TECHNICAL APPENDIX TO PRECAUTIONARY SAVING AND AGGREGATE DEMAND

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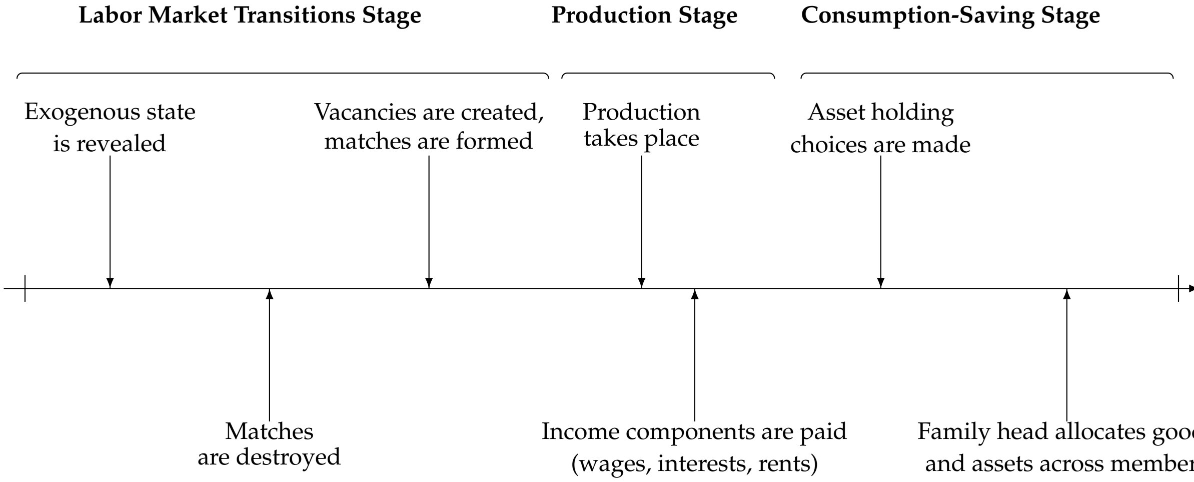


FIGURE 1. Model timeline within a period

A. THE MODEL

A.1. **Generalities.** Let X denote the beginning-of-period vector of aggregate states. Let Γ denote the law of motion X that agents use for form rational expectations :

$$X' = \Gamma(X, \epsilon'),$$

where ϵ' is the innovation to the exogenous aggregate state. The exogenous state is Markovian and includes a stochastic productivity trend e^z, where z drifts at rate $\mu_z \geq 0$. There are $\Omega \in [0,1) \subset \mathbb{R}$ familes of workers and $1-\Omega$ of firm owners, et every family has mass one. The period utility function is $u(c - h\mathbf{c}) = \lim_{\tilde{\sigma}\to\sigma} \frac{(c-h\mathbf{c})^{1-\tilde{\sigma}}-1}{1-\tilde{\sigma}}$, $\sigma > 0$, $h \in (0,1)$, and workers and firm owners subjective discount factors satisfy:

$$0 < \beta^W < \beta^F < e^{(\sigma-1)\mu_z}$$

We let $\mathbf{c}^F$ be the consumption habit of firm owners (equal to the average consumption of firm owners in the previous period) and $\mathbf{c}^W(\aleph)$ the habit level of workers having been continuously unemployed for $\aleph \in \mathbb{Z}_+$ periods.

Also, let $\tilde{\mu}(a, \aleph)$ denote the beginning of period distribution of workers over assets and periods of unemployment when the aggregated state is X. Similarly, let $\mu(a, \aleph)$ denote the distribution of workers after labor market transitions.

Figure 1 synthesises the time frame within a period.

