e,t}) + B_t, \tag{143}$$

$$u'(c_{e,t}) = \beta R_{t+1} u'(c_{u,t+1}), \text{ and } v'(l_{FB}) = w_{FB}u'(c_{FB}). \tag{144}$$

with furthermore the budget constraints: $c_{e,t} = w_t l_{e,t} - a_{e,t}$ and $c_{u,t} = R_t a_{e,t-1}$.

C.2.1 The Dual Approach

We use here the dual approach to solve for the Ramsey program. See Section C.2.2 for the Lagrangian approach and Section C.2.3 for the primal approach. Denoting by $\beta^t\mu_t$, $\beta^t\lambda_{c,t}$, and $\beta^t\lambda_{l,t}$ the Lagrange multipliers on constraints (143)–(144), the FOCs become:

$$\mu_t = \beta\mu_{t+1}(1 + F_{K,t+1}), \tag{145}$$

$$0 = \mu_t(F_{L,t} - w_t) - \lambda_{c,t}w_t u''(c_{e,t}) - \lambda_{l,t}\left(v''(l_{e,t}) - w_t^2 u''(c_{e,t})\right), \tag{146}$$

$$0 = \mu_t - \beta\mu_{t+1}R_{t+1} + \lambda_{c,t}\left(u''(c_{e,t}) + \beta R_{t+1}^2 u''(c_{u,t+1})\right) - \lambda_{l,t}w_t u''(c_{e,t}), \tag{147}$$

$$0 = u'(c_{e,t}) - \mu_t - \lambda_{c,t}u''(c_{e,t}) + \frac{\lambda_{l,t}}{l_{e,t}}\left(u'(c_{e,t}) + l_{e,t}w_t u''(c_{e,t})\right), \tag{148}$$

$$0 = u'(c_{u,t}) - \mu_t + \frac{\lambda_{c,t-1}}{a_{e,t-1}}\left(u'(c_{u,t}) + R_t a_{e,t-1}u''(c_{u,t})\right). \tag{149}$$

Multiplying FOC (149) considered at $t+1$ by βR_{t+1} and subtracting FOCs (148) and (147), we deduce: $\frac{\lambda_{c,t}}{a_{e,t}} = \frac{\lambda_{l,t}}{l_{e,t}}$. Substituting for $\lambda_{l,t}$ into (146)–(149), we obtain using the budget constraints $c_{e,t} = w_t l_{e,t} - a_{e,t}$, and $c_{u,t+1} = R_{t+1}a_{e,t}$, the FOCs $u'(c_{e,t}) = \beta R_{t+1}u'(c_{u,t+1})$ and $v'(l_{e,t}) = w_t u'(c_{e,t})$, the notation ϵ^u, ϵ^v, and $\tilde{\lambda}_{c,t} := \frac{\lambda_{c,t}}{a_{e,t}}u'(c_{e,t})$:

$$0 = \mu_t(\frac{F_{L,t}}{w_t} - 1) - \tilde{\lambda}_{c,t}\left(\epsilon^u(c_{e,t}) + \epsilon^v(l_{e,t})\right), \tag{150}$$

$$0 = \mu_t - \beta R_{t+1}\mu_{t+1} + \tilde{\lambda}_{c,t}\left(\epsilon^u(c_{e,t}) - \epsilon^u(c_{u,t+1})\right), \tag{151}$$

$$0 = u'(c_{e,t}) - \mu_t + \tilde{\lambda}_{c,t}\left(1 - \epsilon^u(c_{e,t})\right), \tag{152}$$

which together with equations (143)–(145) and budget constraints $c_{e,t} = w_t l_{e,t} - a_{e,t}$ and $c_{u,t} = R_t a_{e,t-1}$ fully characterizes the allocation.

At the steady state, the FOC (145) implies $K/L = K_{FB}/L_{FB}$ such that $F_L = w_{FB}$ and $Y = y_{FB}l_e$. Note that we can rule out the case $\mu = 0$ since it would imply $\tilde{\lambda}_c = 0$ and $u'(c_e) = 0$. We thus easily deduce equation (37). Using (144) to express prices, we obtain that the allocation (c_e, c_u, l_e) is determined by the equations of system (140).

C.2.2 The Lagrangian Approach

We check that the Lagrangian approach yields the same FOCs as in (145)–(149). We still denote by $\beta^t\mu_t$, $\beta^t\lambda_{c,t}$, and $\beta^t\lambda_{l,t}$ the Lagrange multipliers on constraints (143)–(144), and additionally define: $\psi_t^e = -u'(c_{e,t}) + (\lambda_{c,t} - \lambda_{l,t}w_t)u''(c_{e,t})$, and $\psi_t^u = -u'(c_{u,t}) - \lambda_{c,t-1}R_t u''(c_{u,t})$, and $\hat{\psi}_t^x = \mu_t + \psi_t^x$ $(x = u, e)$, similar to (28) and (29) when $\lambda_{l,t} = 0$.

We obtain the following FOCs – where we still denote by $\beta^t\mu_t$ the Lagrange multiplier on the government budget constraint: (i) $\mu_t = \beta\mu_{t+1}(1 + F_{K,t+1})$, (ii) $w_t\psi_t^e = -v'(l_{e,t}) - \lambda_{l,t}v''(l_{e,t}) + \mu_t(F_{L,t} - w_t)$, (iii) $\hat{\psi}_t^e = \beta R_{t+1}\hat{\psi}_{t+1}^u$, (iv) $\hat{\psi}_t^e = \frac{\lambda_{l,t}}{l_{e,t}}u'(c_{e,t})$, and (v) $\hat{\psi}_t^u = \frac{\lambda_{c,t-1}}{a_{e,t-1}}u'(c_{u,t})$. This directly yields (145)–(149) after substitution.

C.2.3 The Primal Approach

We check that the primal approach yields the same FOCs as in (145)–(149). Using individual Euler equations, we express prices as a function of allocation and obtain $R_t = \frac{u'(c_{e,t-1})}{\beta u'(c_{u,t})}$ and $w_t = \frac{v'(l_{e,t})}{u'(c_{e,t})}$. The planner's program writes as:

$$\max_{(c_{e,t},l_{e,t},c_{u,t},a_{e,t},B_t)_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t \left(u(c_{e,t}) - v\left(l_{e,t}\right) + u(c_{u,t})\right),$$

$$\text{s.t. } c_{e,t} + c_{u,t} + G_t + a_{e,t} - B_t = a_{e,t-1} - B_{t-1} + F(a_{e,t-1} - B_{t-1}, l_{e,t}), \tag{153}$$

$$v'(l_{e,t})l_{e,t} = u'(c_{e,t})\left(a_{e,t} + c_{e,t}\right), \tag{154}$$

$$u'(c_{e,t-1})a_{e,t-1} = \beta u'(c_{u,t})c_{u,t}. \tag{155}$$

Denoting by μ_t, κ_l, and $\kappa_c$the discounted Lagrange multipliers on constraints (153)–(155), respectively, we obtain the following FOCs:

$$\mu_t = \beta\mu_{t+1}(1 + F_{K,t+1}), \tag{156}$$

$$0 = -u'(c_{e,t}) + \mu_t \frac{F_{L,t}}{w_t} - \kappa_{l,t}u'(c_{e,t})(1 + \frac{l_{e,t}v''(l_{e,t})}{v'(l_{e,t})}), \tag{157}$$

$$0 = 1 - \frac{\mu_t}{u'(c_{e,t})} + \kappa_{l,t}\left(1 + \frac{c_{e,t}u''(c_{e,t})}{u'(c_{e,t})}\right), \tag{158}$$

$$0 = 1 - \frac{\mu_t}{u'(c_{u,t})} + \kappa_{l,t-1}\left(1 + \frac{c_{u,t}u''(c_{u,t})}{u'(c_{u,t})}\right). \tag{159}$$

FOC (156) is identical to (145), while FOCs (158) and (159) are identical to (148) and (149) when setting $\kappa_{l,t} := \frac{\lambda_{c,t}}{a_{e,t}}$ and using the budget constraints. Finally, FOC (158) is identical to (150) when noticing $\tilde{\lambda}_{c,t} = \kappa_{l,t}u'(c_{e,t})$ and using budget constraint.

C.3 Proof of Proposition 6

In the case of the non-utilitarian planner, we can prove the following proposition characterizing the Ramsey allocation (which is again more general than Proposition 6).

Proposition 11 *With an non-utilitarian planner and a CRRA-separable utility function, An interior solution* (c_e, c_u, l_e) *(if it exists) must satisfy the following sets of conditions.*

1. *The allocation is determined by* (c_e, c_u, l_e)*:*

$$\left\{\begin{array}{rcl} \frac{(\omega-1)(1-\sigma)}{\sigma+\frac{1}{\varphi}}\left(\frac{w_{FB}c_e^{-\sigma}}{\chi^{-1}l_e^{\frac{1}{\varphi}}}-1\right) & = & \omega-\left(\frac{c_e}{c_u}\right)^{-\sigma}, \\ \beta c_u^{1-\sigma}+c_e^{1-\sigma} & = & \chi^{-1}l_e^{1+\frac{1}{\varphi}}, \\ G+c_u+c_e & = & y_{FB}l_e. \end{array}\right. \tag{160}$$

2. *The first-best condition is* $\frac{1+\beta}{1-\beta}G \leq \left(\frac{1+\beta}{1-\beta}\frac{\alpha}{\frac{1}{\beta}+\delta-1}-(1-\alpha)\right)\left(\frac{\alpha}{\frac{1}{\beta}+\delta-1}\right)^{\frac{\alpha}{1-\alpha}}\chi^{\varphi}w_{FB}^{\varphi}c_e^{-\varphi\sigma}+\frac{\beta}{1+\beta}(1-\omega^{\frac{1}{\sigma}})c_e$*, where* c_e *is the unit root of* $c\in\mathbb{R}_+\mapsto c(1+\omega^{\frac{1}{\sigma}})+G-y_{FB}\chi^{\varphi}w_{FB}^{\varphi}c^{-\varphi\sigma}$*.*

3. *The Straub–Werning condition is* $\omega<1$*.*

C.3.1 First Best

The first-best allocation is the solution of $\max_{(c_{e,t},c_{u,t},l_{e,t},K_t)_{t\geq 0}}\sum_{t=0}^{\infty}\beta^t(u(c_{e,t})-v(l_{e,t})+\omega u(c_{u,t}))$, subject to the resource constraint. This implies the following FOCs: (i) $u'(c_{e,t})=\omega u'(c_{u,t})$, (ii) $v'(l_{e,t})=F_L(K_{t-1},l_{e,t})u'(c_{e,t})$, and (iii) $\mu_t=\beta(1+F_K)\mu_{t+1}$. We deduce that at the steady state, we have $K/L=K_{FB}/L_{FB}$, $c_{e,FB}+c_{u,FB}+G=y_{FB}l_{FB}$, and $v'(l_{FB})=w_{FB}u'(c_{e,FB})$. We deduce that $c_{e,FB}$ is the root of $c\mapsto c+u'^{-1}(\omega u'(c))+G-y_{FB}v'^{-1}(w_{FB}u'(c))$. With the assumptions on u and v, we deduce that $c_{e,FB}$ and $c_{u,FB}$ exist and are unique.

