The Case for a Financial Approach to Money Demand

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Abstract

The distribution of money across households is much more similar to the distribution of financial assets than to that of consumption levels. This is a puzzle for theories which directly link money demand to consumption. This paper shows that the joint distribution of money and financial assets can be explained in an heterogeneous agent model where both a cash-in-advance constraint and financial adjustment costs, as in the Baumol-Tobin literature, are introduced. Studying each friction in turn, I find that the financial friction explains 85% of total money demand.

JEL codes: E40, E50.

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1 Introduction

Why do households hold money? Various theories of money demand have answered this question by focusing on the transaction role money plays in goods markets (e.g., shopping-time and cash-in-advance (CIA) models), transaction costs in financial markets (Baumol [8]; Tobin [40]) or simply assuming a liquidity role for money, as in the models with money in the utility function (MIUF). These theories are observationally equivalent in aggregate data: they can be realistically calibrated to match various estimates, such as the interest elasticity of money demand. In this paper, I show that microeconomic data can be used to quantify the contribution of the previous frictions to money demand. Indeed, the shape of the distribution of money across households is similar to the distribution of financial wealth but not close to the distribution of consumption levels. Using a heterogeneous agent model, I show that reproducing this money distribution allows us to quantify the contribution of goods-market frictions and financial-market frictions. In addition to its theoretical interest, the ability to reproduce the distribution of money is crucial for the assessment of the real and welfare effects of inflation.

More precisely, in both Italian and US data, the distribution of money (M1) is similar to that of financial wealth, and much more unequally distributed than that of consumption (as measured by the Gini coefficient, for example). In the US in 2004, the Gini coefficients are around 0.3 for the distribution of consumption levels across households, 0.5 for income, 0.8 for net wealth and 0.8 for money. This stylized fact, further detailed below, holds for different definitions of money, various time periods, and after controlling for life-cycle effects. This distribution of money cannot be understood in standard macroeconomic models where money demand is modeled only by frictions on the goods markets, via CIA, MIUF or shopping-time considerations. In these models, real money balances are proportional to consumption, and money holdings and consumption should be equally distributed across households (i.e. have the same Gini coefficient). As shown below, this property holds even when we consider more general transaction technologies in the goods market, which may produce scale economies.

In this paper, I show that a realistic joint distribution of consumption, money and financial assets can be reproduced when a friction on financial markets is introduced in addition to a transaction friction on goods markets. The friction in the goods market considered here is a standard cash-in-advance constraint which states that household must hold money to consume. The friction in financial markets follows the Baumol-Tobin literature. I assume that there exists a fixed adjustment cost for the financial portfolio: money holdings can be freely adjusted, but there is a fixed cost of adjusting the quantity of financial assets. The initial Baumol-Tobin model considered a cost of going to the bank and thus modeled the choice between currency and bank deposits. Following many others, I consider instead a fixed cost of adjusting the financial portfolio, in order to model the choice between money (including bank deposits) and other financial assets. This portfolio-adjustment cost creates a financial motive to hold money: households hold monetary balances to smooth consumption without paying the fixed cost of adjusting the financial portfolio. They only go infrequently on financial markets to replenish their money account, which is the standard result of the Baumol-Tobin model.[1]

This portfolio choice together with the cash-in-advance constraint are introduced into a production economy where infinitely-lived agents face uninsurable income fluctuations and borrowing constraints, a framework often described as the "Bewley-Huggett-Aiyagari" environment. In this type of economy, households choose between two assets with different returns, but also different adjustment costs, in order to smooth uninsurable idiosyncratic income fluctuations. This type of economy does not introduce life-cycle considerations and is thus well-suited for the analysis of heterogeneity within generations. The model is calibrated to reproduce the idiosyncratic income fluctuations faced by US households, as estimated by Heathcote [24]. The average inflation rate is targeted to its average value in 2004 in the US, the year for which the shape of the money distribution is available for US households. The adjustment cost and the severity of the cash-in-advance constraint are chosen to match the average quantity of money held by households in the US economy and the high degree of

[1] This friction alone generates a positive price for money in equilibrium, as the early work of Heller [26] and Chatterjee and Corbae [11] have shown.

inequality in money holdings.

The main result of this paper is that the model generates a realistic joint distribution of money and financial assets, when both frictions on financial and goods market are introduced. Removing each of the two frictions in turn, I find that frictions on the goods market are necessary to explain why many households hold only small amounts of money. The friction on the financial market explains why a few households hold large quantities of money. This last friction is thus required to generate the considerable inequality in money holdings. The explanation of this result is that households go infrequently to financial markets to replenish their money holdings due to the adjustment cost. However, as the opportunity cost of holding money is high, households rapidly decumulate their money holdings, and wait before going back to the financial market. As a result, a few households temporarily hold large quantities of money, which contributes to money inequality. Removing the two motives to hold money in the quantitative exercise, I find that transaction motives account for 15% of the total money stock, whereas financial motives account for 85%, motivating the title of this paper. A few households have to hold large amounts of money to reproduce the observed inequality in money, which is possible only if the financial motive is sufficiently large.

Related Literature

To my knowledge this paper is the first to reproduce a realistic distribution of money. It can be related to two strands of the existing literature. The first is the heterogeneous agents literature, which tries to reproduce inequality in the distribution of various assets as an equilibrium outcome. The second is the literature on money demand, which has a theoretical and an empirical component.

First, as noted by Heathcote, Storesletten and Violante [25], the heterogeneous agents literature has largely bypassed monetary economics, except for few papers listed below. The initial work in the heterogeneous agents literature considered money as the only available asset for self-insurance against idiosyncratic shocks (Bewley [9] and [10]; Scheinkman and Weiss [35]; Imrohoroglu [28]). More recent papers have introduced another financial asset with some additional frictions to justify positive money demand. Imrohoroglu and Prescott [27] use a per-period cost, so that households hold either money or financial assets, but never both, and consider the real effects of various monetary arrangements. Erosa and Ventura [20] introduce a cash-in-advance constraint and a fixed cost of withdrawing money from financial markets to study the inflation tax, but do not characterize the money distribution. Akyol [2] analyzes an endowment economy where the timing of market openings implies that only high-income agents hold money. More recently Algan and Ragot [1] considered the effect of inflation in an incomplete market economy where money is introduced in the utility function. To my knowledge, none of these papers is able to reproduce a realistic distribution of money.[2]

Second, the paper belongs to the literature on money demand, and more specifically to the Allais-Baumol-Tobin model in general equilibrium. Alvarez, Atkeson and Kehoe [3] introduce both a fixed transaction cost and a cash-in-advance constraint in a general-equilibrium setting. To simplify their analysis of the short-run effect of money injections, they assume that markets are complete and, in consequence, that all agents have the same financial wealth. I depart from the complete-market assumption to try to match the money distribution. This paper is also related to the empirical work which has estimated money demand using household data. Mulligan and Sala-i-Martin [32] introduce a fixed adoption cost of the technology to participate in financial markets, in addition to a shopping-time constraint. They estimate the adoption cost via various economic and econometric models using US household data. Attanasio, Guiso and Jappelli [5] estimate a shopping-time model *à la* McCallum and Goodfriend [31], using Italian household data. Finally, Alvarez and Lippi [4] use Italian household data to estimate a model where households face a cash-in-advance constraint, a fixed transaction cost and a stochastic cost of withdrawing money. They show that this stochastic component improves the outcome of the model as compared to a deterministic Baumol-Tobin framework. Although I also use household data, my goal is different: I reproduce a realistic joint distribution of money, wealth, and consumption as a general-equilibrium outcome, and show that simple frictions in financial markets are enough to generate the results.

[2] Recent papers in the search-theoretic literature (Chiu and Molico, [13]) also study inequality in money holdings. At this stage, these papers do not have a realistic financial market environment. The distribution of wealth is thus not consistent with the data.

The paper is organized as follows. Section 2 presents empirical facts about the distribution of money in Italy and the US, and Section 3 shows that the usual assumptions regarding money demand fail to reproduce these facts. Section 4 describes the fixed transaction-cost model, and the parameterization appears in Section 5. Section 6 presents the results and the distribution of money and assets, and Section 7 discusses some robustness tests. Finally, Section 8 concludes.