A.2. **Workers.** Let $\tilde{\boldsymbol{\mu}}(a, \aleph) \in X$ and $\boldsymbol{\mu}(a, \aleph)$ be the economywide distributions of workers over assets $a \in \mathbb{R}$ and length of unemployment spell $\aleph \in \mathbb{Z}_+$ at the beginning and the end of the labor market transition rate, respectively, and $\tilde{\mathbf{n}}^W \equiv \int_a \mathrm{d}\tilde{\boldsymbol{\mu}}(a,0)$ and $\mathbf{n}^W = f(1-\tilde{\mathbf{n}}^W) + (1-s)\tilde{\mathbf{n}}^W$ the corresponding employment rates. The UI scheme is assumed to be balanced in every period, i.e.

$$\tau w \mathbf{n}^W = b^u e^z (1 - \mathbf{n}^W). \tag{A.1}$$

Let $\tilde{\mu}(a,\aleph)$ and $\mu(a,\aleph)$ denote the distribution of workers by types within a representative family of workers and $\tilde{n}^W = \int_{\mathbb{R}} \mathrm{d}\tilde{\mu}(a,0)$ and $n^W = f(1-\tilde{n}^W) + (1-s)\tilde{n}^W$ the corresponding employment rates. The family head solves:

$$V^W(\mu, X) = \max_{(a^{W\prime}(a,\aleph), c^W(a,\aleph))_{\aleph\in\mathbb{Z}_+, a\in\mathbb{R}}} \sum_{\aleph\in\mathbb{Z}_+} \int_{a\in\mathbb{R}} u(c^W(a,\aleph) - h\mathbf{c}^W(\aleph))\mathrm{d}\mu(a,\aleph) + \beta^W \mathbb{E}_{\mu,X} V^W(\mu', X')$$

$$\text{subject to} \quad a^{W\prime}(a,\aleph) + c^W(a,\aleph) = \mathbf{1}_{\aleph=0}(1-\tau)w + \mathbf{1}_{\aleph\geq 1} b^u \mathrm{e}^z + (1+r)a$$

$$a^{W\prime}(a,\aleph) \geq \underline{a}\mathrm{e}^z$$

Due to the assumed risk-sharing arrangement, the wealth of workers who will be employed ($\aleph = 0$) at the beginning of next-period consumption-saving stage will be:

$$A' \equiv \frac{(1-s') \int_{\mathbb{R}} a \mathrm{d}\tilde{\mu}'(a,0) + f' \sum_{\aleph\geq 1} \int_{\mathbb{R}} a \mathrm{d}\tilde{\mu}'(a,\aleph)}{n^{W\prime}},$$

where $n^{W\prime}$ will be the number of employed workers at that time:

$$\begin{aligned} n^{W\prime} &= (1-s') \int_{\mathbb{R}} \mathrm{d}\tilde{\mu}'(a,0) + f' \sum_{\aleph\geq 1} \int_{\mathbb{R}} \mathrm{d}\tilde{\mu}'(a,\aleph) = (1-s')\tilde{n}^{W\prime} + f'(1-\tilde{n}^{W\prime}) \\ &= (1-s')n^W + f'(1-n^W) \end{aligned}$$

A.2.1. ***Restatement of workers' problem.*** From Proposition 1 in Section 2.1 of the main text, we now that the cross-sectional distribution of workers μ has a unique mass point in a for all $\aleph \geq 0$. Hence the cross-sectional distribution μ is summarized by the value of these mass points $a(\aleph)$ and their associated number of workers $n(\aleph)$, where $n^W \equiv n(0)$ (i.e., the number of employed workers in the family). It follows that the problem of the workers can be rewritten as:

$$\begin{aligned} \hat{V}^W\left((a(\aleph), n(\aleph))_{\aleph\geq 0}, X\right) = \max_{(a^{W\prime}(\aleph), c^W(\aleph))_{\aleph\in\mathbb{Z}_+}} \Bigg\{ & \sum_{\aleph\geq 0} n(\aleph) u(c^W(\aleph) - h\mathbf{c}^W(\aleph)) \\ & + \beta^W \mathbb{E}_{\mu,X} \hat{V}^W\left((a'(\aleph), n'(\aleph))_{\aleph\geq 0}, X'\right) \Bigg\} \end{aligned}$$