If the first-best is decentralized, budget constraints imply: $2a_{u,FB}=\frac{K}{L}l_{FB}-\frac{\beta}{1-\beta}G-\frac{\beta}{1+\beta}(w_{FB}l_{FB}-c_{e,FB}+c_{u,FB})$. The condition $a_{u,FB}\geq 0$ implies the first-best condition in Proposition 11 (Item 2).

C.3.2 Binding Credit Constraints

Since unemployed agents are credit-constrained, the planner's program can be written as: $\max_{(c_{e,t},l_{e,t},c_{u,t},a_e,R_t,w_t,B_t)_{t\geq 0}}\sum_{t=0}^{\infty}\beta^t\left(u(c_{e,t})-v(l_{e,t})+\omega u(c_{u,t})\right)$, subject to constraints (143)–(144) and budget constraints: $c_{e,t}=w_tl_{e,t}-a_{e,t}$ and $c_{u,t}=R_ta_{e,t-1}$. For the sake of simplicity, we only use here on the dual approach. Denoting by $\beta^t\mu_t$, $\beta^t\lambda_{c,t}$, and $\beta^t\lambda_{l,t}$ the

Lagrange multipliers on constraints (143)–(144), FOCs are:

$$\mu_t = \beta\mu_{t+1}(1 + F_{K,t+1}), \tag{161}$$

$$0 = \mu_t(F_{L,t} - w_t) - \lambda_{c,t}w_t u''(c_{e,t}) - \lambda_{l,t}\left(v''(l_{e,t}) - w_t^2 u''(c_{e,t})\right), \tag{162}$$

$$\begin{aligned} 0 &= \mu_t - \beta\mu_{t+1}R_{t+1} - u'(c_{e,t}) + \beta R_{t+1}\omega u'(c_{u,t+1}) \\ &\quad + \lambda_{c,t}\left(u''(c_{e,t}) + \beta R_{t+1}^2 u''(c_{u,t+1})\right) - \lambda_{l,t}w_t u''(c_{e,t}), \end{aligned} \tag{163}$$

$$0 = u'(c_{e,t}) - \mu_t - \lambda_{c,t}u''(c_{e,t}) + \frac{\lambda_{l,t}}{l_{e,t}}\left(u'(c_{e,t}) + l_{e,t}w_t u''(c_{e,t})\right), \tag{164}$$

$$0 = \omega u'(c_{u,t}) - \mu_t + \frac{\lambda_{c,t-1}}{a_{e,t-1}}u'(c_{u,t})(1 + \frac{c_{u,t}u''(c_{u,t})}{u'(c_{u,t})}). \tag{165}$$

Multiplying FOC (165) considered at $t+1$ by βR_{t+1} and subtracting FOCs (164) and (163), we deduce: $\frac{\lambda_{c,t}}{a_{e,t}} = \frac{\lambda_{l,t}}{l_{e,t}}$. Plugging this into (162)–(165), we obtain using $\tilde{\lambda}_{c,t} := \frac{\lambda_{c,t}}{a_{e,t}}u'(c_{e,t})$:

$$0 = \mu_t(\frac{F_{L,t}}{w_t} - 1) - \tilde{\lambda}_{c,t}\left(\epsilon^u(c_{e,t}) + \epsilon^v(l_{e,t})\right), \tag{166}$$

$$0 = \mu_t - \beta R_{t+1}\mu_{t+1} + (\omega - 1)u'(c_{e,t}) + \tilde{\lambda}_{c,t}\left(\epsilon^u(c_{e,t}) - \epsilon^u(c_{u,t+1})\right), \tag{167}$$

$$0 = \omega\mu_t - \beta R_{t+1}\mu_{t+1} + \tilde{\lambda}_{c,t}\left(1 - \epsilon^u(c_{u,t+1}) - \omega(1 - \epsilon^u(c_{e,t}))\right). \tag{168}$$

We thus obtain at the steady state $K/L = K_{FB}/L_{FB}$ from equation (161) and by combination of (167)–(168):

$$\frac{\omega - \beta R}{\frac{F_L}{w} - 1} = \frac{\omega(1 - \epsilon^u(c_e)) - (1 - \epsilon^u(c_u))}{\epsilon^u(c_e) + \epsilon^v(l_e)},$$

which is equation (37). Using the individual FOCs stating that $\beta R = \left(\frac{c_e}{c_u}\right)^{-\sigma}$ and $wc_e^{-\sigma} = \chi^{-1}l_e^{\frac{1}{\varphi}}$, we obtain:

$$\frac{(\omega - 1)(1 - \sigma)}{\sigma + \frac{1}{\varphi}}\left(\frac{w_{FB}c_e^{-\sigma}}{\chi^{-1}l_e^{\frac{1}{\varphi}}} - 1\right) = \omega - \left(\frac{c_e}{c_u}\right)^{-\sigma},$$

while the two other equations of (160) come from the aggregate budget constraint $c_e + c_u/R = w_{FB}l_e$ and from the resource constraint. Finally solving (123)–(168) at the steady state yields $\frac{\mu}{u'(c_e)u'(c_u)} = \frac{1-\omega}{u'(c_u)-u'(c_e)}$, which implies $\omega < 1$ for the Straub-Werning condition.

D Simple Model Dynamics After a Period-0 Shock

D.1 Model Linearization and Proof of Proposition 7

Defining $\theta = \frac{1}{1+\varphi}\frac{\beta}{1+\beta}$, FOCs (69) and (70) and government budget constraint (68) become:

$$\mu_t = \beta(1 + \alpha K_t^{\alpha-1}\chi^{(1-\alpha)\varphi} w_{t+1}^{(1-\alpha)\varphi} - \delta)\mu_{t+1}, \tag{169}$$

$$0 = 1 - \mu_t w_t (\chi w_t)^{\varphi} (1-\theta) + \frac{\varphi}{1+\varphi}\mu_t (1-\alpha) K_{t-1}^{\alpha} (\chi w_t)^{\varphi(1-\alpha)}, \tag{170}$$

$$K_{t-1}^{\alpha}(\chi w_t)^{\varphi(1-\alpha)} = G_t + K_t - (1-\delta)K_{t-1} + \frac{1}{\mu_t} + (1-\theta) w_t (\chi w_t)^{\varphi}. \tag{171}$$

We deduce R_t from $1 = R_t \mu_t \theta w_{t-1}(\chi w_{t-1})^{\varphi}$ (i.e., FOC (71)) and B_t from $B_t = \theta w_t (\chi w_t)^{\varphi} - K_t$ (i.e., financial market clearing). We denote by a hat the proportional deviation to the steady-state value. The linearization of equations (169)–(171) yields:

$$\widehat{\mu}_t = E_t \widehat{\mu}_{t+1} + (1-\beta(1-\delta))((\alpha-1)\widehat{K}_t + (1-\alpha)\varphi E_t \widehat{w}_{t+1}), \tag{172}$$

$$0 = -\alpha \widehat{K}_{t-1} + (A-1)\widehat{\mu}_t + ((\varphi+1)(A-1) + 1 + \varphi\alpha)\widehat{w}_t, \tag{173}$$

$$0 = \frac{G}{Y}\widehat{G}_t + \frac{\alpha}{\frac{1}{\beta} - 1 + \delta}\left(\widehat{K}_t - \beta^{-1}\widehat{K}_{t-1}\right) - (A-1)(1-\alpha)\varphi\left(\frac{\widehat{\mu}_t}{1+\varphi} - \widehat{w}_t\right), \tag{174}$$

where τ^L is defined in (87) and $A := (1 + \frac{1}{\varphi(1+\beta)})(1-\tau^L) > 1$. The inequality $A > 1$ comes from the Straub-Werning condition.

In the remainder, we will focus on full capital depreciation: $\delta = 1$.

Dynamic system. In that case, we obtain from (172)–(174):

$$E_t\left[\widehat{\mu}_{t+1}\right] = r_\mu \widehat{\mu}_t + t_\mu \widehat{K}_t, \tag{175}$$

$$\widehat{K}_t = r_K \widehat{\mu}_t + t_K \widehat{K}_{t-1} + s_K \widehat{G}_t, \tag{176}$$

where we have defined:

$$r_\mu = \frac{(1+\varphi)(A-1) + 1 + \alpha\varphi}{(1+\alpha\varphi)A}, \; t_\mu = (1-\alpha)\frac{(1+\varphi)(A-1)+1}{(1+\alpha\varphi)A}, \; t_K = \frac{1}{\beta}\frac{1}{r_\mu}, \tag{177}$$

$$r_K = \frac{1-\alpha}{\alpha\beta}(A-1)\frac{\varphi}{1+\varphi}\left(1 + \frac{(1+\varphi)(A-1)}{(1+\varphi)(A-1) + 1 + \varphi\alpha}\right), \; s_K = -\frac{G}{\alpha\beta Y}. \tag{178}$$

Since $A > 1$, it can be checked that the coefficients t_K, r_K, t_μ are positive, while $r_\mu > 1$ and $s_K < 0$. Note that all these coefficients are defined at the steady-state and are independent of the values $\widehat{G}_0, \rho_G$ defining the dynamics of the shock $\widehat{G}_t$.

Deriving a simplified dynamic system. Using an identification method, we look for coefficients ρ_K, σ_K, ρ_μ, σ_μ, such that, for $t > 1$:

$$\widehat{K}_t = \rho_K \widehat{K}_{t-1} + \sigma_K \widehat{G}_t \tag{179}$$

$$\widehat{\mu}_t = \rho_\mu \widehat{K}_{t-1} + \sigma_\mu \widehat{G}_t. \tag{180}$$

Combining (175)–(176) yields: $E_t\widehat{K}_{t+1} - (t_K + r_\mu + r_K t_\mu)\widehat{K}_t + r_\mu t_K \widehat{K}_{t-1} = (s_K \rho_G - r_\mu s_K)\widehat{G}_t$. Using (179), we obtain that ρ_K must solve the following equation:

$$\rho_K^2 - (t_K + r_\mu + r_K t_\mu)\rho_K + r_\mu t_K = 0, \tag{181}$$

whose discriminant is: $D = (t_K + r_\mu + r_K t_\mu)^2 - 4 r_\mu t_K$. Since $t_K, r_\mu, r_K, t_\mu \geq 0$, we have $D \geq (t_K + r_\mu)^2 - 4 r_\mu t_K = (t_K - r_\mu)^2 > 0$, where the strict inequality comes from $t_K = \frac{1}{\beta r_\mu} > 0$. Equation (181) thus admits two distinct roots, which are:

$$\rho_{K,1} = \frac{t_K + r_\mu + r_K t_\mu + \sqrt{D}}{2} \text{ and } \rho_{K,2} = \frac{t_K + r_\mu + r_K t_\mu - \sqrt{D}}{2}. \tag{182}$$

Since $(t_K + r_\mu + r_K t_\mu)^2 > D > 0$, we deduce that $0 < \rho_{K,2} < \rho_{K,1}$.