2 The Distribution of Money

This section presents some empirical facts about the distribution of money and assets in Italy and the US. Although the model below will be calibrated using US data, I use Italian data to check that the properties of the distribution of money are similar across countries. In the following, I use a narrow definition of money, M1, to emphasize the distinction between money and other financial assets. The robust stylized fact is that the distribution of money is similar to the distribution of assets. The same analysis has been carried out for various monetary aggregates and the results are quantitatively similar. As a summary of the following analysis, Figure 1 depicts the Lorenz curves of the money, income and net worth[3] distributions in the 2004 Survey of Consumer Finance, and those of consumption, income, net worth, and money distributions in data from the 2004 Italian Survey of Households' Income and Wealth. In both cases, I only consider households whose head is aged between 35 and 44 to avoid life-cycle effects. Money is more unequally distributed than are income and net wealth in both countries.

2.1 Italian Data

This section uses the 2004 Italian Survey of Households' Income and Wealth to examine the distribution of money. This periodic survey provides data for various deposit accounts, currency, income and wealth in the Italian population. Each survey is conducted on a sample of about 8,000 households,

[3] As is fairly usual, I use net worth as a summary statistic for all types of assets. The Lorenz curve of financial assets is very similar to that of net wealth.

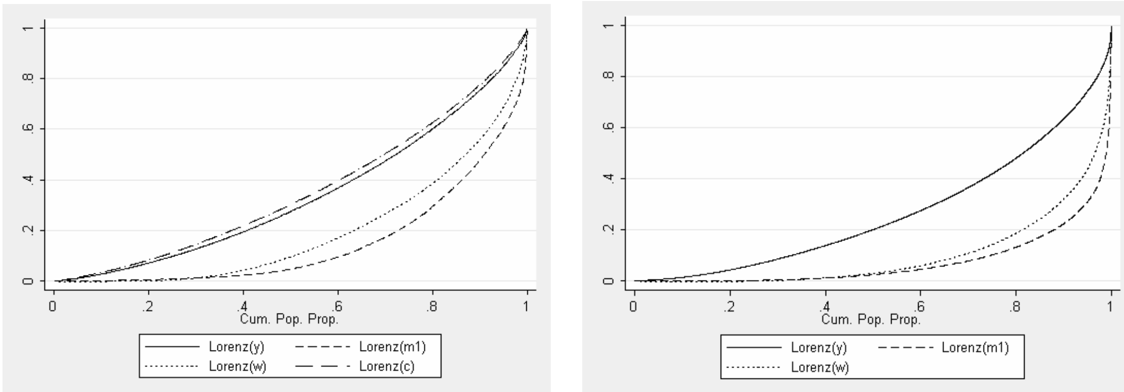


Figure 1: Lorenz Curves of Income (y), Money (m1), Wealth (w) and Consumption (c), in Italy (left) and the US (right), for households whose head is aged between 35 and 44.

and provides representative weights. A number of recent papers have used this data set to analyze money demand at the household level (Attanasio, Guiso and Jappelli [5]; and Alvarez and Lippi [4], amongst others).

Table 1: Distribution of Money and Wealth, Italy 2004

Gini coefficient of	Cons.	Income	Net W.	Money
Total Population	.30	.35	.59	.68
Popn., 35≤Age≤44	.29	.32	.61	.70
Popn., 35≤Age≤44, 99%.	.27	.31	.57	.63

Table 1 shows the Gini coefficient of the distributions of consumption, income, net worth and money (in columns) for three different types of households (in rows). The first column presents the Gini coefficient for total consumption, and the first row shows the figure for the whole population. This is fairly low, at .30. To avoid life-cycle effects the second line focuses on households whose head is aged between 35 and 44. The Gini coefficient is almost unchanged at .29. The second column shows the results for the distribution of income. The Gini coefficient is a little higher than that of consumption at .35, falling to .32 for the 35-44 age group. The third column performs the same exercise for the distribution of net wealth. This is more dispersed than consumption or income: the Gini coefficient for net worth is .59, increasing slightly to .61 for the 35-44 age group.

I use Italian data to construct the quantity of money (M1) held by each households, as the sum of the amount held in currency and in checking accounts. Although checking accounts are interest-bearing in Italy, the interest rate is low enough for this aggregation to be relevant: the average interest rate on checking accounts is below 1%, whereas the average yearly yield of Italian 10-year securities was over 4% in 2004. The last column of Table 1 shows the distribution of money. The Gini coefficient is very high here, at .68, and increases to .70 for the 35-44 age group. As a robustness check, I consider the distribution of money without including the 1% of the households who hold the most money. Some households may hold money in their checking accounts for a few days prior to buying very expensive durable goods (such as houses). If the survey interview occurs during this period, we will observe high levels of money balances that are not relevant.[4] The Gini coefficient on money holdings falls from .70 to .63 after this exclusion, thus remaining high.

The distribution of money is thus similar to that of net wealth, and is very different from that of consumption. For space reasons, this section has characterized the distribution by the Gini coefficient. However, other measures of inequality yield the same results. This can be seen graphically in Figure 1, which shows the four Lorenz curves for the population aged between 35 and 44.

Table 2 presents the empirical correlations between money holdings, consumption levels, income and wealth. Money is positively correlated with consumption, income and wealth, with a coefficient of between .2 and .3. The correlation between the ratio of money over total financial assets and wealth is negative. That is, the share of money in the financial portfolio falls with wealth. This property of the money/wealth distribution had previously been noted in US data by Erosa and Ventura [20].

[4] I carry out this exercise even though it is problematic to justify the exclusion of this 1% of households. If households keep money to buy a house over a period of one week, and buy a new house as often as every five years, the probability that they will be observed with this money the day of the interview is only $(1/52) * (1/5) = 0.4\%$.

Table 2: Empirical Correlations, Italy 2004, 35≤age≤44

Money & Income	.21
Money & Consumption	.27
Money & Net Wealth	.30
(Money/Fin. W.) & Net .W.	-0.13

2.2 US Data

US data do not allow us to carry out the same detailed analysis: Income, money and financial wealth come from by the Survey of Consumer Finance (SCF), and the distribution of consumption can be found in the survey of Consumer Expenditures (CE). Hence, we cannot calculate the correlation between consumption and money. I use a conservative definition of money, which is the amount held in checking accounts. This is the only fraction of M1 which is available in the data. I also provide statistics for the amount held in all transaction accounts, which corresponds to the M2 aggregate.

The distribution of income in the SCF[5] 2004 is given in the first column of Table 3. The Gini coefficient is .54 and decreases to .47 (second row) if we consider households whose head is aged between 35 and 44. It decreases further to .41 if we exclude the 1% money-richest households (in the third row).

The results for the distribution of net wealth are given in column 2. The values of the Gini coefficient are very similar between specifications, and range between .81 and .73.

The Gini coefficient of the distribution of money[6] held in checking accounts is given in column 3: this is very high at .81. Excluding life-cycle effects in the second row, the Gini coefficient increases to .83. Finally the third row excludes the 1% money-richest households: the Gini coefficient falls, but

[5] The same exercise can be carried out for a number of years of the SCF. The results are quantitatively similar.

[6] This considerable inequality in money holdings does not depend on the year of the survey. The Gini coefficient for the amount in checking accounts is .74 for the SCF 2001 survey and .72 for the SCF 1998 survey. Note that the nominal interest rate (the Fed's fund rate) was above 5% on average so that the opportunity cost of holding this liquidity was high during this period.

remains high at .75. The fourth column performs the same analysis for money held in all transaction accounts, such as checking, savings and money market accounts. The Gini coefficient here is of the same order of magnitude, and falls from .85 to .79. excluding the 1% money-richest households. The Gini coefficient for money is higher than that of the distribution of income for all definitions of money and for all sets of households. As a result, the distribution of money is much closer to the distribution of net wealth than to the distribution of income.

Table 3: Distribution of Money and Wealth, US 2004 labelDistrib1

Gini coefficient of	Income	Net. W	Check. Acc.	Trans.Acc.
Total Population	.54	.81	.81	.85
Pop., 35≤Age≤44	.47	.80	.83	.85
Pop., 35≤Age≤44, 99%.	.41	.73	.75	.79

The correlation between money (checking account), income and other assets is presented in Table 4. Money is positively correlated with both income and net wealth: richer households hold more money on average. The last line of Table 4 shows the correlation between the ratio of money in financial wealth and total net wealth. This correlation is negative. As in the Italian data, richer households hold more money, but as a smaller percentage of their financial wealth.