$$\begin{aligned} \text{subject to} \quad & a^{W\prime}(\aleph) + c^W(\aleph) = b^u \mathrm{e}^z + (1+r)a \quad \text{for } \aleph \geq 1, \\ & a^{W\prime}(0) + c^W(0) = (1-\tau)w + (1+r)A, \quad \text{for } \aleph = 0 \\ & a^{W\prime}(\aleph) \geq \underline{a}\mathrm{e}^z \end{aligned}$$

Importantly, the law of motion for μ implies that the relevant elements of μ' in $\hat{V}^{W\prime}(\cdot)$ are given by:

$$\text{For } \aleph \geq 1: \begin{cases} a'(\aleph) = a^{W\prime}(\aleph-1) \\ n'(1) = s'n(0), \text{ and } n'(\aleph) = (1-f')\,n(\aleph-1) \text{ for } \aleph \geq 2 \end{cases}$$

$$\text{For } \aleph = 0: \begin{cases} A' = \frac{1}{n'(0)}[(1-s')a^{W\prime}(0) + f'\sum_{\aleph\geq 1} a^{W\prime}(\aleph)n(\aleph)] \\ n'(0) = (1-s')n(0) + f'(1-n(0)) \end{cases}$$

The Lagrangian function is as follows:

$$\begin{aligned} L\left(A, n(0), (a(\aleph), n(\aleph))_{\aleph\geq 1}, X\right) &= n(0)u(c^W(0) - h\mathbf{c}(0)) + \sum_{\aleph\geq 1} n(\aleph)u(c^W(\aleph) - h\mathbf{c}(\aleph)) \\ &+ \beta^W \mathbb{E}_{\mu,X} V^W\left(A', n'(0), (a'(\aleph), n'(\aleph))_{\aleph\geq 1}, X'\right) \\ &- \Lambda(0)\left[a^{W\prime}(0) + c^W(0) - (1-\tau(X))w(X) - (1+r)A\right] \\ &- \sum_{\aleph\geq 1} \Lambda(\aleph)[a^{W\prime}(\aleph) + c^W(\aleph) - b^u e^z - (1+r)a] + \sum_{\aleph\geq 0} \Gamma(\aleph)(a^{W\prime}(\aleph) - \underline{a}e^z) \end{aligned}$$

A.2.2. ***First-order and envelope conditions***. The first-order conditions are as follows. The FOCs w.r.t the $c^W(\aleph)$s are:

$$n(\aleph)u_c(c^W(\aleph) - h\mathbf{c}(\aleph)) = \Lambda(\aleph), \text{ for } \aleph = 0, 1... \tag{A.2}$$

The FOCs w.r.t. $a^{W\prime}(0)$ is:

$$\begin{aligned} \Lambda(0) - \Gamma(0) &= \beta^W \mathbb{E}_{\mu,X}\left[\frac{\partial \hat{V}^{W\prime}}{\partial A'}\frac{\partial A'}{\partial a^{W\prime}(0)} + \frac{\partial \hat{V}^{W\prime}}{\partial a'(1)}\frac{\partial a'(1)}{\partial a^{W\prime}(0)}\right] \\ &= \beta^W \mathbb{E}_{\mu,X}\left[\frac{\partial \hat{V}^{W\prime}}{\partial A'}\frac{(1-s')n(0)}{n'(0)} + \frac{\partial \hat{V}^{W\prime}}{\partial a'(1)}\right] \end{aligned} \tag{A.3}$$