Proof of Proposition 7. Let us now prove that condition (41) is a necessary and sufficient condition for equilibrium stability. Since $0 < \rho_{K,2} < \rho_{K,1}$ and $\rho_{K,2}\rho_{K,1} = \beta^{-1} > 1$, we must have $\rho_{K,1} > 1$, which imposes that $\rho_K = \rho_{K,2}$. The Blanchard-Kahn condition for the system stability requires $\rho_{K,2} < 1$. Note that in the limit case when the equilibrium does not exist (i.e., $\tau^K = \overline{\tau}^K_{SW} = \frac{1}{1-\beta}$), and which corresponds to $A = 1$, it is straightforward to check that $\rho_{K,2} = 1$ and that the dynamic system is not stable. The condition $\rho_{K,2} < 1$ is equivalent to $J := t_K + r_\mu + r_K t_\mu - r_\mu t_K - 1 > 0$. Using equations (177)–(178), we can show that:

$$\begin{aligned} \frac{J}{J_0} &= (\beta(1+\varphi)(A-1) + (1+\alpha\varphi)(\beta - A)) \\ &+ \frac{1-\alpha}{\alpha(1+\varphi)}((1+\varphi)(A-1)+1)\left(2(1+\varphi)(A-1) + 1 + \varphi\alpha\right), \\ \text{where: } J_0 &= \frac{\varphi(1-\alpha)(A-1)}{\beta(1+\alpha\varphi)A((1+\varphi)(A-1)+1+\varphi\alpha)}. \end{aligned}$$

Since $A > 1$, $J_0 > 0$ and the sign of J is the one of $P(A-1)$, which is defined as:

$$P(A-1) = \frac{1+\alpha\varphi}{1+\varphi}(-(1-\beta)(1+\varphi) + \frac{1-\alpha}{\alpha}) + (A-1)^2 \frac{1-\alpha}{\alpha} 2(1+\varphi) \\ + (A-1)\left(-(1-\beta)(1+\varphi) + \frac{1-\alpha}{\alpha} + 2(1+\alpha\varphi)\frac{1-\alpha}{\alpha}\right).$$

A necessary condition for $P(A-1) > 0$ for all $A > 1$ is $P(0) \geq 0$. However, $P(0) \geq 0 \Rightarrow P'(0) > 0$. So, since $P''(0) \geq 0$, $P(0) \geq 0$ is a necessary and sufficient condition for $P(A-1) > 0$ for $A > 1$. The condition $P(0) \geq 0$ is equivalent to (41), which concludes the proof of condition (41). Note that a sufficient condition for stability is $\overline{g}_1 < 0$ since it implies (41).

D.2 Characterizing the Dynamics of Capital and Public Debt and Proof of Proposition 8

Characterization of the system (179)–(180). We deduce from (175)–(176) that $(r_\mu - \rho_K)\rho_\mu = -t_\mu \rho_K$. Since $r_\mu > 1$, $t_\mu > 0$, and $\rho_K \in (0,1)$, we deduce that $\rho_\mu < 0$. Regarding parameters σ_K and σ_μ, we have from (175)–(176):

$$\sigma_K = r_K \sigma_\mu + s_K, \tag{183}$$

$$r_\mu \sigma_\mu = (\rho_\mu - t_\mu)\sigma_K + \sigma_\mu \rho_G. \tag{184}$$

Equation (184) implies $(r_\mu - \rho_G)\sigma_\mu = (\rho_\mu - t_\mu)\sigma_K$. Using $r_\mu > 1 > \rho_G$ and the definition of ρ_μ implying that $\rho_\mu - t_\mu = r_\mu \rho_\mu / \rho_K < 0$, we deduce that σ_μ and σ_K have opposite signs. Using $r_K > 0$ and $s_K < 0$ in equation (183), we deduce that $\sigma_\mu > 0 > \sigma_K$.

The role of the shock persistence ρ_G. Combining (183) and (184) yields: $(r_\mu + (t_\mu - \rho_\mu) r_K)\sigma_\mu = (\rho_\mu - t_\mu) s_K + \sigma_\mu \rho_G$, or using the implicit function theorem: $(r_\mu - \rho_G + (t_\mu - \rho_\mu) r_K)\frac{\partial \sigma_\mu}{\partial \rho_G} = \sigma_\mu$, since only σ_μ (and σ_K) depend on ρ_G. Since $r_\mu > 1 > \rho_G$, and $\sigma_\mu, t_\mu, r_K > 0 > \rho_\mu$, we deduce using the previous equation and (183) that: $\frac{\partial \sigma_\mu}{\partial \rho_G} > 0$ and $\frac{\partial \sigma_K}{\partial \rho_G} > 0$. The previous derivative, and equation (180), imply $\widehat{\mu}_0 = \sigma_\mu \widehat{G}_0$, which implies that for the same initial shock $\widehat{G}_0$, the increase in $\widehat{\mu}_0$ is higher, the higher the persistence:

$$\left.\frac{\partial \widehat{\mu}_0}{\partial \rho_G}\right|_{\widehat{G}_0} > 0 \tag{185}$$

Then, from (173), we have: $\widehat{w}_0 = -\frac{A-1}{((\varphi+1)(A-1)+1+\varphi\alpha}\widehat{\mu}_0$, which implies $\frac{\partial \widehat{w}_0}{\partial \rho_G}\Big|_{\widehat{G}_0} < 0$. Finally, from $\frac{\partial \sigma_K}{\partial \rho_G} > 0$, we deduce $\frac{\partial \widehat{K}_0}{\partial \rho_G} < 0$.

Dynamic of the capital stock. By induction, (39) and (179) imply: $\widehat{G}_t = \rho_G^t \widehat{G}$ and $\widehat{K}_t = \sigma_K \phi(t)\widehat{G}_0$, with $\phi(t) = \frac{\rho_K^{t+1}-\rho_G^{t+1}}{\rho_K-\rho_G}$ if $\rho_K \neq \rho_G$, or $(t+1)\rho_G^t$ if $\rho_K = \rho_G$. We have $\phi(0) = 1$, $\phi(\infty) = 0$. Moreover, $\phi'(t_m) = 0$ iff:

$$t_m + 1 = \begin{cases} \frac{\ln(-\ln(\rho_K))-\ln(-\ln(\rho_G))}{\ln(\rho_G)-\ln(\rho_K)} > 0 & \text{if } \rho_K \neq \rho_G, \\ -\frac{1}{\ln(\rho_G)} > 0 & \text{if } \rho_K = \rho_G. \end{cases}$$

It is direct to check that $\phi'(t) > 0$ iff $t < t_m$. The capital response is procyclical (it has the sign of $\widehat{G}_0$). When $\widehat{G}_0 > 0$, capital increases until date t_m before decreasing and converging back to its steady-state value.

We now investigate the impact of ρ_G on t_m. Defining $r_G := -\ln(\rho_G)$ and $r_K := -\ln(\rho_K)$, we obtain: $\frac{\partial t_m}{\partial r_G} = \frac{\frac{r_G-r_K}{r_G}-(\ln(r_G)-\ln(r_K))}{(r_G-r_K)^2}$ if $\rho_K \neq \rho_G$. By the Taylor-Lagrange theorem, there exists $r \in (r_K, r_G)$, such that: $\ln(r_K) - \ln(r_G) = \frac{r_K-r_G}{r_G} - \frac{(r_K-r_G)^2}{2r^2}$, from which we deduce:

$$\frac{\partial t_m}{\partial r_G} = \begin{cases} \frac{-\frac{(r_K-r_G)^2}{2r^2}}{(r_G-r_K)^2} < 0 & \text{if } \rho_K \neq \rho_G, \\ -\frac{1}{r_G^2} < 0 & \text{if } \rho_K = \rho_G. \end{cases}$$

So t_m decreases with r_G and increases with ρ_G: the more persistent ρ_G, the longer the impact of capital dynamics.

Dynamics of public debt. Regarding public debt, the financial market clearing implies that $B_t = \frac{\beta}{1+\beta}\frac{\chi^\varphi}{1+\varphi}w_t^{1+\varphi} - K_t$. Defining $\alpha_B := \frac{1}{B}\frac{\beta}{1+\beta}\frac{\chi^\varphi}{1+\varphi}w^{1+\varphi}$, we have: $\widehat{B}_t = \alpha_B \widehat{w}_t - (\alpha_B - 1)\widehat{K}_t$. Using equations (70), (179), and (180), one finds $\widehat{B}_t = \Theta^K \widehat{G}_0 \rho_K^t - \Theta^G \widehat{G}_0 \rho_G^t$, with:

$$\Theta^K := \left(\alpha_B \frac{\alpha - (A-1)\,\rho_\mu}{(\varphi+1)(A-1)+1+\varphi\alpha} - (\alpha_B - 1)\,\rho_K\right)\frac{\sigma_K}{\rho_K-\rho_G}, \tag{186}$$

$$\Theta^G := \left(\alpha_B \frac{\alpha - (A-1)\,\rho_\mu}{(\varphi+1)(A-1)+1+\varphi\alpha} - (\alpha_B - 1)\,\rho_K\right)\frac{\sigma_K}{\rho_K-\rho_G} + \alpha_B \frac{A-1}{(\varphi+1)(A-1)+1+\varphi\alpha}\sigma_\mu + (\alpha_B - 1)\,\sigma_K. \tag{187}$$

Proof of Proposition 8. At impact ($t = 0$), we have:

$$B\widehat{B}_0 = -\left(\frac{\beta}{1+\beta}\chi^{\varphi}w^{1+\varphi}\frac{A-1}{(\varphi+1)(A-1)+1+\varphi\alpha}\sigma_{\mu}(\rho_G) + \sigma_K(\rho_G)K\right)\hat{G}_0(\rho_G), \quad (188)$$

where we have explicitly noted the dependence on ρ_G. Recall that $\frac{\partial\sigma_\mu}{\partial\rho_G} > 0$, $\frac{\partial\sigma_K}{\partial\rho_G} > 0$, and since the $N\hat{P}V_0$ is fixed and $\hat{G}_0$ endogenous, $\left.\frac{\partial\widehat{G}_0}{\partial\rho_G}\right|_{N\hat{P}V} < 0$.

As a consequence, if the public debt is positive at the steady state ($B > 0$ equivalent to $\bar{g}_1 < 0$ – see Section A.11), then for a positive exogenous initial shock, $\hat{G}_0 > 0$, $\frac{\partial\sigma_\mu}{\partial\rho_G} > 0$, $\frac{\partial\sigma_K}{\partial\rho_G} > 0$ imply $\frac{\partial\widehat{B}_0}{\partial\rho_G} < 0$. The higher the shock persistence, the greater the variation of public debt at impact decreases: $\left.\frac{\partial\widehat{B}_0}{\partial\rho_G}\right|_{\hat{G}_0} < 0$.