Table 4: Empirical Correlations

US, 2004, 35≤age≤44	
Money & Income	.12
Money & Net Wealth	.17
(Money/Fin. W.) & Net Wealth	-0.08

Table 5 below presents some additional properties of the joint distribution of money and assets in the US economy, which will be used to illustrate the model's outcome. The table shows the fraction of total wealth and total money held by the richest 1% of the population (line 1), the richest 10% (line 2), the richest 20% (line 3) and the poorest 40% (line 4). First, the richest households hold a significant fraction of money, whereas the 40% poorest households hold a much lower fraction. Second, we can check that the proportion of money in total wealth is higher for poorer than for richer households. Poor households hold relatively more money than financial assets, but they hold a smaller fraction of the total quantity of money.

Table 5: Asset Holding Distribution

US, 35≤Age≤44

	Fract. of Wealth	Fract. of Checking
Wealth 99-100	32.7%	10.9%
Wealth 90-100	70.0%	60.8%
Wealth 80-100	82.4%	72.6%
Wealth 0-40	1.03%	5.4%

Table 6 summarizes the distribution of money in the US in 2004, for the relevant age group. The first line gives the fraction of the population which holds the least money, the second line shows the fraction of total money held by this group. For instance, the 60% of the population who hold the least money, holds 3.6% of the total money in checking accounts. This table shows that the money is very unequally distributed, as the top 5% of the money distribution hold 69.2% of the total amount of checking-account money. This empirical distribution will be used to assess the ability of the model to reproduce a relevant money distribution.

Finally, the distribution of consumption can be obtained from the survey of Consumer Expenditures (CE). Krueger and Perri [29] note that the distribution of consumption is much less unequally distributed than that of income. The consumption Gini coefficient is around 0.27 and changes only little over time. I calculate the same Gini coefficient for total consumption using the NBER extract

Table 6: Money Distribution

US, 2004, $35 \leq$Age≤ 44							
Fract of Pop.	40%	50%	60%	70%	80%	90%	95%
Fract.of Checking	0.9%	1.7%	3.6%	7.1%	12.2%	21.1%	30.8%

of the Consumer Expenditures survey in 2002, which is the latest year available. I find a Gini coefficient of .28. There is substantial empirical debate about the quality of the data and the estimated changes in consumption inequality (Attanasio, Battistin, Ichimura [6]). The consensus view is that consumption levels are less unequally distributed than is income. As a result, the distribution of money is much closer to the distribution of total wealth than to that of consumption.

To summarize these US and Italian findings: 1) inequality in money holdings is more similar to inequality in net wealth and very different from inequality in consumption; 2) money is positively correlated with wealth, income and consumption levels; and 3) the ratio of money over financial assets falls with wealth.

3 Some Difficulties in Linking Money and Consumption

Simple models of money demand. Simple models linking money demand to consumption cannot reproduce the shape of the distribution of money. These models assume that the real money holdings of a household i, m^i, are simply proportional to consumption, c^i

$$m^i = Ac^i$$

where A is a scaling factor, the same for all households, which may depend on the nominal interest rate, real wages and preference parameters. This form is used for instance in Cooley and Hansen [14] to assess the welfare cost of inflation. It is also found in all models with money-in-the utility function (MIUF), where the utility function is homothetic in money and consumption in the sense of Chari, Christiano and Kehoe [12], which is the benchmark case in this literature. It also pertains in a simple specification of the shopping-time model (McCallum and Goodfriend [31]).

In these models, the distributions of money and consumption are homothetic, and their Gini coefficients are the same. This is at odds with the data, as shown in Section 2.

Economies of scale in the transaction technology. A number of authors have noted that the share of money holdings in total wealth falls with total wealth, and have concluded that the transaction technology exhibits scale economies: richer households, even if they consume more, need less money because they buy more goods via credit. Dotsey and Ireland [19] provide a microfoundation of this transaction technology, which uses the flexibility provided by the definition of cash and credit goods in Stokey and Lucas [36]. Erosa and Ventura [20] use this formulation in a heterogenous-agents setting. This implies that the quantity of money and the consumption level of household i satisfy the following relationship:

$$\frac{m^i}{c^i} = A\left(c^i\right)^{-\theta} \text{ with } \theta > 0 \tag{1}$$

However, this specification is not able to reproduce a realistic distribution of money. With moderate returns (a low value of θ), the distribution of money is more equally distributed than the distribution of consumption, because households with more consumption hold fewer real balances. A more dispersed distribution of money can only be obtained via a implausibly high increasing returns in the transaction technology. In this case, households who consume the most hold almost no money, whereas households who consume little hold higher levels of money balances. However, one implication of this assumption is that consumption and money should be *negatively* correlated, as higher consumption implies lower money holdings and vice versa. This correlation is rejected by the data.

To illustrate, I consider the distribution of consumption of Italian households aged between 35 and 44. I generate fictitious money distributions with various transaction technologies, using the general form of the transaction technology (1) for various values of θ. I finally analyze the distributional properties of the joint distribution of money and consumption.

Table 7 presents the value of the Gini coefficient and the correlation between money and con-

Table 7: Properties of the Distribution of Money for Different Transaction Technologies

Values of θ	Data	0	0.5	1	2	3.7
Gini of consumption	.29	.29	.29	.29	.29	.29
Gini of Money	.70	.29	14	0	0.30	.70
Corr. Money Consumpt.	.27	1	.97	0	-0.63	-0.36

sumption for various values of θ. For values of θ less than 1, the distribution of money is more equally distributed than is the distribution of consumption. To obtain a more inequal distribution of money, the returns on the transaction technology must be higher than 1, but the correlation between money and consumption then becomes negative, which is at odds with the data.

The same type of experiment can be carried out with the US Data. Using the distribution of money, I generate a fictitious distribution of consumption using (1). I determine the value of θ for which the distribution of consumption is realistic in terms of the Gini coefficient. Again, we need a value of θ of over 3 to obtain a Gini coefficient above .47, which is the Gini coefficient on income.

Finally, note that the microfoundation of money demand with scale economies in Dotsey and Ireland [19] requires increasing returns to scale to obtain the correct sign on the interest elasticity of money demand.

To summarize, economies of scale in the transaction technology alone can not generate a realistic distribution of money. This is because money is at the same time positively correlated with consumption and much more dispersed than consumption levels. The following model proves than we can obtain a realistic distribution of money by focusing on transaction frictions in the *financial* market in addition to the frictions in the *goods* markets. The correlation between money and consumption will appear as an outcome, rather than as a specific utility function imposed on households.

Unobserved Heterogeneity. The discussion above made no reference to unobserved heterogeneity. The relationship between money demand and consumption could indeed take the form

$$m^i = A^i c^i \tag{2}$$

where any heterogeneity in A^i could yield considerable dispersion in money holdings.[7] Nevertheless, explanations based on unobserved heterogeneity are not satisfactory. The extent of unobserved heterogeneity needed to match the data is considerable. Using Italian data, for which data on consumption are available, the Gini coefficient over the A^i coefficients is 0.66, and is thus greater than the Gini coefficient on consumption or income. This value is so high because the correlation between money and consumption is low. As a result, the heterogeneity assumed is of the order of magnitude of the value to be explained. The strategy of this paper is to focus on a structural model to reproduce the distribution of money across households as an equilibrium outcome, without assuming any unobserved heterogeneity.

4 The Model

The economy is populated by a unit mass of households and a representative firm. There is a consumption-investment good and there are two assets: money and a riskless asset issued by firms. Time is discrete and $t = 0, 1, ..$ denotes the period. There is no aggregate uncertainty, but households face idiosyncratic productivity shocks. These shocks are not insurable, and households can partially self-insure by holding money or riskless assets. Households must pay a fixed cost λ in terms of the final good[8] to enter the financial market in order to adjust their financial position, and pay no cost to adjust their monetary holdings. Moreover, households must hold money in order to consume according to a simple transaction technology.