The FOCs w.r.t. the $a^{W\prime}(\aleph)$s, $\aleph = 1, 2...$ are:

$$\begin{aligned} \Lambda(\aleph) - \Gamma(\aleph) &= \beta^W \mathbb{E}_{\mu,X}\left[\frac{\partial \hat{V}^{W\prime}}{\partial \partial A'}\frac{\partial A'}{\partial a^{W\prime}(\aleph)} + \frac{\partial \hat{V}^{W\prime}}{\partial a'(\aleph+1)}\frac{\partial a'(\aleph+1)}{\partial a^{W\prime}(\aleph)}\right] \\ &= \beta^W \mathbb{E}_{\mu,X}\left[\frac{\partial \hat{V}^{W\prime}}{\partial A'}\frac{f'n(\aleph)}{n'(0)} + \frac{\partial \hat{V}^{W\prime}}{\partial a'(\aleph+1)}\right] \end{aligned} \tag{A.4}$$

We now derive the envelope conditions. The marginal value of wealth A is given by:

$$\frac{\partial \hat{V}^W}{\partial A} = (1+r)\Lambda(0) = (1+r)n(0)u_c(c^W(0) - h\mathbf{c}(0)) \tag{A.5}$$

Similarly, the marginal value of a unit of wealth $a(\aleph)$, $\forall \aleph \geq 1$ is given by:

$$\frac{\partial \hat{V}^W(\cdot)}{\partial a(\aleph)} = (1+r)\Lambda(\aleph) = (1+r)n(\aleph)u_c(c^W(\aleph) - h\mathbf{c}(\aleph)). \tag{A.6}$$

A.2.3. ***Euler conditions.*** We are now in a position to derive the Euler conditions for employed ($\aleph = 0$) and unemployed ($\aleph \geq 1$) workers, depending on whether the debt limit is binding ($\Gamma(\aleph) > 0$) or not binding ($\Gamma(\aleph) = 0$). Let us start with employed workers. First, using (A.2) and (A.3) we get:

$$u_c(c^W(0) - h\mathbf{c}(0)) = \frac{\Lambda(0)}{n(0)} = \beta^W \mathbb{E}_{\mu,X}\left[\frac{\partial \hat{V}^{W\prime}}{\partial A'} \times \frac{1-s'}{n'(0)} + \frac{1}{n(0)} \times \frac{\partial \hat{V}^{W\prime}}{\partial a'(1)}\right] + \frac{\Gamma(0)}{n(0)}$$

Now, using the envelope conditions for $\partial \hat{V}^W / \partial A$ and $\partial \hat{V}^W / \partial a(1)$ (equations (A.5) and (A.6), respectively), with a one-period lead, we obtain:

$$\begin{aligned}
&u_c(c^W(0) - h\mathbf{c}(0)) - \frac{\Gamma(0)}{n(0)} \\
&= \beta^W \mathbb{E}_{\mu,X}\left[(1+r')\left(\frac{(1-s')n'(0)u_c(c^{W\prime}(0) - h\mathbf{c}'(0))}{n'(0)} + \frac{n'(1)u_c(c^{W\prime}(1) - h\mathbf{c}(1))}{n(0)}\right)\right] \\
&= \beta^W \mathbb{E}_{\mu,X}\left[(1+r')\left((1-s')u_c(c^{W\prime}(0) - h\mathbf{c}'(0)) + s'u_c(c^{W\prime}(1) - h\mathbf{c}(1))\right)\right]
\end{aligned}$$

We now turn to unemployed workers and follow the same steps as for employed workers. Using (A.2) and (A.4) we get:

$$u_c(c^W(\aleph) - h\mathbf{c}(\aleph)) = \frac{\Lambda(\aleph)}{n(\aleph)} = \beta^W \mathbb{E}_{\mu,X}\left[\frac{\partial \hat{V}^{W\prime}}{\partial A'} \frac{f'}{n'(0)} + \frac{1}{n(\aleph)} \frac{\partial \hat{V}^{W\prime}}{\partial a'(\aleph+1)}\right] + \frac{\Gamma(\aleph)}{n(\aleph)}$$