In the case of a constant $N\hat{P}V_0$, we have: $B\left.\frac{\partial\widehat{B}_0}{\partial\rho_G}\right|_{N\hat{P}V_0} = \left.\frac{\partial\widehat{B}_0}{\partial\rho_G}\right|_{\hat{G}_0} + \frac{B\widehat{B}_0}{\hat{G}_0(\rho_G)}\left.\frac{\partial\widehat{G}_0}{\partial\rho_G}\right|_{N\hat{P}V_0}$. If in addition to $B > 0$, we also have $\widehat{B}_0 > 0$, we deduce since $\left.\frac{\partial\widehat{B}_0}{\partial\rho_G}\right|_{\hat{G}_0} < 0$ and $\left.\frac{\partial\widehat{G}_0}{\partial\rho_G}\right|_{N\hat{P}V} < 0$: $B\left.\frac{\partial\widehat{B}_0}{\partial\rho_G}\right|_{N\hat{P}V_0} < 0$.

E Numerical Examples for Separable Utility Functions

E.1 The KPR Utility Function

We consider the calibration of Table 4. The preference parameters (σ and γ) are in the same ballpark as to those of Dyrda and Pedroni (2022). The other parameter (β, α and δ) are set to standard values. In the equilibrium with binding credit constraints for

Parameters	Value
discount factor β	0.96
capital share α	0.36
capital depreciation rate δ	0.025
steady-state public spending G	0.5
inverse of IES, σ	2.0
consumption share, γ	0.6

Table 4: Calibration of an economy with a KPR utility function.

unemployed agents, this calibration generates the allocation and prices described in Table

5 – where we do not repeat that for unemployed, labor supply and asset holdings are null.

Allocation and taxes		
employed agents	consumption c_e	0.644
	labor supply l_e	0.719
unemployed agents	consumption c_u	0.462
taxes	capital tax τ^K	58.982%
	labor tax τ^L	7.582%

Table 5: Allocation in the economy with the calibration of Table 4.

E.2 The Fishburn Utility Function

As in the KPR case, we provide a numerical example rather than algebra derivation. We consider the calibration of Table 6. The other parameters (β, α, δ, and G) are identical to those of Table 4. Note that γ and σ do not play any role in this case.

Parameters	Value
labor scaling factor χ	1.0
Frish elasticity φ	0.5
utility consumption threshold $\underline{c}$	0.7

Table 6: Calibration of an economy with a Fishburn utility function. Other parameters as in Table 4.

This calibration generates the allocation and prices described in Table 7.

Allocation and taxes		
employed agents	consumption c_e	0.934
	labor supply l_e	1.171
unemployed agents	consumption c_u	0.697
taxes	capital tax τ^K	9.160%
	labor tax τ^L	0.007%

Table 7: Allocation in the economy with the calibration of Table 6.

Consumption levels are consistent with the threshold $\underline{c}$, since $c_u < \underline{c} < c_e$. Tax rates are positive, and the post-tax gross rate R verifies $0 < \beta R < 1$. Since the function is DRRA, the equilibrium existence involves positive taxes and positive NDG. The equilibrium with binding credit constraint exists.[30]

E.3 The Stone-Geary Utility Function

We use the calibration of Table 8, while other parameters (β, α, δ, χ, and φ) are identical to those of Table 6.

Parameters	Value
utility consumption threshold $\underline{c}$	1.0
steady-state public spending G	4.3278
inverse of IES σ	1.0

Table 8: Calibration with a Stone-Geary utility function. Other parameters as in Table 6.

We obtain the allocation of Table 9, featuring positive taxes. As in the Fishburn case, the equilibrium with binding credit constraints exists and features positive capital and labor taxes. With a DRRA utility function, the NDG is positive.

Allocation and prices		
employed agents	consumption c_e	1.087
	labor supply l_e	2.910
unemployed agents	consumption c_u	1.085
taxes	capital tax τ^K	0.626
	labor tax τ^L	0.552

Table 9: Allocation of the economy with the calibration of Table 8.

E.4 The CARA Utility Function

We use the calibration of Table 10. The other parameters (β, α, δ, and φ) are identical to those of Table 6. Note that $\underline{c}$ does not play any role in this case.

The allocation featuring positive capital taxes is summarized in Table 11. The equilibrium with binding credit constraint for unemployed agents thus exists with CARA utility

[30]We have also checked that: (i) the public spending is too high for the first-best equilibrium to exist, (ii) the calibration fulfills the Blanchard-Kahn conditions, and (iii) our main result of Proposition 8 still holds. We also did so for other specifications (Stone-Geary, CARA and non-Utilitarian planner), even though we do not mention it below.

Parameters	Value
labor scaling factor χ	0.58811
steady-state public spending G	0.01
absolute risk aversion γ	1.0

Table 10: Calibration with a CARA utility function. Other parameters as in Table 6.

function. A particularity of this equilibrium comes from the IRRA property of CARA utilities. Indeed, IRRA utilities imply negative NDG and hence a negative labor tax for the capital tax to be positive. 7

Allocation and prices		
employed agents	consumption c_e	0.160
	labor supply l_e	0.139
unemployed agents	consumption c_u	0.141
taxes	capital tax τ^K	0.473
	labor tax τ^L	-0.295

Table 11: Allocation of the economy with the calibration of Table 10.

E.5 The non-Utilitarian Planner

We consider the calibration of Table 12.

Parameters	Value
utility consumption threshold $\underline{c}$	0.0
steady-state public spending G	0.8
weight of unemployed agents ω	0.99

Table 12: Calibration with a non-utilitarian planner. Other parameters as in Table 6.

Note that $\underline{c} = 0$ since we focus on the CRRA case. The allocation featuring positive capital taxes is summarized in Table 13. The equilibrium with binding credit constraint for unemployed agents thus exists with CRRA utility function and non-Utilitarian planner.

Allocation and prices		
employed agents	consumption c_e	1.000
	labor supply l_e	1.251
unemployed agents	consumption c_u	0.995
taxes	capital tax τ^K	0.125
	labor tax τ^L	0.052

Table 13: Allocation of the economy with the calibration of Table 12.

F The General Model

F.1 Deriving FOCs

The Lagrangian of the Ramsey program (44)–(50) can be written as:

$$\begin{aligned}\mathcal{L} = &\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i)\ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_{c,t}^i - R_t \lambda_{c,t-1}^i\right) U_c(c_t^i, l_t^i)\ell(di) \\ &+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \lambda_{l,t}^i \left(U_l(c_t^i, l_t^i) + (1-\tau_t) w_t (y_t^i)^{1-\tau_t} (l_t^i)^{-\tau_t} U_c(c_t^i, l_t^i)\right) \ell(di) \\ &- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Bigg(G_t + (1-\delta) B_{t-1} + (R_t - 1 + \delta) \int_i a_{t-1}^i \ell(di) + w_t \int_i (y_t^i l_t^i)^{1-\tau_t} \ell(di) \\ &\qquad - (\int_i a_{t-1}^i \ell(di) - B_{t-1})^\alpha (\int_i y_t^i l_t^i \ell(di))^{1-\alpha} - B_t \Bigg).\end{aligned} \tag{189}$$

FOC with respect to savings choices. Using $\frac{\partial c_t^j}{\partial a_t^i} = -1_{i=j}$ and $\frac{\partial c_{t+1}^j}{\partial a_t^i} = R_{t+1} 1_{i=j}$, and the notation ψ_t^i of (51), deriving (189) with respect to a_t^i yields:

$$\psi_t^i = \beta \mathbb{E}_t \left[R_{t+1} \psi_{t+1}^i \right] + \beta \mathbb{E}_t \left[\mu_{t+1} (1 + \tilde{r}_{t+1} - R_{t+1}) \right].$$

FOC with respect to labor supply. Deriving (189) with respect to l_t^i yields:

$$\psi_{l,t}^i = (1-\tau_t) w_t y_t^i (y_t^i l_t^i)^{-\tau_t} \hat{\psi}_t^i + \mu_t F_{L,t} y_t^i - \lambda_{l,t}^i (1-\tau_t) \tau_t w_t y_t^i (y_t^i l_t^i)^{-\tau_t} U_c(c_t^i, l_t^i)/l_t^i, \tag{190}$$

where: $\psi_{l,t}^i = -\omega_t^i U_l(c_t^i, l_t^i) - \lambda_{l,t}^i U_{ll}(c_t^i, l_t^i) + (\lambda_{c,t}^i - R_t \lambda_{c,t-1}^i - \lambda_{l,t}^i (1-\tau_t) w_t (y_t^i)^{1-\tau_t} (l_t^i)^{-\tau_t}) U_{cl}(c_t^i, l_t^i)$ is similar to (51) but for labor supply and not for consumption.

FOC with respect to the interest rate. Deriving (189) with respect to R_t yields:

$$0 = \int_j \left(\hat{\psi}_t^j a_{t-1}^j + \lambda_{c,t-1}^j U_c(c_t^j, l_t^j)\right) \ell(dj).$$

FOC with respect to the wage rate. Deriving (189) with respect to w_t yields:

$$0 = \int_j (y_t^j l_t^j)^{1-\tau_t} \left(\hat{\psi}_t^j + \lambda_{l,t}^j (1-\tau_t) U_c(c_t^j, l_t^j)/l_t^j\right) \ell(dj).$$

FOC with respect to public debt. Deriving (189) with respect to B_t yields:

$$\mu_t = \beta(1 + \tilde{r}_{t+1})\mu_{t+1}.$$

FOC with respect to progressivity.

$$0 = \int_j (y_t^j l_t^j)^{1-\tau_t} \left(\hat{\psi}_t^j + \lambda_{l,t}^j (1-\tau_t) U_c(c_t^j, l_t^j)/l_t^j\right) \ln(y_t^j l_t^j)(dj)$$
$$+ \int_j \lambda_{l,t}^j (y_t^j l_t^j)^{1-\tau_t} (U_c(c_t^j, l_t^j)/l_t^j) \ell(dj).$$

F.2 Consistency of the Two Approaches

We verify here that the two approaches we consider (the analytical one of Section 3.1 and the quantitative one of Section 4) yield consistent results. We proceed in two steps. First, in Section F.2.1, we check that the application of the Lagrangian approach to the environment of Section 3.1 delivers the same FOCs as in the analytical approach (equations (103)–(105)). Second, in Section F.2.2, we compare the quantitative outcomes of the two approaches and show that the analytical solution is the limit of the general solution when the transition matrix converges to the anti-diagonal matrix.