4.1 Households

There is a continuum of length 1 of infinitely-lived households who enjoy utility from consumption c and disutility from hours worked n. For simplicity only, I follow Greenwood, Hercowitz and Huffman

[7] This reduced form formulation can be obtained with a MIUF, cash-in-advance or shopping-time framework. (see Feenstra [21], and Croushore [16], for example).

[8] The results do not change significantly if we assume that this cost is paid in labor, and thus affects labor supply.

[22] and Domeij and Heathcote [18] in assuming the following functional form for the period utility function (see also Heathcote [24], for a discussion of the properties of this functional form):

$$u(c,n) = \frac{1}{1-\gamma}\left[\left(c - \psi\frac{n^{1+\frac{1}{\varepsilon}}}{1+1/\varepsilon}\right)^{1-\gamma} - 1\right]$$

In this specification, ε is the Frisch elasticity of labor supply, ψ scales labor supply, and γ is the risk-aversion coefficient. In each period, a household i can be in one of three states according to its labor market status. Productivity e_t^i is then either e^1, e^2 or e^3. For instance, a household with productivity e^1 which works n_t hours earns labor income of $e^1 n_t w_t$, where w_t is the after-tax wage by efficiency unit. Labor productivity e_t^i follows a three-state first-order Markov chain with a transition matrix denoted T. $N_t = [N_t^1, N_t^2, N_t^3]'$ is the distribution vector of households according to their state on the labor market in period $t = 0, 1....$ The distribution in period t is $N_0 T^t$. Given standard conditions, which will be fulfilled here, the transition Matrix T has an unique ergodic set $N^* = \{N_1^*, N_2^*, N_3^*\}$ such that $N^* T = N^*$. To simplify the dynamics, I assume that the economy starts with the distribution N^* of households.

The variables a_t^i and m_t^i denote respectively the *real* quantity of financial assets and money held at the end of period $t-1$, and r_t is the after-tax real interest rate on the riskless asset between $t-1$ and t. Note that we denote a_{t+1}^i and m_{t+1}^i as the real quantity of financial assets and money chosen in period t, for symmetry in the notation. P_t denotes the money price of one unit of the investment-consumption good, and $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate between periods $t-1$ and t. The real income at the *beginning* of period t of a household holding a_t^i and m_t^i is thus $\frac{m_t^i}{\Pi_t} + (1+r_t)\,a_t^i$.

Households pay proportional taxes on capital and labor income: τ_t^{cap} is the tax rate on capital and τ_t^{lab} is the tax rate on labor. The variables $\tilde{w}_t$ and $\tilde{r}_t$ are respectively the real wage and the real interest rate before taxes:

$$w_t = \left(1 - \tau_t^{lab}\right)\tilde{w}_t \text{ and } r_t = \left(1 - \tau_t^{cap}\right)\tilde{r}_t$$

In period t, each household can choose to participate or not in the financial market. If the household participates, it pays a cost of λ and can freely use the total monetary and financial resources $\frac{m_t^i}{\Pi_t} +$

$(1+r_t)\,a_t$ to consume the amount c_t^i, and to save the quantities a_{t+1}^i of financial assets and m_{t+1}^i of money. If the household does not participate, it can only use its monetary revenue m_t^i/Π_t to consume c_t^i and to keep a fraction m_{t+1}^i in money. It is assumed that financial wealth is reinvested in financial assets:[9] $a_{t+1}^i = (1+r_t)\,a_t^i$. This participation choice is summarized by the dummy variable I_t^i, which equals 1 when the household participates and 0 otherwise.

Households must hold cash before consuming. I follow Lucas [30] in assuming that financial markets and money markets open before the goods market. As a consequence, and with our choice of notation, households face the following cash-in-advance constraint:

$$c_t \leq \theta m_{t+1}$$

Here m_{t+1} is the quantity of money decided in period t and θ is a technology parameter which reflects the consumption velocity of money.[10]

Note that there are two reasons to hold money in this model. First, money is necessary to consume because of the cash-in-advance constraint: this summarizes the transactions role of money in the goods market. Second, money can be also held for "financial motives", which is to avoid the portfolio adjustment cost in financial markets.

Last, no private households can issue money $m_{t+1}^i \geq 0$, and households face a simple borrowing limit when participating in financial markets: $a_{t+1}^i \geq 0$, for $t = 0, 1..$ and $i \in [0,1]$.

The program of household i can be summarized as follows:

$$\max_{\{m_{t+1}^i, a_{t+1}^i, c_t^i, n_t^i, I_t^i\}_{t=0,1..}} E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^i, n_t^i\right)$$

[9] This is the standard assumption made by Romer [34] for instance. The quantitative results do not change if interest is paid in money.

[10] This timing convention is more convenient here than that of Svensson [39]. In the latter, households must choose their money holdings one period before consuming. As a consequence, households cannot adjust their money holdings after their idiosyncratic productivity shock, or adjust their consumption within the period, but would be able to adjust their financial portfolio. This would create a discrepancy between money and financial assets, which is problematic in the context of the current paper.

subject to

$$
\begin{aligned}
c_t^i + m_{t+1}^i + I_t^i\left(a_{t+1}^i - (1+r_t)\, a_t^i + \lambda\right) &= e_t^i w_t n_t^i + \frac{m_t^i}{\Pi_t} \\
\left(1 - I_t^i\right)\left(a_{t+1}^i - (1+r_t)\, a_t^i\right) &= 0 \\
c_t &\leq \theta m_{t+1} \\
c_t^i, n_t^i, m_{t+1}^i, a_{t+1}^i &\geq 0, I_t^i \in \{0,1\} \\
a_0^i, m_i^0 &\text{ given}
\end{aligned}
$$

Recursive Formulation

The program of the households can be written recursively as follows (see Bai [7], for a proof of the existence of Bellman equations in this type of economy). Define $V_t^{par}\left(a_t^i, m_t^i, e_t^i\right)$ as the maximum utility that a household with productivity e_t^i can reach in period t if it participates in financial markets at period t and holds amounts m_t^i and a_t^i of monetary and financial wealth respectively; $V_t^{ex}\left(a_t^i, m_t^i, e_t^i\right)$ is the analogous utility if the household does not participate.

The Bellman value $V_t^{par}\left(a_t^i, m_t^i, e_t^i\right)$ then satisfies

$$
\begin{aligned}
V_t^{par}\left(m_t^i, a_t^i, e_t^i\right) = \max_{a_{t+1}^i, m_{t+1}^i, n_t^i, c_t^i} & \{u\left(c_t^i, n_t^i\right) \\
& + \beta E_t \max\{V_{t+1}^{par}\left(m_{t+1}^i, a_{t+1}^i, e_{t+1}^i\right), V_{t+1}^{ex}\left(m_{t+1}^i, a_{t+1}^i, e_{t+1}^i\right)\}\}
\end{aligned}
$$

subject to

$$
\begin{aligned}
m_{t+1}^i + a_{t+1}^i + c_t^i = w_t e_t^i n_t^i + \frac{m_t^i}{\Pi_t} - \lambda + (1+r_t)\, a_t^i \\
c_t \leq \theta m_{t+1} \\
c_t^i, n_t^i, a_{t+1}^i, m_{t+1}^i \geq 0
\end{aligned}
$$

When participating in financial markets, the household faces a single budget constraint, where the participation cost λ has to be paid. The household maximizes its current utility anticipating that next period's participation decision will be made next period, when next period's idiosyncratic shock is revealed. The expectation operator E is then taken over the idiosyncratic shock.