Again, using the envelope conditions for $\partial \hat{V}^W / \partial A$ and $\partial \hat{V}^W / \partial a(\aleph+1)$ (equations (A.5) and (A.6), respectively) with a one-period lead, we obtain, $\forall \aleph \geq 1$:

$$\begin{aligned}
&u_c(c^W(\aleph) - h\mathbf{c}(\aleph)) - \frac{\Gamma(\aleph)}{n(\aleph)} \\
&= \beta^W \mathbb{E}_{\mu,X}\left[(1+r')\left(\frac{f'n'(0)u_c(c^{W\prime}(0) - h\mathbf{c}^{W\prime}(0))}{n'(0)} + \frac{n'(\aleph+1)u_c(c^{W\prime}(\aleph+1) - h\mathbf{c}^{W\prime}(\aleph+1))}{n(\aleph)}\right)\right] \\
&= \beta^W \mathbb{E}_{\mu,X}\left[(1+r')\left(f'u_c(c^{W\prime}(0) - h\mathbf{c}^{W\prime}(0)) + (1-f')\,u_c(c^{W\prime}(\aleph+1) - h\mathbf{c}^{W\prime}(\aleph+1))\right)\right]
\end{aligned}$$

Now, defining the relevant intertemporal marginal rates of substitution (IMRS) as follows:

$$M^{W\prime}(0) = \beta^W \frac{(1-s')u_c(c^{W\prime}(0) - h\mathbf{c}^{W\prime}(0)) + s'u_c(c^{W\prime}(1) - h\mathbf{c}^{W\prime}(1))}{u_c(c^W(0) - h\mathbf{c}^W(0))}, \tag{A.7}$$

$$M^{W\prime}(\aleph) = \beta^W \frac{(1-f')u_c(c^{W\prime}(\aleph+1) - h\mathbf{c}^{W\prime}(\aleph+1)) + f'u_c(c^{W\prime}(0) - h\mathbf{c}^{W\prime}(0))}{u_c(c^W(\aleph) - h\mathbf{c}^W(\aleph))}, \ \forall \aleph \geq 1, \tag{A.8}$$

we can rewrite the Euler conditions as, $\forall \aleph \in \mathbb{Z}_+$:

$$\mathbb{E}_{\mu,X}\left[M^{W\prime}(\aleph)(1+r')\right] = 1 - \frac{\Gamma(\aleph)}{u_c(c^W(\aleph) - h\mathbf{c}(\aleph))n(\aleph)} \leq 1. \tag{A.9}$$

When the debt limit is not binding for $\aleph$-type workers, then $\Gamma(\aleph) = 0$, $a^{W\prime}(\aleph) > \underline{a}e^z$ and $\mathbb{E}_{\mu,X}\left[M^{W\prime}(\aleph)(1+r')\right] = 1$. When the debt limit is binding for $\aleph$-type workers, then $\Gamma(\aleph) > 0$, $a^{W\prime}(\aleph) = \underline{a}e^z$ and $\mathbb{E}_{\mu,X}\left[M^{W\prime}(\aleph)(1+r')\right] < 1$ (see Proposition 2 in the main paper). The two-wealth state case analysed from Section 2.2 of the main paper onwards corresponds to the case where, at every point in time, $\mathbb{E}_{\mu,X}\left[M^{W\prime}(0)\,(1+r')\right] = 1$ and $\mathbb{E}_{\mu,X}\left[M^{W\prime}(\aleph)(1+r')\right] < 1 \ \forall \aleph \geq 1$.