F.2.1 Checking that FOCs are Identical

We check here that the FOCs of the Ramsey program derived in the general case of Section 4.1 (i.e., equations (53)–(57)) exactly simplify to the FOCs derived in the specific case of Section 3.1 (i.e., equations (69)–(72)). The larger number of equations in the first case comes from the definitions of Lagrange multipliers. We start with expressing ψ_t^i and $\psi_{l,t}^i$ in the context of the GHH utility function. We denote $C = c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}$. Since $U(c,l) = \ln\left(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}\right)$, we have: $U_c(c,l) = \frac{1}{C}$, $U_{cc}(c,l) = -\frac{1}{C^2}$, $U_l(c,l) = -\chi^{-1}l^{1/\varphi}\frac{1}{C}$, $U_{ll}(c,l) = -\frac{\chi^{-1}l^{1/\varphi-1}}{C}\left(\frac{1}{\varphi} + \frac{\chi^{-1}l^{1/\varphi}}{C}\right)$, and $U_{cl}(c,l) = \frac{\chi^{-1}l^{1/\varphi}}{C^2}$. Plugging this into the definitions of ψ_t^i and $\psi_{l,t}^i$, and using the labor Euler equation (11) stating that $\chi^{-1} l_t^{i,1/\varphi} = y_t^i w_t$, we deduce that the expressions of ψ_t^i and $\psi_{l,t}^i$ become:

$$\psi_t^i C_t^i = 1 + \left(\lambda_{c,t}^i - R_t \lambda_{c,t-1}^i\right) \frac{1}{C_t^i}, \tag{191}$$

$$\psi_{l,t}^i C_t^i = y_t^i w_t \left(1 + \frac{\lambda_{l,t}^i}{\varphi l_t^i} + (\lambda_{c,t}^i - R_t \lambda_{c,t-1}^i) \frac{1}{C_t^i}\right). \tag{192}$$

We now turn to the FOCs. Note that FOC (54) is exactly the same as FOC (70), while FOC (57) has no equivalent in the simplified version since the progressivity parameter τ_t is set to zero. The definition of $\psi_{l,t}^i$ becomes with $\tau_t = 0$: $\psi_{l,t}^i = w_t y_t^i \psi_t^i + \mu_t (F_{L,t} - w_t) y_t^i$. Equations (191) and (192) become: $\frac{\lambda_{l,t}^i}{\varphi l_t^i} \frac{y_t^i}{C_t^i} = \mu_t (\frac{F_{L,t}}{w_t} - 1) y_t^i$, which is equivalent to $0 = 0$ for unemployed agents since their productivity is null. For employed agents, it becomes:

$$\lambda_{e,l,t} = \varphi \mu_t l_{e,t} C_{e,t} \frac{\tau_t^L}{1 - \tau_t^L}. \tag{193}$$

The three remaining FOCs are equations (53), (55), and (56). Taking advantage of the deterministic transitions, as well as the fact that unemployed agents are credit-constrained with null productivity, these FOCs can also be written as:

$$\psi_{e,t} - \mu_t = \beta R_{t+1} (\psi_{u,t+1} - \mu_{t+1}), \tag{194}$$

$$\mu_t C_{u,t} = \psi_{u,t} C_{u,t} + \frac{\lambda_{e,c,t-1}}{a_{e,t-1}}, \tag{195}$$

$$\mu_t C_{e,t} = \psi_{e,t} C_{e,t} + \frac{\lambda_{e,l,t}}{l_{e,t}}, \tag{196}$$

while similarly expressions of ψ_t^i in (191) can further be specified as:

$$\psi_{e,t} C_{e,t} = 1 + \frac{\lambda_{e,c,t}}{C_{e,t}}, \tag{197}$$

$$\psi_{u,t} C_{u,t} = 1 - R_t \lambda_{e,c,t-1} \frac{1}{C_{u,t}}. \tag{198}$$

Combining (195) and (198) with $a_{e,t-1} = \frac{C_{u,t}}{R_t}$ (unemployed budget constraint) gives:

$$\mu_t C_{u,t} = 1, \tag{199}$$

with the expression of $C_{u,t} = R_t \frac{\beta}{1+\beta} \frac{w_{t-1} (\chi w_{t-1})^{\varphi}}{1+\varphi}$ identical to FOC (71).

Using the consumption Euler equation (19) stating that $\frac{1}{C_{e,t}} = \beta R_{t+1} \frac{1}{C_{u,t+1}}$, the budget constraints (21) and (22) implying that $C_{u,t} = \beta R_t C_{e,t-1}$, and (199) meaning that $1 =$

$\beta\mu_{t+1}R_{t+1}C_{e,t}$, we deduce from (194) and (197):

$$\frac{\lambda_{e,c,t}}{C_{e,t}} = \frac{\beta}{1+\beta}(\mu_t C_{e,t} - 1). \tag{200}$$

Finally, we turn to FOC (196). Combined with the expressions of $\lambda_{e,l,t}$ in (193), $\psi_{e,t}$ in (197), and of $\lambda_{e,c,t}$ in (200), this becomes:

$$C_{e,t}\mu_t \left(1 - (1+\beta)\varphi\frac{\tau_t^L}{1-\tau_t^L}\right) = 1. \tag{201}$$

Using the budget constraint (21) stating that $C_{e,t} = \frac{w_t(\chi w_t)^\varphi}{(1+\beta)(1+\varphi)}$, equation (201) becomes FOC (72) and hence FOC (69). This completes the proof that the generic FOCs of Section 4.1 exactly imply the FOCs (69)–(72).

F.2.2 Comparing the Quantitative Outcomes of the two Approaches

We show that the analytical solution can be computed as the limit of the quantitative model where the transition matrix converges to the anti-diagonal matrix of Assumption A. We thus consider a specification of the quantitative model that is similar to the one of the analytical model: a GHH utility function, a linear labor tax, a two-state productivity process, and a zero credit constraint. We consider the transition matrix Π_ε defined for any $\varepsilon \in [0,1]$ as: $\Pi_\varepsilon = \begin{bmatrix} \varepsilon & 1-\varepsilon \\ 1-\varepsilon & \varepsilon \end{bmatrix}$, which for $\varepsilon = 0$ corresponds to the anti-diagonal case of Assumption A.

We use the same calibration as in Figure 1, namely: $\alpha = 0.3$, $\beta = 0.7$, $\varphi = 0.3$, $\delta = 1$, $G = 0.01$, $\chi = 1$. This calibration guarantees the existence of a positive debt and a positive capital tax in the analytical model (when $\varepsilon = 0$). We compute the optimal steady-state fiscal policy as a function of ε with the truncation approach, as in Section 4. We plot the results in Figure 6. The first observation is for low values of ε (from 10^{-6} to 10^{-10}): the outcomes of the two models are very similar. The quantitative resolution is thus consistent with the analytical method. The second observation is when ε increases beyond 10^{-5}, the capital tax diminishes sharply, while the labor tax goes up. This result is consistent with intuition. Indeed, in this very stylized setup, a higher ε means that a higher share of the population remains unemployed with a null income. Their sole resource is their savings. Diminishing the capital tax fosters savings and enables agents to better self-insure them-selves against the null income risk. Increasing the labor tax enables the government to balance its budget – since public spending remains fixed.

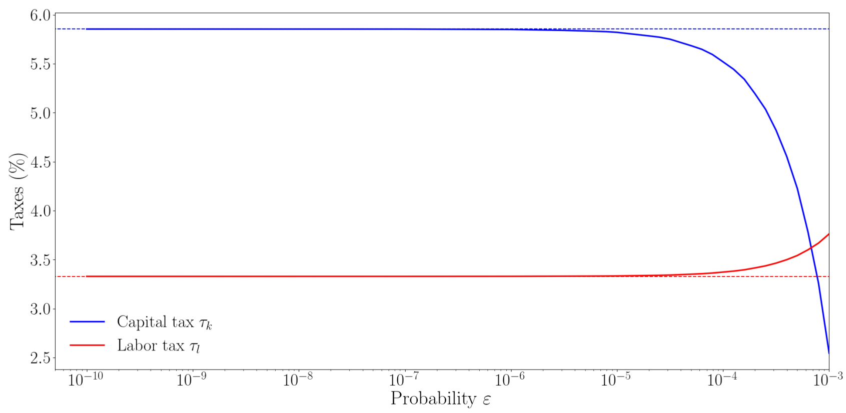

Figure 6: Comparison of the results of the quantitative model (plain lines) to those of the analytical model (dashed lines).

G The Ramsey Program on the Truncated Model

G.1 Formulation

We define the set of $(\xi_h^{u,0})_h$ such that:

$$\sum_{y^t \in \mathcal{Y}^t \left| (y^t_{t-N+1},\ldots,y^t_t)=h \right.} u(c_t(y^t)) = \xi_h^{u,0} u\left(\sum_{y^t \in \mathcal{Y}^t \left| (y^t_{t-N+1},\ldots,y^t_t)=h \right.} c_t(y^t) \right),$$

or compactly: $\xi_h^{u,0} u(c_{t,h}) := \sum_h u(c_t^i)$. Similarly, we define $(\xi_h^{v,0})$, $(\xi_h^{u,1})$, (ξ_h^{τ}), and $(\xi_h^{v,1})$ such that: $\xi_h^{v,0} v(l_{t,h}) := \sum_h v(l_t^i)$, $\xi_h^{u,1} u'(c_{t,h}) := \sum_h u'(c_t^i)$, $\xi_h^{\tau} \sum_h (y_0^h l_{t,h})^{1-\tau_t} := \sum_h (y_t^i l_t^i)^{1-\tau_t}$, and $\xi_{t,s^N}^{v,1} v'(l_{t,h}) := (1-\tau_t) w_t \xi_h^{\tau} (l_{t,h} y_h)^{1-\tau_t} \xi_h^{u,1} (u'(c_{t,h})/l_{t,h})$. The Ramsey problem is then:

$$\max_{\left(r_t, \tilde{w}_t, \tilde{r}_t, \tau_t^K, \tau_t, \kappa_t, B_t, K_t, L_t, (a_t^i, c_t^i, l_t^i, \nu_t^i)_i\right)_{t \geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \sum_h S_{t,h} \omega_h (\xi_h^{u,0} u(c_{t,h}) - \xi_h^{v,0} v(l_{t,h})) \right],$$

$$G_t + T_t + (1+r_t) B_{t-1} + r_t K_{t-1} + w_t \xi_h^{\tau} \sum_h (l_{t,h} y_h)^{1-\tau_t} \ell(di) = F(K_{t-1}, L_t, z_t) + B_t, \quad (202)$$

$$\text{for all } h \in \mathcal{Y}\text{: } c_{t,h} + a_{t,h} = w_t \xi_h^{\tau} (l_{t,h} y_h)^{1-\tau_t} + (1+r_t) \tilde{a}_{t,h} + T_t, \quad (203)$$

$$a_{t,h} \geq 0, \ \nu_{t,h}(a_{t,h} + \bar{a}) = 0, \ \nu_{t,h} \geq 0, \quad (204)$$

$$\xi_h^{u,E} u'(c_{t,h}) = \beta \mathbb{E}_t \left[\sum_{\tilde{h} \in \mathcal{H}} \Pi_{t,h\tilde{h}} \xi_{\tilde{h}}^{u,E} u'(c_{t+1,\tilde{h}})(1 + r_{t+1}) \right] + \nu_{t,h}, \quad (205)$$

$$\xi_h^{v,1} v'(l_{t,h}) = (1-\tau_t) w_t \xi_h^{\tau} (l_{t,h} y_h)^{1-\tau_t} \xi_h^{u,1} (u'(c_{t,h})/l_{t,h}), \quad (206)$$

$$K_t + B_t = \sum_h S_{t,h} a_{t,h}, \ L_t = \sum_h S_{t,h} y_h l_{t,h},$$

with $\tilde{a}_{t,h} = \sum_{\tilde{h}\in\mathcal{H}} \Pi_{\tilde{h}h,t} \frac{S_{t-1,\tilde{h}}}{S_{t,h}} a_{t-1,\tilde{h}}$ and $S_{t,h} = \sum_{\tilde{h}\in\mathcal{H}} \Pi_{\tilde{h}h,t} S_{t-1,\tilde{h}}$.