The value $V_t^{ex}\left(a_t^i, m_t^i, e_t^i\right)$ satisfies

$$
\begin{aligned}
V_t^{ex}\left(m_t^i, a_t^i, e_t^i\right) = & \max_{m_{t+1}^i, n_t^i, c_t^i} \{u\left(c_t^i, n_t^i\right) \\
& +\beta E \max\{V_{t+1}^{par}\left(m_{t+1}^i, a_{t+1}^i, e_{t+1}^i\right), V_{t+1}^{ex}\left(m_{t+1}^i, a_{t+1}^i, e_{t+1}^i\right)\}\}
\end{aligned}
$$

subject to

$$
\begin{aligned}
m_{t+1}^i + c_t^i &= w_t e_i^t n_t^i + \frac{m_t^i}{\Pi_t} \\
a_{t+1}^i &= a_t^i\left(1 + r_t\right) \\
c_t &\leq \theta m_{t+1} \\
c_t^i, n_t^i, m_{t+1}^i &\geq 0
\end{aligned}
$$

When not participating in financial markets, the household cannot change its financial position, but does anticipate that it may or may not participate next period

Finally, the maximum utility that a household with productivity e_t^i and assets m_t^i and a_t^i can reach is

$$
V_t\left(m_t^i, a_t^i, e_t^i\right) = \max\{V_t^{par}\left(m_t^i, a_t^i, e_t^i\right), V_t^{ex}\left(m_t^i, a_t^i, e_t^i\right)\}
$$

According to this expression, the household either chooses to participate, in which case $I_t^i = 1$, or not, $I_t^i = 0$.

The solution of the household's problem produces a set of optimal decision rules which are functions of productivity, the set of assets and $E = \{e^1, e^2, e^3\}$:

$$
\left.
\begin{aligned}
c_t(.,.,.) &: \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow \mathbb{R}^+ \\
a_{t+1}(.,.,.) &: \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow \mathbb{R}^+ \\
m_{t+1}(.,.,.) &: \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow \mathbb{R}^+ \\
n_t(.,.,.) &: \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow [0,1] \\
I_t(.,.,.) &: \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow \{0,1\}
\end{aligned}
\right\} t = 0, 1, ...
$$

4.2 Firms

The consumption-investment good is produced by a representative firm in a competitive market. Capital depreciates at a rate of δ and is installed one period before production. We denote by K_t and L_t the aggregate capital and aggregate effective labor used in production in period t. Output Y_t is given by

$$Y = F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha},\ 0 < \alpha < 1$$

Effective labor supply is:

$$L_t = e^1 L_t^1 + e^2 L_t^2 + e^3 L_t^3$$

where L_t^1, L_t^2 and L_t^3 is the aggregate labor supply of workers of productivity $1, 2$ and 3 respectively. Profit maximization yields the following relationships

$$\tilde{w}_t = F'_L(K_t, L_t) \tag{3}$$

$$\tilde{r}_t + \delta = F'_K(K_t, L_t) \tag{4}$$

where $\tilde{w}_t$ and $\tilde{r}_t$ are before-tax real wages per efficient unit and the real interest rate.

4.3 Monetary Policy

At each period t, monetary authorities create an amount of new money Δ_t. Let M_t be the total amount of nominal money in circulation at the *end* of period t. The law of motion of the nominal quantity of money is thus

$$M_t = M_{t-1} + \Delta_t \tag{5}$$

The real value of the inflation tax is thus Δ_t / P_t.

I focus below on stationary equilibria where monetary authorities create a quantity of money proportional to the total nominal quantity of money of the previous period, with a coefficient of π. In this case $\Delta_t = \pi M_{t-1}$ and the revenue from the inflation tax is $\pi M_{t-1}/P_t$.

4.4 Government

The Government finances a public good, which costs G_t units of goods in period t. It receives the inflation tax Δ_t/P_t and the proportional taxes on capital and labor income, with coefficients τ_t^{cap} and τ_t^{lab} respectively. It is assumed that the Government does not issue any debt. Its budget constraint is

$$G_t = \tau_t^{cap}\tilde{r}_t K_t + \tau_t^{lab}\left(L_t^1 e_t^1 + L_t^2 e_t^2 + L_t^3 e_t^3\right)\tilde{w}_t + \frac{\Delta_t}{P_t} \tag{6}$$

where L_t^1, L_t^2 and L_t^3 are total labor supply of type $1, 2$ and 3 households respectively.

4.5 Market Clearing

Denote $\Phi_t : \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow [0,1]$ as the joint distribution of households over financial assets, money holdings and productivity in period t. Money and capital market equilibria state that money is held by households at the end of each period, and that financial savings are lent to the representative firm. These can be written as, for $t \geq 0$:

$$M_t = \int_{\mathbb{R}^+\times\mathbb{R}^+\times E} P_t m_{t+1}(a,m,e)\, d\Phi_t(a,m,e) \tag{7}$$

$$K_{t+1} = \int_{\mathbb{R}^+\times\mathbb{R}^+\times E} a_{t+1}(a,m,e)\, d\Phi_t(a,m,e) \tag{8}$$

Goods-market equilibrium requires that the amount produced is either consumed by the State, invested in the firm, consumed by the households, or destroyed in the transaction cost. This can be written as

$$\begin{aligned} G_t + K_{t+1} + \int_{\mathbb{R}^+\times\mathbb{R}^+\times E} c_t(a,m,e)\, d\Phi_t(a,m,e) \\ + \lambda \int_{\mathbb{R}^+\times\mathbb{R}^+\times E} I_t(a,m,e)\, d\Phi_t(a,m,e) = F(K_t, L_t) + (1-\delta) K_t \end{aligned} \tag{9}$$

4.6 Equilibrium

For a given path of Government spending $\{G_t^{cap}\}_{t=0..\infty}$ and money creation $\{\Delta_t\}_{t=0..\infty}$, an equilibrium in this economy is a sequence of decision rules $c_t(.,.,.)$, $a_t(.,.,.), m_t(.,.,.), n_t(.,.,.)$ $I_t(.,.,.)$ defined over $\mathbb{R}^+ \times \mathbb{R}^+ \times \{e^1, e^2, e^3\}$ for $t = 0..\infty$, sequences of prices $\{P_t\}_{t=0..\infty}$, $\{\tilde{\omega}\}_{t=0..\infty}$ and $\{\tilde{r}\}_{t=0..\infty}$, and sequences of taxes $\{\tau_t^{lab}\}_{t=0..\infty}$ and $\{\tau^{cap}\}_{t=0..\infty}$ such that:

1. The functions $c_t(.,.,.)$, $a_{t+1}(.,.,.), m_{t+1}(.,.,.), n_t\,(.,.,.)$ $I_t(.,.,.)$ solve the household's problem for a sequence of prices $\{P_t\}_{t=0..\infty}$, $\{\tilde{\omega}\}_{t=0..\infty}$ and $\{\tilde{r}\}_{t=0..\infty}$, and taxes $\{\tau_t^{lab}\}_{t=0..\infty}$ and $\{\tau^{cap}\}_{t=0..\infty}$.

2. The joint distribution Φ_t over productivity and wealth evolves according to the decision rules and the transition matrix T.

3. Factor prices are competitively determined by firm optimal behavior (3)-(4).

4. The quantity of money in circulation follows the law of motion (5).

5. Markets clear: equations (7)-(9).

6. Tax rates $\{\tau_t^{lab}\}_{t=0..\infty}$ and $\{\tau_t^{cap}\}_{t=0..\infty}$ are such that the government budget (6) is balanced.

A *stationary* equilibrium is an equilibrium where the nominal money growth rate, the values $G, \tau^{lab}, \tau^{cap}$, r, w, the gross inflation rate $\Pi = \frac{P_t}{P_{t-1}}$, the joint distribution Φ and the decision functions $c(.,.,.)$, $a(.,.,.), m(.,.,.), n\,(.,.,.)$ $I(.,.,.)$ are time-invariant. In such an equilibrium, the aggregate real variables are constant whereas the nominal variables all grow at the same rate.

5 Parameterization

The model period is one quarter. Table 8 summarizes the parameter values at a quarterly frequency in the stationary benchmark equilibrium.

Table 8: Parameter Values

α	β	ψ	σ	ε	τ^{cap}	τ^{lab}	δ	π	λ	θ
0.36	0.99	117	1	0.3	0.397	0.296	0.015	0.007	0.20	0.035

Preference and fiscal parameters

The preference and technology parameters have been set to standard values. The capital share is fixed at $\alpha = 0.36$ (Cooley and Prescott [15]) and the depreciation rate is $\delta = 0.015$, such that the annual depreciation rate is 6% (Stokey and Rebelo [37]). The discount factor β is set to 0.99, to obtain a realistic annaul capital-output ratio of around 3. The risk-aversion parameter, σ, is set to 1. The Frisch Elasticity of labor supply ε is estimated to be between 0.1 and 1. I use a conservative value of 0.3. Given this value, ψ is set such that aggregate effective labor supply is close to 0.33. The fiscal parameters are calibrated to match the actual tax distortions in the US economy. Following Domeij and Heathcote [18], the average tax rate on capital income τ^{cap} is 39.7 percent, whereas the average tax rate on labor income τ^{lab} is 26.9 percent. The implied government consumption to annual output ratio is 0.24, which is a little higher than, but not too dissimilar to, the U.S. average of 0.19 over the 1990-1996 period.