A.3. **Firm owners.** There is a mass $1-\Omega$ of firm owners. In the rest of the paper, the superscript F is assigned to the variables corresponding to the firm owner. They solve the problem

$$V^F(n^F,k,a^F,i,X)=\max_{a^{F\prime},i',v,c^F,k'}\{u(c^F-h\mathbf{c}^F)+\beta^F\mathbb{E}[V^F(n^{F\prime},k',a^{F\prime},i',X')]\},$$

$$\begin{aligned}\text{subject to}\quad & k'=(1-\delta)k+\mathrm{e}^{\varphi_i}(1-S(i'/i))i',\\ & c^F+i'+a^{F\prime}=\psi w(X)n^F+[r_k(X)v-\eta(v)]k+(1+r(X))a^F+\Upsilon(X),\\ & n^{F\prime}=(1-s(X'))n^F+f(X')(1-n^F),\\ & X'=\Gamma(X,\epsilon').\end{aligned}$$

We introduce the Lagrange coefficients $\zeta^F(n^F,k,a^F,i,X)$ and $\Lambda^F(n^F,k,a^F,i,X)$. To simplify the notations we simply write ζ^F for $\zeta^F(n^F,k,a^F,i,X)$ and Λ^F for $\Lambda^F(n^F,k,a^F,i,X)$. The Lagrangian is

$$\begin{aligned}L^F(n^F,k,a^F,i,X)=u(c^F-h\mathbf{c}^F)&\\ -\zeta^F(k'-(1-\delta)k-\mathrm{e}^{\varphi_i}(1-S(i'/i))i')&\\ -\Lambda^F(c^F+i'+a^{F\prime}-\psi w(X)n^F-[r_k(X)v-\eta(v)]k-(1+r(X))a^F-\Upsilon(X))&\\ +\beta^F\mathbb{E}[V^F(n^{F\prime},k',a^{F\prime},i',X')].&\end{aligned}$$

The first order conditions, with respect to $a^{F\prime}$, i', v, c^F, and k' respectively, are

$$\Lambda^F=\beta^F\mathbb{E}[V^F_{a^F}(n^{F\prime},k',a^{F\prime},i',X')],$$

$$\Lambda^F-\zeta^F\mathrm{e}^{\varphi_i}\left[1-S\left(\frac{i'}{i}\right)-S'\left(\frac{i'}{i}\right)\left(\frac{i'}{i}\right)\right]=\beta^F\mathbb{E}[V^F_{i'}(n^{F\prime},k',a^{F\prime},i',X')],$$

$$\begin{aligned}r_k(X)&=\eta'(v),\\ u'(c^F-h\mathbf{c}^F)&=\Lambda^F,\end{aligned}$$

$$\zeta^F=\beta^F\mathbb{E}[V^F_k(n^{F\prime},k',a^{F\prime},i',X')].$$

The envelope conditions are

$$V^F_k(n^F,k,a^F,i,X)=\zeta^F(1-\delta)+\Lambda^F[r_k(X)v-\eta(v)],$$

$$V^F_{a^F}(n^F,k,a^F,i,X)=\Lambda^F(1+r(X)),$$

$$V^F_i(n^F,k,a^F,i,X)=\zeta^F\mathrm{e}^{\varphi_i}S'\left(\frac{i'}{i}\right)\left(\frac{i'}{i}\right)^2.$$

As a consequence, we find

$$\Lambda^F = \beta^F \mathbb{E}[\Lambda^{F\prime}(1+r(X'))],$$

$$\Lambda^F - \zeta^F e^{\varphi_i}\left[1 - S\left(\frac{i'}{i}\right) - S'\left(\frac{i'}{i}\right)\left(\frac{i'}{i}\right)\right] = \beta^F \mathbb{E}\left[\zeta^{F\prime} e^{\varphi_i'} S'\left(\frac{i''}{i'}\right)\left(\frac{i''}{i'}\right)^2\right],$$

$$r_k(X) = \frac{\partial \eta(v)}{\partial v},$$

$$u'(c^F - h\mathbf{c}^F) = \Lambda^F,$$

$$\zeta^F = \beta^F \mathbb{E}\left\{\zeta^{F\prime}(1-\delta) + \Lambda^{F\prime}\left[r_k'(X')v' - \eta(v')\right]\right\},$$

where i'' is next period investment. Define

$$p_k(n^F, k, a^F, i, X) = \frac{\zeta^F}{\Lambda^F} \text{ and}$$

$$M^{F\prime}(n^F, k, a^F, i, n^{F\prime}, k', a^{F\prime}, i', X, X') = \beta^F \frac{\Lambda^{F\prime}}{\Lambda^F} = \beta^F \frac{e^{\varphi_c'} u'(c^{F\prime} - h\mathbf{c}^{F\prime})}{u'(c^F - h\mathbf{c}^F)}.$$