G.2 Factorization

We now factorize the Ramsey problem of Section G.1. The new Ramsey objective is:

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{h\in\mathcal{H}} \Bigg[S_{t,h} \Bigg(\left(\omega_h \xi_h^{u,0} u(c_{t,h}) - \xi_h^{v,0} v(l_{t,h}) \right) - \lambda_{c,t,h} \xi_h^{u,E} u'(c_{t,h}) \Bigg)$$
$$+ \tilde{\lambda}_{c,t,h}(1+r_t)\xi_h^{u,E} u'(c_{t,h}) - \lambda_{l,t,h} \left(v'(l_{t,h}) - (1-\tau_t) w_t \xi_h^{\tau} (y_0^h l_{t,h})^{1-\tau_t} \xi_h^{u,1} u'(c_{t,h})/l_{t,h} \right) \Bigg],$$

with $\tilde{\lambda}_{t,h} = \frac{1}{S_{t,h}} \sum_{\tilde{h}\in\mathcal{H}} S_{t-1,\tilde{h}} \lambda_{t-1,\tilde{h}} \Pi_{t,\tilde{h},h}$.

G.3 FOCs of the Planner

The FOCs of the Ramsey program can finally be written as follows:

$$\hat{\psi}_{t,h} := \omega_h \xi_h^{u,0} u'(c_{t,h}) - \mu_t \tag{207}$$
$$- (\lambda_{c,t,h}\xi_h^{u,E} - (1+r_t)\tilde{\lambda}_{c,t,h}\xi_h^{u,E} - \lambda_{l,t,h}\xi_h^{\tau}(1-\tau_t) w_t (y_0^h l_{t,h})^{1-\tau_t} \xi_h^{u,1}/l_{t,h}) u''(c_{t,h}),$$

$$\hat{\psi}_{t,h} = \beta \mathbb{E}_t \left[(1+r_{t+1}) \sum_{\tilde{h}\in\mathcal{H}} \Pi_{t,h\tilde{h}} \hat{\psi}_{t+1,\tilde{h}} \right] \text{ if } \nu_h = 0 \text{ and } \lambda_{t,h} = 0 \text{ otherwise,} \tag{208}$$

$$\hat{\psi}_{t,h} = \frac{1}{(1-\tau_t) w_t y_0^h \xi_h^{\tau} (y_0^h l_{t,h})^{-\tau_t}} (\omega_h \xi_h^{v,0} v'(l_{t,h}) + \lambda_{l,t,h} \xi_h^{v,1} v''(l_{t,h})) \tag{209}$$
$$+ \lambda_{l,t,h} \tau_t \xi_h^{u,1} (u'(c_{t,h})/l_{t,h}) - \mu_t \frac{F_{L,t}}{(1-\tau_t) w_t \xi_h^{\tau} (y_0^h l_{t,h})^{-\tau_t}},$$

$$0 = \sum_{h\in\mathcal{Y}} S_{t,h} \left(\hat{\psi}_{t,h} \tilde{a}_{t,h} + \tilde{\lambda}_{c,t,h} \xi_h^{u,E} u'(c_{t,h}) \right), \tag{210}$$

$$0 = \sum_{h\in\mathcal{Y}} S_{t,h} \xi_h^{\tau} (l_{t,h} y_0^h)^{1-\tau_t} \left(\hat{\psi}_{t,h} + \lambda_{l,t,h} (1-\tau_t) \xi_h^{u,1} (u'(c_{t,h})/l_{t,h}) \right), \tag{211}$$

$$\mu_t = \beta \mathbb{E} \left[\mu_{t+1} \left(1 + F_{K,t+1} - \delta \right) \right], \tag{212}$$

$$0 = \sum_{h\in\mathcal{Y}} S_{t,h} \lambda_{l,t,h} \xi_h^{\tau} (l_{t,h} y_h)^{1-\tau_t} \xi_h^{u,1} (u'(c_{t,h})/l_{t,h}) \tag{213}$$
$$+ \sum_{h\in\mathcal{Y}} S_{t,h} \left(\hat{\psi}_{t,h} + \lambda_{l,t,h} (1-\tau_t) \xi_h^{u,1} (u'(c_{t,h})/l_{t,h}) \right) \ln\left(l_{t,h} y_h \right) \xi_h^{\tau} (l_{t,h} y_h)^{1-\tau_t}.$$

The solution of the Ramsey program for planner in the truncated model is characterized by equations (202)–(213). These equations can be simulated in two steps. First, we need to solve the model at the steady state (and more precisely, to obtain the expressions of the ξs and of the weight ωs that enable one to reproduce the actual US fiscal system at the steady state – for further explanation see Section 4.1). Second, the dynamic system (202)–(213) can then be simulated around the previous steady state using perturbation method (via Dynare for instance).

H Computing the Steady State

In this section, we provide closed-form formulas for preference multipliers ξs (Section H.1) and the Pareto weights ωs (Section H.2). We start with some notation:

$\circ$ is the Hadamard product, $\otimes$ is the Kronecker product, $\times$ is the usual matrix product.

For any vector V, we denote by $diag\,(V)$ the diagonal matrix with V on the diagonal.

The matrix representation consists in stacking together the equations characterizing the steady state, so as to provide a convenient matrix notation for solving the steady state. It also provides an efficient solution to compute the values for the coefficients (ξ_h) and (ω_h). We assume as given an indexing of histories over $\mathcal{H}$ of cardinal N_{tot}.

H.1 Computing the ξs

We denote with a bold letter the vector associated to a given variable: e.g., $\boldsymbol{S}$ is N_{tot}-vector of steady-state history sizes: $\boldsymbol{S} = (S_h)_{h\in\mathcal{H}}$. Similarly, $\boldsymbol{a}$, $\boldsymbol{c}$, $\boldsymbol{l}$, and $\boldsymbol{\nu}$ are the N_{tot}-vectors of end-of-period wealth, consumption, labor supply, and Lagrange multipliers, respectively. These vectors are known from the steady-state equilibrium of the Bewley model. We also define $\boldsymbol{P}$ as the diagonal matrix having 1 on the diagonal at h if and only if the history h is not credit-constrained (i.e., $\nu_h = 0$), and 0 otherwise. We also define $\boldsymbol{I}$ as the $(N_{tot} \times N_{tot})$-identity matrix. Noting $\mathbf{\Pi}$ as the transition matrix across histories, we obtain the following steady-state relationships. We have:

$$\boldsymbol{S} = \Pi\boldsymbol{S}, \tag{214}$$

$$\boldsymbol{S} \circ \boldsymbol{c} + \boldsymbol{S} \circ \boldsymbol{a} = (1+r)\Pi\left(\boldsymbol{S} \circ \boldsymbol{a}\right) + w\boldsymbol{S} \circ \xi^{\tau} \circ \left(\boldsymbol{y} \circ \boldsymbol{l}\right)^{1-\tau} + T\mathbf{1}, \tag{215}$$

$$(\boldsymbol{I} - \boldsymbol{P})\boldsymbol{a} = \mathbf{0}_{N_{tot}\times 1}, \tag{216}$$

which correspond to: the definition of history sizes, the individual budget constraint (203), the definition of credit-constrained histories (204). Denoting by $\boldsymbol{D_x}$ the diagonal $(N_{tot} \times N_{tot})$-matrix with the N_{tot}-vector $\boldsymbol{x}$ on the diagonal, the Euler equation (205) becomes: $\boldsymbol{D}_{u'(\boldsymbol{c})}\boldsymbol{\xi}^{u,E} = \beta(1+r)\boldsymbol{\Pi}^\top \boldsymbol{D}_{u'(\boldsymbol{c})}\boldsymbol{\xi}^{u,E} + \boldsymbol{\nu}$, implying that:

$$\boldsymbol{\xi}^{u,E} = \left[\left(\boldsymbol{I} - \beta(1+r)\boldsymbol{\Pi}^\top\right)\boldsymbol{D}_{u'(\boldsymbol{c})}\right]^{-1}\boldsymbol{\nu}, \tag{217}$$

while the labor supply FOC similarly gives:

$$\boldsymbol{\xi}^{v,1} = (1-\tau)w(\boldsymbol{y}\circ\boldsymbol{l})^{1-\tau}\circ\boldsymbol{\xi}^{\tau}\circ\boldsymbol{\xi}^{u,1}\circ u'(\boldsymbol{c})./(\boldsymbol{l}\circ v'(\boldsymbol{l})). \tag{218}$$

H.2 Finding the Constraints on the Pareto Weights ω

We now construct the constraints that the Pareto weights ω must fulfill for the steady-state allocation to be optimal for the observed instruments of the planner. More precisely, we show that there are two vectors $\boldsymbol{L}_2, \boldsymbol{L}_3$ such that all the FOCs of the planner are fulfilled when $\boldsymbol{L}_2\boldsymbol{\omega} = 0$ and $\boldsymbol{L}_3\boldsymbol{\omega} = 0$. The derivation of these vectors is not complicated, but tedious. We define the following quantities:

$$\begin{aligned}
&\bar{\boldsymbol{\omega}} := \boldsymbol{S}\circ\omega,\ \bar{\boldsymbol{\lambda}}_c := \boldsymbol{S}\circ\lambda_c\ \bar{\boldsymbol{\psi}} := \boldsymbol{S}\circ\hat{\boldsymbol{\psi}},\\
&\bar{\boldsymbol{\lambda}}_l := \boldsymbol{S}\circ\boldsymbol{\lambda}_t,\ \boldsymbol{S}\circ\tilde{\boldsymbol{\lambda}}_c := \boldsymbol{\Pi}\bar{\boldsymbol{\lambda}}_c,\ \bar{\boldsymbol{\Pi}} := \boldsymbol{S}\circ\boldsymbol{\Pi}^\top\circ(1./\boldsymbol{S}),\\
&\tilde{\boldsymbol{\xi}}^{u,1} := \boldsymbol{\xi}^{u,1}./\boldsymbol{l},\ \tilde{\boldsymbol{\xi}}^{v,1} := \boldsymbol{\xi}^{v,1}./((1-\tau)w\boldsymbol{\xi}^{\tau}\circ\boldsymbol{y}^{1-\tau}\circ\boldsymbol{l}^{-\tau}),\ \tilde{\boldsymbol{\xi}}^{v,0} = \boldsymbol{\xi}^{v,0}./((1-\tau)w\boldsymbol{\xi}^{\tau}\circ\boldsymbol{y}^{1-\tau}\circ\boldsymbol{l}^{-\tau}).
\end{aligned}$$