The household productivity process

Different models of the income process are now used in the literature. Our modeling strategy is to use a simple process which yields realistic distributions for consumption, income and wealth. I consequently use that in Domeij and Heathcote [18], with endogenous labor used at a quarterly frequency. They estimate a three-state Markov process, which reproduces the process for logged labor earnings using PSID data. The Markov chain is estimated under two constraints: (i) The first-order autocorrelation in annual labor income is 0.9; and (ii) The standard deviation of the residual in the wage equation is 0.224. These values are consistent with estimations found in the literature (Storesletten, Telmer and Yaron, [38], amongst others).

The transition matrix is

$$T = \begin{bmatrix} 0.974 & 0.026 & 0 \\ 0.0013 & 0.9974 & 0.0013 \\ 0 & 0.026 & 0.974 \end{bmatrix}$$

The three productivity levels are $e^1 = 4.74$, $e^2 = 0.848$, $e^3 = 0.17$. The long-run distribution of productivity across the three states is $N^* = [0.045 \quad 0.91 \quad 0.045]'$. This parameterization yields a realistic distribution of both wealth and consumption, which is very useful for the issue that we address.

Monetary Parameters

The other parameters concern monetary policy and the transaction cost. First, I take the US annual inflation rate in 2004, 2.8 percent. Consequently, the quarterly inflation rate is $\pi = 0.007$. The two remaining parameters concern the portfolio adjustment cost λ and the transaction constraint θ. I have optimized over the values of these two parameters to find 1) a realistic ratio of aggregate money to aggregate income and 2) a realistic equilibrium money distribution. Money is defined as above as the amount in checking accounts in SCF 2004. The ratio of money over income is 8% for households between 35 and 44 years old. I choose the parameters to obtain a money distribution as close as possible to the empirical one given in Table 6. I find a portfolio adjustment cost $\lambda = 0.2$ and a transaction cost $\theta = 0.035$. A robustness check is provided below. To summarize the results analyzed in detail below, a large value of λ is necessary to obtain considerable inequality in money holdings. A low but positive transaction parameter θ is required to reproduce the money holdings of the poorest households. Scaling by average income per capita in the US of $43000, I find an annual transaction cost for financial markets for the riskless asset of around $1500. To my knowledge, there is no consensus in the empirical literature regarding the level of such costs. The empirical strategy of Mulligan and Sala-i-Martin [32] and Paiella [33] only provides the median cost or the lower bound of the participation cost. Some insights can be obtained from the literature which estimates the cost of participating in the risky-asset market. Vissing-Jorgensen [41] estimates this participation cost to be as high as 1100 dollars in order to understand the transaction decisions of 95% of non-participants, whereas a cost of 260 dollars suffices to explain the choices of 75% of non-participants. In consequence, although the cost is towards the top-end of these estimates, it is not inconsistent with current empirical results.

I now present the result of the model. I first present the outcome of the general model. I then remove successively the frictions on the goods and on the financial markets. This exercise will allow me to quantify the contribution of each motive for total money holdings.

6 Results of the General Model

This section first presents the household policy rules and then the properties of the distribution of consumption, income, money and total wealth.

6.1 Households Behavior

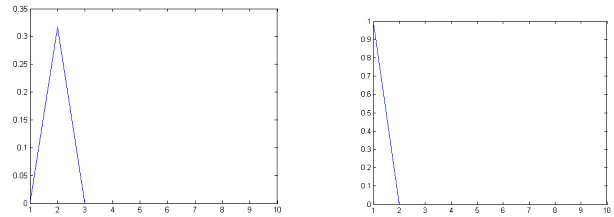
Money Holdings. Table 9 first represents the quantity of money held by each type of household. The inequality between types is not particularly high. For instance, type 3 households, the low income households, hold a higher fraction of money than their share in the total population. The inequality in money holdings will thus come from the heterogeneity within types of households.

Table 9: Money Holdings pet Type of Agents

Agents	Type 1	Type 2	Type 3
Fraction of Money	26.3%	68.5%	5.2%
Faction of Pop	4.5%	91%	4.5%

To provide a better understanding of the sources of inequality in money holdings, Figures 2-4 depict the saving behavior in money and financial assets of an agent who is always of type $1, 2$ or 3 respectively, with the same initial portfolio composed of 1 unit of financial assets. Note that, in this exercise, household productivity does not change, whereas some change is expected to occur from the transition matrix T. As a consequence, these Figures should be read as particular household histories.

The choice of a type-3 household, which has the lowest labor income, is presented in Figure 2. The household decumulates all financial wealth down to 0 in one period. It first transfers a part of its wealth into money in the second period and then holds just a small amount of money which is necessary in order to consume, because of the transaction constraint on the goods market. These agents thus hold only little money after a short period.



Money Fin. Asset

Figure 2: Portfolio Evolution of type 3 Households

The portfolio choice of a type-2 household is shown in Figure 3. When a threshold for financial assets is reached, the household participates only infrequently. When the household participates in financial markets, it replenishes its money holdings by selling financial assets. This buffer stock is quickly reduced to a small amount consistent with the transaction constraint on the goods market, after 3 periods. Indeed, the opportunity cost of holdings money is high, as its return is negative. The amount of money held thus exhibits an uneven pattern: only few agents hold a large quantity of money, which explains the inequality in money holdings.

Money Fin. Asset

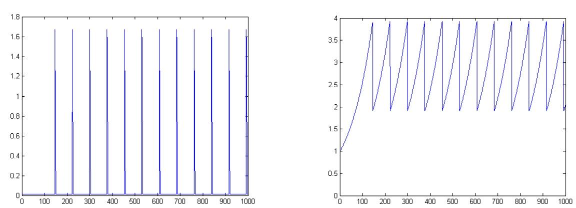


Figure 3: Portfolio Evolution of Type 2 Households

Finally, in Figure 4, we see that the behavior of a type-1 household, with the highest labor income, consists of different phases. These households save and accumulate high buffer stocks. First, the type-1 household accumulates some money and participates often (every three periods) in the financial market to buy some assets. This yields a rapid accumulation of financial assets. After a while, the type-1 household no longer participates in the financial market and only holds the quantity of money necessary to consume because of the transaction constraint. The household lets the amount of financial assets accumulate via the interest paid. Third, the households participate in financial markets to sell some assets to obtain money and to consume. Fourth, the household holds only financial assets, and is then rich enough not to care about the adjustment cost.

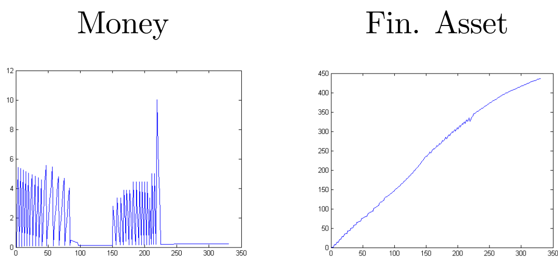


Figure 4: Portfolio Evolution of Type 1 Households

In conclusion, households participate infrequently in financial markets, which creates inequality in money holdings across households. Moreover, more productive households often hold more money than less productive ones, but this ranking is not constant and depends on household wealth. This will explain the correlation between money and wealth given in the next section.

Participation Decisions. Households' participation decisions help to explain infrequent financial market participation. Figure 5 represents the participation decision in financial markets for type-2 households (which are the most numerous). The decision rule of other agents is discussed below.