With these definitions, planner's FOCs (207)–(213) become:

$$\begin{aligned}
\bar{\boldsymbol{\psi}} &= \bar{\boldsymbol{\omega}}\circ\boldsymbol{\xi}^{u,0}\circ u'(\boldsymbol{c}) - \mu\boldsymbol{S} && (219)\\
&\quad - \left(\bar{\boldsymbol{\lambda}}_c\circ\boldsymbol{\xi}^{u,E} - (1+r)\boldsymbol{\Pi}\bar{\boldsymbol{\lambda}}_c\circ\boldsymbol{\xi}^{u,E} - (1-\tau)w\bar{\boldsymbol{\lambda}}_l\circ\boldsymbol{\xi}^{\tau}\circ(\boldsymbol{y}\circ\boldsymbol{l})^{1-\tau}\circ\tilde{\boldsymbol{\xi}}^{u,1}\right)\circ u''(\boldsymbol{c})\\
\boldsymbol{P}\bar{\boldsymbol{\psi}} &= \beta(1+r)\boldsymbol{P}\bar{\boldsymbol{\Pi}}\bar{\boldsymbol{\psi}}, && (220)\\
0 &= \bar{\boldsymbol{\psi}} - \tau\tilde{\boldsymbol{\xi}}^{u,1}\circ u'(\boldsymbol{c})\circ\bar{\boldsymbol{\lambda}}_l + \mu F_L\boldsymbol{S}./((1-\tau)w\boldsymbol{\xi}^{\tau}\circ\boldsymbol{y}^{-\tau}\circ\boldsymbol{l}^{-\tau}) && (221)\\
&\quad - \bar{\boldsymbol{\omega}}\circ\tilde{\boldsymbol{\xi}}^{v,0}\circ v'(\boldsymbol{l}) - \bar{\boldsymbol{\lambda}}_l\circ\tilde{\boldsymbol{\xi}}^{v,1}\circ v''(\boldsymbol{l})\\
\tilde{\boldsymbol{a}}^\top\bar{\boldsymbol{\psi}} &= -\left(\boldsymbol{\xi}^{u,E}\circ u'(\boldsymbol{c})\right)^\top\Pi\bar{\boldsymbol{\lambda}}_c, && (222)\\
0 &= \left(\boldsymbol{\xi}^{\tau}\circ(\boldsymbol{y}\circ\boldsymbol{l})^{1-\tau}\right)^\top\bar{\boldsymbol{\psi}} + (1-\tau)\left(\boldsymbol{\xi}^{\tau}\circ(\boldsymbol{y}\circ\boldsymbol{l})^{1-\tau}\circ\tilde{\boldsymbol{\xi}}^{u,1}\circ u'(\boldsymbol{c})\right)^\top\bar{\boldsymbol{\lambda}}_l, && (223)\\
0 &= \left(\ln(\boldsymbol{y}\circ\boldsymbol{l})\circ\boldsymbol{\xi}^{\tau}\circ(\boldsymbol{y}\circ\boldsymbol{l})^{1-\tau}\right)^\top\bar{\boldsymbol{\psi}} && (224)\\
&\quad + \left((\boldsymbol{1} + (1-\tau)\ln(\boldsymbol{y}\circ\boldsymbol{l}))\circ\boldsymbol{\xi}^{\tau}\circ(\boldsymbol{y}\circ\boldsymbol{l})^{1-\tau}\circ\tilde{\boldsymbol{\xi}}^{u,1}\circ u'(\boldsymbol{c})\right)^\top\bar{\boldsymbol{\lambda}}_l,\\
0 &= (\boldsymbol{I} - \boldsymbol{P})\bar{\boldsymbol{\lambda}}_c. && (225)
\end{aligned}$$

Expressing Lagrange multipliers as a function of $\bar{\boldsymbol{\omega}}$. Equation (221) yields:

$$\bar{\boldsymbol{\lambda}}_l = \boldsymbol{M}_0\bar{\boldsymbol{\omega}} + \boldsymbol{M}_1\bar{\boldsymbol{\psi}} + \mu\boldsymbol{V}_0, \tag{226}$$
$$\text{with: } \boldsymbol{M}_0 = -\boldsymbol{M}_1\boldsymbol{D}_{\tilde{\boldsymbol{\xi}}^{v,0}\circ v'(\boldsymbol{l})},$$
$$\boldsymbol{M}_1 = \boldsymbol{D}^{-1}_{\tilde{\boldsymbol{\xi}}^{v,1}\circ v''(\boldsymbol{l})+\tau\tilde{\boldsymbol{\xi}}^{u,1}\circ u'(\boldsymbol{c})},$$
$$\boldsymbol{V}_0 = F_L\boldsymbol{M}_1\boldsymbol{S}./((1-\tau)w\boldsymbol{\xi}^{\tau}\circ\boldsymbol{y}^{-\tau}\circ\boldsymbol{l}^{-\tau}).$$

Then equation (219) implies:

$$\bar{\boldsymbol{\psi}} = \hat{\boldsymbol{M}}_0\bar{\boldsymbol{\omega}} + \hat{\boldsymbol{M}}_1\bar{\boldsymbol{\lambda}}_c + \hat{\boldsymbol{M}}_2\bar{\boldsymbol{\lambda}}_l - \mu\boldsymbol{S}, \tag{227}$$
$$\text{with: } \hat{\boldsymbol{M}}_0 = \boldsymbol{D}_{\boldsymbol{\xi}^{u,0}\circ u'(\boldsymbol{c})},\ \hat{\boldsymbol{M}}_1 = -\boldsymbol{D}_{\boldsymbol{\xi}^{u,E}\circ u''(\boldsymbol{c})}(\boldsymbol{I}-(1+r)\boldsymbol{\Pi}),$$
$$\hat{\boldsymbol{M}}_2 = (1-\tau)w\boldsymbol{D}_{\boldsymbol{\xi}^{\tau}\circ(\boldsymbol{y}\circ\boldsymbol{l})^{1-\tau}\circ\tilde{\boldsymbol{\xi}}^{u,1}\circ u''(\boldsymbol{c})}.$$

With (226) and (227), we get $(\boldsymbol{I}-\hat{\boldsymbol{M}}_2\boldsymbol{M}_1)\bar{\boldsymbol{\psi}}=(\hat{\boldsymbol{M}}_0+\hat{\boldsymbol{M}}_2\boldsymbol{M}_0)\bar{\boldsymbol{\omega}}+\hat{\boldsymbol{M}}_1\bar{\boldsymbol{\lambda}}_c+\mu(\hat{\boldsymbol{M}}_2\boldsymbol{V}_0-\boldsymbol{S})$:

$$\bar{\boldsymbol{\psi}} = \boldsymbol{M}_3\bar{\boldsymbol{\omega}} + \boldsymbol{M}_4\bar{\boldsymbol{\lambda}}_c + \mu\boldsymbol{V}_1, \tag{228}$$
$$\text{with: } \boldsymbol{M}_2 = \boldsymbol{I}-\hat{\boldsymbol{M}}_2\boldsymbol{M}_1,\ \boldsymbol{M}_4 = \boldsymbol{M}_2^{-1}\hat{\boldsymbol{M}}_1,$$
$$\boldsymbol{M}_3 = \boldsymbol{M}_2^{-1}(\hat{\boldsymbol{M}}_0+\hat{\boldsymbol{M}}_2\boldsymbol{M}_0),\ \boldsymbol{V}_1 = \boldsymbol{M}_2^{-1}(\hat{\boldsymbol{M}}_2\boldsymbol{V}_0-\boldsymbol{S}).$$

Then, using (220), (225), and (228), we get: $((\boldsymbol{I}-\boldsymbol{P})+\boldsymbol{P}(I-\beta(1+r)\bar{\boldsymbol{\Pi}})\boldsymbol{M}_4)\bar{\boldsymbol{\lambda}}_c = -\boldsymbol{P}(I-\beta(1+r)\bar{\boldsymbol{\Pi}})\boldsymbol{M}_3\bar{\boldsymbol{\omega}} - \mu\boldsymbol{P}(I-\beta(1+r)\bar{\boldsymbol{\Pi}})\boldsymbol{V}_1$ and:

$$\bar{\boldsymbol{\lambda}}_c = \boldsymbol{M}_5\bar{\boldsymbol{\omega}} + \mu\boldsymbol{V}_2, \tag{229}$$
$$\text{with: } \tilde{\boldsymbol{R}}_5 = -((\boldsymbol{I}-\boldsymbol{P})+\boldsymbol{P}(I-\beta(1+r)\bar{\boldsymbol{\Pi}})\boldsymbol{M}_4)^{-1}\boldsymbol{P}(I-\beta(1+r)\bar{\boldsymbol{\Pi}}),$$
$$\boldsymbol{M}_5 = \tilde{\boldsymbol{R}}_5\boldsymbol{M}_3,\ \boldsymbol{V}_2 = \tilde{\boldsymbol{R}}_5\boldsymbol{V}_1.$$

We then use equation (222), which becomes $0 = \tilde{\boldsymbol{a}}^{\top}(\boldsymbol{M}_3\bar{\boldsymbol{\omega}} + \boldsymbol{M}_4\bar{\boldsymbol{\lambda}}_c + \mu\boldsymbol{V}_1) + (\boldsymbol{\xi}^{u,E}\circ u'(\boldsymbol{c}))^{\top}\Pi\bar{\boldsymbol{\lambda}}_c$, or after using (229):

$$\mu = -\boldsymbol{L}_1\bar{\omega}, \tag{230}$$
$$\text{with: } C_1 = \tilde{\boldsymbol{a}}^{\top}(\boldsymbol{V}_1+\boldsymbol{M}_4\boldsymbol{V}_2) + (\boldsymbol{\xi}^{u,E}\circ u'(\boldsymbol{c}))^{\top}\boldsymbol{\Pi}\boldsymbol{V}_2,$$
$$\boldsymbol{L}_1 = \left(\tilde{\boldsymbol{a}}^{\top}(\boldsymbol{M}_3+\boldsymbol{M}_4\boldsymbol{M}_5) + (\boldsymbol{\xi}^{u,E}\circ u'(\boldsymbol{c}))^{\top}\boldsymbol{\Pi}\boldsymbol{M}_5\right)/C_1.$$

We deduce that from (226), (228), (229), and (230):

$$\bar{\boldsymbol{\lambda}}_c = (\boldsymbol{M}_5 - \boldsymbol{V}_2\boldsymbol{L}_1)\bar{\boldsymbol{\omega}}, \tag{231}$$

$$\bar{\boldsymbol{\psi}} = \boldsymbol{M}_6\bar{\boldsymbol{\omega}}, \tag{232}$$

$$\bar{\boldsymbol{\lambda}}_l = \hat{\boldsymbol{M}}_6\bar{\boldsymbol{\omega}}, \tag{233}$$

with: $\boldsymbol{M}_6 = \boldsymbol{M}_3 + \boldsymbol{M}_4(\boldsymbol{M}_5 - \boldsymbol{V}_2\boldsymbol{L}_1) - \boldsymbol{V}_1\boldsymbol{L}_1$ and $\hat{\boldsymbol{M}}_6 = \boldsymbol{M}_0 + \boldsymbol{M}_1\boldsymbol{M}_6 - \boldsymbol{V}_0\boldsymbol{L}_1$.