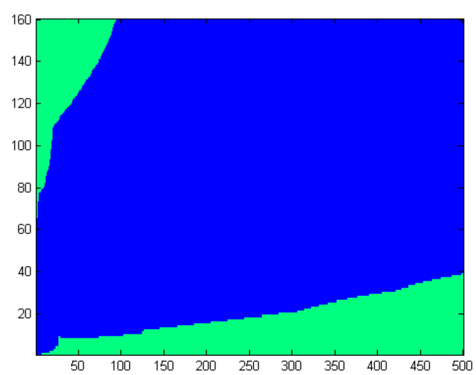


Figure 5: Participation Decision of Type 2 Households. a is on the x-axis and m is on the y-axis

The graph should be read as follows: The $x-$axis measures the quantity of financial assets held at the beginning of the period, denoted by a. The $y-axis$ represents the real quantity of money held at the beginning of the period, denoted by m. Each point on the graph is thus a beginning-of-period portfolio (a, m). The graph plots $I(a, m, 2)$ which is the dummy variable indicating the participation decision. The dark area represents the set of values, (a, m), for which $I(a, m, 2) = 0$, that is the set of initial portfolios for which the household chooses not to participate in financial markets. The lighter area represents the set of values (a, m) for which $I(a, m, 2) = 1$, that is the set of portfolios for which households choose to participate in financial markets.

Households holding a high quantity of money and a small quantity of financial assets (a low, m high) and households holding a small quantity of money and high quantity of assets (a high, m low) both participate in financial markets. Households who are inbetween do not participate. Households with a large amount of money and few assets participate to actually save in financial assets and dis-save money: These households hold a large quantity of money and want to transfer it to their financial account to benefit from the remuneration of financial savings. Households with little money and many financial assets participate to dis-save in financial assets and save in money: These households prefer to increase their money stock in the current period to avoid paying portfolio adjustment costs in the future. There is a large inaction region, where households choose not to participate in financial markets. In this case, they smooth consumption only with money and let the remuneration of financial savings accumulate on their financial account. This participation decision is very similar to those obtained in the (S, s) models first studied in Grossman and Laroque [23], among others, with one asset. Households thus hold both money and financial assets in equilibrium, although the (marginal) return on money is lower than that on financial assets.

Finally, the participation decision of type-1 and type-2 households are comparable. Type-1 households participate more often to save in financial assets and less often to save in money, because they have a higher labor income. The reverse is true for type-3 households.

6.2 The Distribution of Money and Financial Assets

The distribution of consumption, money and financial wealth is summarized in Table 10, which presents the associated Gini coefficients.

Table 10: Gini coefficients

	Consumption	Money	Wealth
US Data (Age 35-44)	.28	.83	.80
Model	.35	.80	.84

First note that the model performs quantitatively well in reproducing the inequality in the distribution of consumption, income, money and wealth. The Gini of the total wealth distribution is 0.84. The Gini coefficient for money is 0.80, which is similar to that actually observed in the US economy. The Gini coefficient for consumption is a little higher than its empirical counterpart.

Table 11 presents the summary statistics for the distribution of money. The model does a good job in reproducing the distribution of money of the households who hold the lowest quantity of money. The bottom 50% of the population holds 1.7% of the money stock in the US data, whereas it holds 1.3% of the total money stock in the model. Moreover, the bottom 80% of the population holds 12.2% of the money stock in the US data and roughly the same amount, 13.3%, in the model. Although the model is able to reproduce the considerable inequality in money holdings, and thus high Gini coefficients, it does not fully capture the empirical inequality in money holdings at the top of the distribution.[11].

Table 12 below investigates other distributional properties of the model. As in Table 5 above, the table shows the fraction of wealth and money held by various subpopulations, ranked by their wealth. The right-hand side of the table presents the values produced by the model. For ease of

[11] The difficulty in fully capturing inequality at the top of the distribution is well known in this class of models (see DeNardi, 2004, for instance)

Table 11: Money Holding Distribution

US, 2004, 35≤Age≤44

Fract of Pop.	40%	50%	60%	70%	80%	90%	95%
US Data	0.9%	1.7%	3.6%	7.1%	12.2%	21.1%	30.8%
Model	1.1%	1.3%	1.6%	3.0%	13.3%	35.6%	58%

comparison, the left-hand side reproduces the empirical counterparts in the US in 2004. The model performs relatively well in reproducing the wealth and money holdings of the poorest households.

Table 12: The Distribution of Asset Holdings

	US Data, 35≤Age≤44		Model	
	Fract. of Wealth	Fract. of Checking	Fract. of Wealth	Fract. of Money
Wealth 90-100	70.0%	60.8%	74.6%	26.2%
Wealth 80-100	82.4%	72.6%	91.5%	67.4%
Wealth 0-40	1.0%	5.4%	0.3%	9.3%

Table 13 presents the correlations between money, income and financial wealth generated by the model. The left-hand side shows the values in US data for the relevant age groups, and the right-hand side shows the model results. All of the model correlations have the right signs. Further, the correlation between money and wealth is roughly the same in the model (.22) and in the data (.17). Last, the model is able to reproduce the sign of the correlation between wealth and the ratio of money to total wealth, but the negative correlation is too high in the model, -0.32, compared to that in the data, -0.08.

These results present the best equilibrium money distribution I was able to obtain. Both the financial and the transaction motives shape the distribution of money. To see this, the next two

Table 13: Empirical Correlations,

	US Data	Model
Money & Income	.12	.
Money & Consumption	-	.43
Money & Wealth	.17	.22
(Money/ Wealth) & Wealth	-.08	-.32

Sections close off in turn the financial and transaction motives to show how they matter for the distribution of money.

7 Analysis of the frictions on goods and financial markets

The previous section has shown that transaction frictions on both goods and financial markets can produce a realistic money distribution. In this section, I study each friction separately.

7.1 Frictions on the Goods Market Only

I first remove the friction on the financial market by setting to 0 the value of the portfolio-adjustment cost. As a consequence, households hold money only because of the transaction friction on the goods market. The other parameters provided in Table 8 remain unchanged. The resulting distribution of consumption, money and financial wealth is summarized in Table 14.

Table 14: Gini coefficients

	Consumption	Money	Wealth
US Data (Age 35-44)	.28	.83	.80
Model	.37	.37	.81

As expected, this specification of the model cannot reproduce a realistic distribution of money:

the Gini coefficient on money holdings is the same as that on consumption, 0.37. This can be seen in Table 15, which presents the results concerning the money distribution. Money is too equally distributed. For instance, the bottom 50% of the money distribution hold only 1.7% of the money stock in the data as compared to 26% in the model.

This exercise confirm that introducing only transaction motives for holding money does not

Table 15: Money Holding Distribution

US, 2004, 35≤Age≤44

Fract of Pop.	40%	50%	60%	70%	80%	90%	95%
US Data	0.9%	1.7%	3.6%	7.1%	12.2%	21.1%	30.8%
Model	17%	26%	32%	40%	51%	64%	77%

produce a satisfactory money distribution. This result is robust to changes in the value of θ.

7.2 Frictions on Financial Markets Only

I now close the transaction motive by setting $\theta = 0$. In this economy, households only hold money to avoid the portfolio-adjustment cost in financial markets. The other parameters provided in Table 8 remain unchanged. The distribution of consumption, money and financial wealth is summarized in Table 16. The model is able to reproduce realistic levels of inequality for consumption, money and wealth. Nevertheless, the underlying shape of the money distribution is not realistic.

Table 16: Gini coefficients

	Consumption	Money	Wealth
US Data (Age 35-44)	.28	.83	.80
Model	.34	.87	.83

Table 17 below shows that too many households do not hold money in this economy. Section 6.1 discussed why this comes about. Households hold money for only a few periods after participating in financial markets, because the opportunity cost of holding money is high. They thus rapidly drive their money balances down to 0. Only few households hold large quantities of money, which drives this result. But, as a consequence, too many households do not hold any money.

Table 17: /The Distribution of Money Holding

US, 2004, 35≤Age≤44

Fract of Pop.	40%	50%	60%	70%	80%	90%	95%
US Data	0.9%	1.7%	3.6%	7.1%	12.2%	21.1%	30.8%
Model	0%	0%	0%	0%	0.2%	24%	51%

The result from the comparison of the two previous economies is that frictions on the goods markets have to be introduced to explain why many households hold a small amount of money. The friction on the financial market is necessary to understand why few households hold large amounts of money.

7.3 An Alternative Calibration

This section provides an alternative calibration where transaction motives have a higher weight, to show that this calibration does not match the empirical shape of the money distribution. I consider a transaction parameter θ equal to 0.055 and a portfolio adjustment cost λ of 0.17. The other parameters provided in Table 8 remain unchanged. With these values, I obtain a quantity of money over income equal to 8%. Table 18 presents the new monetary parameters.

Table 18: Parameter Values

λ	θ
0.17	0.055

First, this alternative calibration provides inequality figures for consumption and financial wealth close to those obtained in the benchmark calibration, as can be seen in Table 19. The inequality in money holdings is smaller than that in the data, because the transaction motive has more weight and yields less inequality in money holdings.

Table 19: Gini coefficients

	Consumption	Money	Wealth
US Data (Age 35-44)	.28	.83	.80
Model	.35	.72	.84

This can also be seen in the shape of the money distribution given in Table 20. In this calibration money is too equally distributed. For instance, the first 80% of money distribution holds 21.3% of the money stock in this calibration, whereas they actually hold 12.2% in the US data. The corresponding figure was 13.3% in the benchmark calibration.

Table 20: The Money Holding Distribution

US, 2004, 35≤Age≤44

Fract of Pop.	40%	50%	60%	70%	80%	90%	95%
US Data	0.9%	1.7%	3.6%	7.1%	12.2%	21.1%	30.8%
Model	2%	2.52%	4.26%	10.3%	21.3%	44.3%	64%

I conclude from this exercise that frictions on the goods market must be small enough to produce a high enough figure for inequality in money holdings.

7.4 A Quantification of Money Demand Motives

I can now provide a quantification of the contribution of each friction to money demand. I can compare the quantity of money obtained in three different economies: first, that in the benchmark economy; second that in the economy with frictions on the goods market only; and third that in the economy with frictions on the financial market only.

The results are summarized[12] in Table 21. The first line refers to the benchmark economy where both frictions are introduced: $\lambda = .2$; $\theta = .0355$. The second column gives the value of the real amount of money in this economy: .39. The third column provides the value of the quantity of money over annualized GDP. We here find the value of 8%, which was the value targeted. The second line refers to the economy with frictions on the goods market only: $\lambda = 0$. The quantity of money falls to the value of .02, which corresponds to a value of money over GDP of only .5%. The real quantity of money obtained in this economy corresponds to only 5.1% of the quantity of money obtained in the benchmark economy. This first comparison leads to the conclusion that financial frictions represent roughly 95% of total money demand in the benchmark economy.

The third line is the economy with only financial motives, where the friction on the goods market has been removed , $\theta = 0$. We find a quantity of money equal to .33, which corresponds to 6.5% of money over GDP. This amount of money corresponds to 85% of the quantity of money obtained in the benchmark economy. This suggests that frictions on the financial markets explain 85% of total money demand.

Table 21: Quantification of Money Demand Motives

Economies	Money	Money/GDP
$\lambda = .2$; $\theta = .0355$	.39	8%
$\lambda =$; $\theta = .0355$	.02	.5%
$\lambda = .2$; $\theta = 0$	.33	6.5%

An additional insight from these comparisons is that both frictions interact in a non-linear way. The money obtained when both frictions are introduced is greater than the sum of the quantity of money for each friction separately. Households which go infrequently to financial markets have to

[12] Quarterly GDP in the three economies are very close to 1.25. This allows us to simply compare the value of money over GDP in the three experiments.

hold a higher quantity of money when frictions on the goods market are introduced, because they now have to hold more money to be sure of fulfilling the cash-in-advance constraint in future periods.

Comparing these three economies, we deduce that the amount of money demanded due to financial motives represents between 85% and 95% of the total quantity of money. This result has been anticipated since Section 2, where the empirical money distributions were presented: considerable departures from the transaction motive on the goods market are necessary to reproduce both high inequality in money holdings and low inequality in consumption.

8 Conclusion

I first document that the distribution of money across households is similar to the distribution of financial assets, and very different from the distribution of consumption. This fact appears as a puzzle for theory of money demand which directly links money demand and consumption. The contribution of this paper is to show that the distribution of money can be reproduced as an equilibrium outcome when transaction frictions are introduced on both the goods and financial markets. The friction on the goods market is a standard cash-in-advance constraint, and the friction on the financial markets is a portfolio adjustment, as in the Baumol-Tobin literature.

It is shown that both motives are necessary to obtain a realistic shape of the money distribution and a high value of inequality in money holdings. The transaction motive is necessary to explain why many people hold a small amount of money. The financial motive appears important to explain why a few people hold large amounts of money. Considering the transaction and the financial motives in turn, it is found that the financial motive alone explains more than 85% of the quantity of money in circulation. One path for future research would be to search for simple shortcuts to introduce financial frictions in simpler macroeconomic models.

A Appendix

A.1 Equilibrium Values

The following table provides the equilibrium values of the model, at quarterly frequency

K	M	Y	C	L	r	w
14.69	0.39	1.25	0.67	0.31	0.95%	1.87

A.2 Computational Strategy

The computational strategy for the stationary equilibrium of the type of model used in this paper is now well-defined. I describe here the main steps.

1) I first consider a given real interest rate r and tax rates over labor τ^{lab} and capital τ^{cap}. These define the post-tax real interest rate and wage, r and w.

2) I then iterate over the six value functions $\{V_j^{par}(.,.,e), V_j^{ex}(.,.,e)\}_{e=1,2,3}$, using the value-function iteration procedure, as previously used in Algan and Ragot (2010). This method is known to be slow but does not require differentiability, which is not ensured in this model. More precisely, I iterate over the value functions, $\{V_j^{par}(.,.,e), V_j^{ex}(.,.,e)\}_{e=e_1,e_2,e_3}$ solving the following maximization, for $e = e_1, e_2, e_3$:

$$V_{j+1}^{par}(m, a, e) = \max_{a', m', n, c} \{u(c, n)$$

$$+ \beta E \max\{V_j^{par}(m', a', e'), V_j^{ex}(m', a', e')\}\}$$

$$m' + a' + c = wen + \frac{m}{\Pi} - \lambda + (1 + r) a$$

$$c \leq \theta m' \text{ and } c, n, a', m' \geq 0$$

and

$$
\begin{aligned}
V_{j+1}^{ex}(m,a,e) &= \max_{m',n,c}\{u(c,n) \\
&\quad +\beta E\max\{V_j^{par}(m',a',e'),V_j^{ex}(m',a',e')\}\} \\
m'+c &= wen+\frac{m}{\Pi} \\
a' &= a(1+r) \\
c_t &\leq \theta m' \text{ and } c,n,m'\geq 0
\end{aligned}
$$

I first solve the maximizations assuming that the cash-in-advance constraint $c \leq \theta m'$ does not bind. If it is not the case, I solve the maximizations imposing $c = \theta m'$.

To initialize the process I simply set $V_0^{par}(.,.,.) = V_0^{ex}(.,.,.) = 0$. Note that the inflation rate Π is given and is an exogenous parameter.

I consider 20×800 portfolios values for (m,a), and the maximization over $m^{'}, a'$ is taken over a square grid with 18000×720000 values. Convergence is ensured when $\left\|V_j^{\kappa}(.,.,e) - V_{j+1}^{\kappa}(.,.,e)\right\| < 10^{-5}$ for all $e = e_1, e_2, e_3$ and all $\kappa = par, ex$. After convergence, I can extrapolate policy functions using the optimal value functions.

3) I find the stationary distribution by iterating the policy functions. I first consider a initial distribution Φ_j over a grid of initial portfolios and agent types (m,a,e). I then apply the policy rules to each portfolio and the transitions probabilities over types T to obtain a new distribution Φ_{j+1} (m',a',e),. When a targeted portfolio(m',a') is not on the grid, I reallocate it to its three closest portfolios. Convergence is obtained when $\|\Phi_{j+1}(.,.,e) - \Phi_j(.,.,e)\| < 10^{-6}$ for $e = e_1, e_2, e_3$.

4) After convergence, I can compute the ex-post capital stock and the real interest rate. Moreover, using the budget constraint of the State (6), I can deduce the level of public spending. I iterate over r and the taxes τ^{cap} to τ^{lab} until the capital market clears. I check that the level of public spending is realistic.

Finally, the code is written in FORTRAN.

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