Constructing the linear constraints on Pareto weights. We use equations (224) and (223), in which we substitute the expressions (232) and (233) of $\bar{\boldsymbol{\psi}}$ and $\bar{\boldsymbol{\lambda}}_l$. We obtain:

$$\boldsymbol{L}_2\bar{\boldsymbol{\omega}} = 0, \text{ and } \boldsymbol{L}_3\bar{\boldsymbol{\omega}} = 0, \tag{234}$$

with $\boldsymbol{L}_2 = (\ln(\boldsymbol{y} \circ \boldsymbol{l}) \circ \boldsymbol{\xi}^\tau \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau})^\top \boldsymbol{M}_6 + ((\mathbf{1} + (1-\tau)\ln(\boldsymbol{y} \circ \boldsymbol{l})) \circ \boldsymbol{\xi}^\tau \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau} \circ \tilde{\boldsymbol{\xi}}^{u,1} \circ u'(\boldsymbol{c}))^\top \hat{\boldsymbol{M}}_6$ and $\boldsymbol{L}_3 = (\boldsymbol{\xi}^\tau \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau})^\top \boldsymbol{M}_6 + (1-\tau)(\boldsymbol{\xi}^\tau \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau} \circ \tilde{\boldsymbol{\xi}}^{u,1} \circ u'(\boldsymbol{c}))^\top \hat{\boldsymbol{M}}_6$.

Estimating the Pareto weights. We assume that each history with the same current productivity has the same weight. As a consequence, there are $|\mathcal{Y}|$ different Pareto weights, $\boldsymbol{\omega}^s$, for all histories. We define $\boldsymbol{M}_7$ as the $N_{tot} \times |\mathcal{Y}|$ matrix, whose element m_{hy} is 1 if history h has current productivity y. We thus have $\bar{\boldsymbol{\omega}} = \boldsymbol{D}_{\boldsymbol{S}}\boldsymbol{M}_7\boldsymbol{\omega}^s$. The Pareto weights are chosen to minimize the distance to the utilitarian SWF, such that planner's FOCs – i.e., equalities (234) – hold. Formally, they solve the following program:

$$\min_{\boldsymbol{\omega}^s} \left\|\boldsymbol{\omega}^s - \mathbf{1}_{|\mathcal{Y}|}\right\|^2,$$

$$\text{s.t. } \boldsymbol{L}_2\boldsymbol{D}_{\boldsymbol{S}}\boldsymbol{M}_7\boldsymbol{\omega}^s = 0, \tag{235}$$

$$\boldsymbol{L}_3\boldsymbol{D}_{\boldsymbol{S}}\boldsymbol{M}_7\boldsymbol{\omega}^s = 0. \tag{236}$$

We now solve this linear-quadratic problem. Denoting by μ_2 and μ_3 the Lagrange multipliers on the two constraints (235) and (236), the FOCs are: $\boldsymbol{\omega}^s - \mathbf{1}_{|\mathcal{Y}|} = \sum_{k=2}^3 \mu_k(\boldsymbol{L}_k\boldsymbol{D}_{\boldsymbol{S}}\boldsymbol{M}_7)^\top$, or after multiplying by $\boldsymbol{L}_k\boldsymbol{D}_{\boldsymbol{S}}\boldsymbol{M}_7$ ($k = 2, 3$) and using (235) and (236):

$$\begin{bmatrix} \mu_2 \\ \mu_3 \end{bmatrix} = \boldsymbol{M}_8^{-1}\boldsymbol{V}_8, \tag{237}$$

$$\text{where: } \boldsymbol{V}_8 = -\begin{bmatrix} \boldsymbol{L}_2\boldsymbol{D}_{\boldsymbol{S}}\boldsymbol{M}_7\mathbf{1}_{|\mathcal{Y}|} \\ \boldsymbol{L}_3\boldsymbol{D}_{\boldsymbol{S}}\boldsymbol{M}_7\mathbf{1}_{|\mathcal{Y}|} \end{bmatrix}, \; \boldsymbol{M}_8 = \begin{bmatrix} \boldsymbol{L}_2\boldsymbol{D}_{\boldsymbol{S}}\boldsymbol{M}_7 \\ \boldsymbol{L}_3\boldsymbol{D}_{\boldsymbol{S}}\boldsymbol{M}_7 \end{bmatrix} \begin{bmatrix} \boldsymbol{L}_2\boldsymbol{D}_{\boldsymbol{S}}\boldsymbol{M}_7 \\ \boldsymbol{L}_3\boldsymbol{D}_{\boldsymbol{S}}\boldsymbol{M}_7 \end{bmatrix}^\top,$$

which finally yields:

$$\boldsymbol{\omega}^s = \mathbf{1}_{|\mathcal{Y}|} + \begin{bmatrix} \boldsymbol{L}_2 \boldsymbol{D}_{\boldsymbol{S}} \boldsymbol{M}_7 \\ \boldsymbol{L}_3 \boldsymbol{D}_{\boldsymbol{S}} \boldsymbol{M}_7 \end{bmatrix}^\top (\boldsymbol{M}_8^{-1} \boldsymbol{V}_8).$$

I Robustness Check for the Truncation Length

Figures 7 and 8 compare the simulation outcomes for the main variables for two truncation lengths, $N = 20$ and $N = 25$. Figure 7 considers the low persistence $\rho_G = 0.7$, while Figure 8 considers the high persistence in turn $\rho_G = 0.97$. In both cases, the two simulation results are undistinguishable.

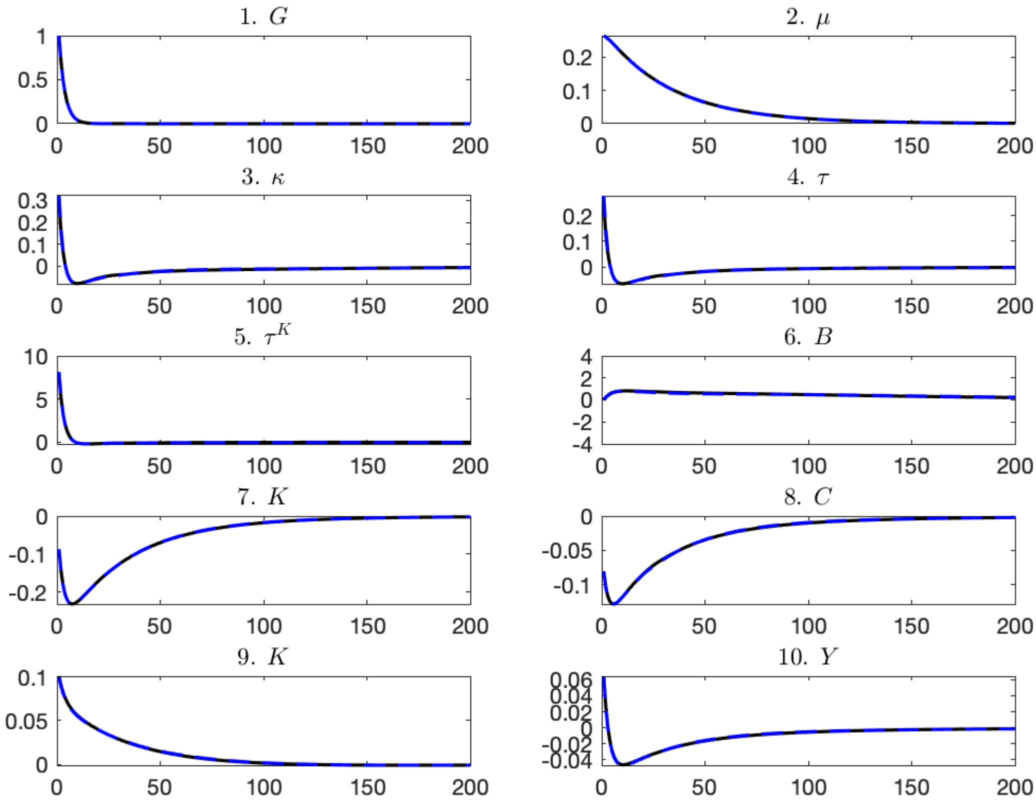

Figure 7: Comparison of the results of the quantitative model for $N = 20$ (black solid line), and $N = 25$ (blue dashed line) for the main variables for $\rho_G = 0.7$.

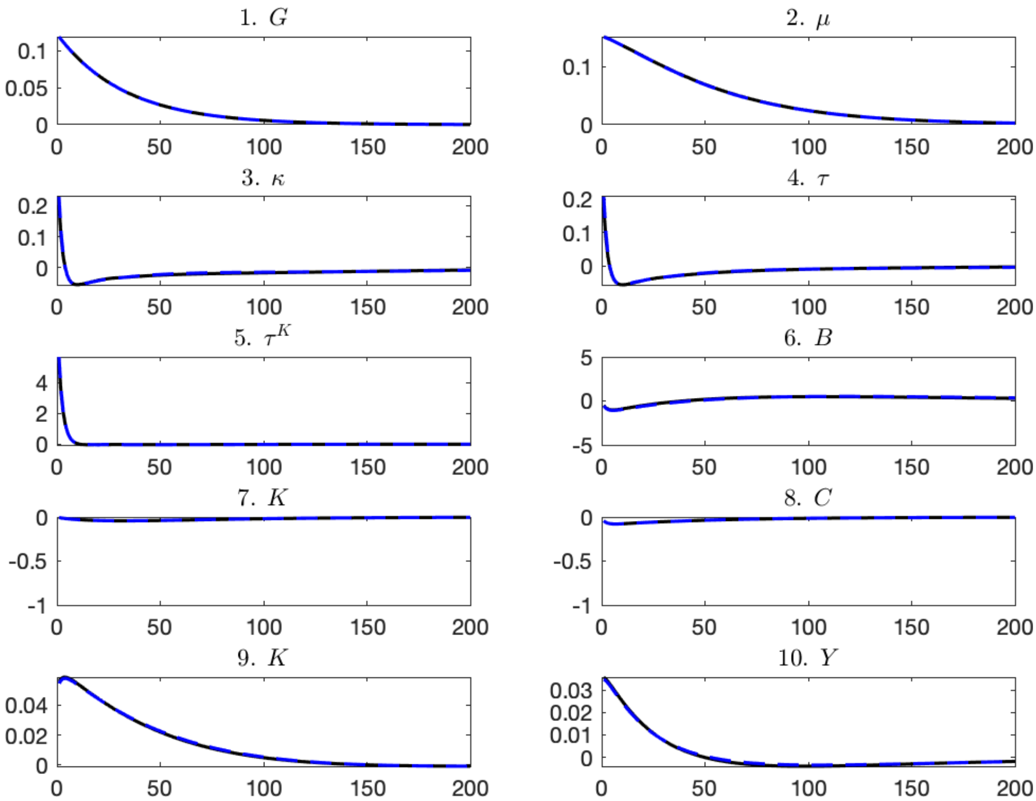

Figure 8: Comparison of the results of the quantitative model for $N = 20$ (black solid line), and $N = 25$ (blue dashed line) for the main variables for $\rho_G = 0.97$.