We find the set of equations

$$1 = \mathbb{E}[M^{F\prime}(1+r(X'))],$$

$$1 = p_k e^{\varphi_i}\left[1 - S\left(\frac{i'}{i}\right) - S'\left(\frac{i'}{i}\right)\left(\frac{i'}{i}\right)\right] + \mathbb{E}\left[M^{F\prime} p_k' e^{\varphi_i'} S'\left(\frac{i''}{i'}\right)\left(\frac{i''}{i'}\right)^2\right],$$

$$r_k(X) = \eta'(v), \text{ and}$$

$$p_k = \mathbb{E}\left\{M^{F\prime}[r_k(X')v' - \eta(v') + (1-\delta)p_k']\right\}$$

where, for the sake of simplicity, we only write $M^{F\prime}$ for $M^{F\prime}(n^F, k, a^F, i, n^{F\prime}, k', a^{F\prime}, i', X, X')$. The other conditions are

$$n^{F\prime} = n^F(1 - s(X')) + f(X')(1 - n^F),$$

$$c^F + i' + a^{F\prime} = \psi w(X) n^F - \left[r_k(X)v - \eta(v)\right]k - (1 + r(X))a^F - \Upsilon(X),$$

$$k' = (1-\delta)k + e^{\varphi_i}\left(1 - S\left(\frac{i'}{i}\right)\right)i'.$$

and

$$M^{F\prime} = \beta^F \frac{u'(c^{F\prime} - hc^F)}{u'(c^F - h\mathbf{c}^F)}, \tag{A.10}$$

A.4. **Firms.** There are four types of firms.

A.4.1. *Final goods firms.* The final good is produced by a continuum of identical and competitive producers that combine intermediate goods, uniformly distributed on the unit interval $i \in [0,1]$ according to the production function

$$y = \left(\int_0^1 y_i^{(\theta-1)/\theta} \mathrm{d}i \right)^{\theta/(\theta-1)}, \tag{A.11}$$

where $\theta > 1$ is the cross partial elasticity of substitution between any two wholesale goods. Let p_i denote the real price of wholesale good i in terms of the final good. This price is taken as given by final goods firms. The program of the representative final good producer is thus

$$\max_y \left\{ y - \int_0^1 p_i(X) y_i \mathrm{d}i \right\}.$$

subject to (A.11). From the optimal choices of final good firms, one can deduce the demand function for the intermediate good firms, $i \in [0,1]$

$$y_i(X, p_i) = p_i^{-\theta} y(X),$$

where $y(X)$ is the total demand for final goods, and where

$$1 = \left(\int_0^1 p_i^{1-\theta}(X) \mathrm{d}i \right)^{1/(1-\theta)}$$

is the nominal price of final goods obtained from imposing zero profits on the final good producers.

A.4.2. *Intermediate goods firms.* Intermediate firm $i \in [0,1]$ produces with the technology

$$y_i = x_i - \kappa_y \mathrm{e}^z, \tag{A.12}$$

Firm i' current real profit is given by

$$\Xi(p_i, X) = (p_i - p_m(X)) y_i(X, p_i) - p_m(X) \kappa_y \mathrm{e}^z, \tag{A.13}$$

where $p_m(X)$ is the real price of wholesale goods in term of final goods (which is taken as given by wholesale goods firms). The firms that do not reoptimise their price set it according to the rule

$$p_i(X, p_{i,-1}) \equiv \frac{(1+\bar{\pi})^{1-\gamma}(1+\boldsymbol{\pi}_{-1})^{\gamma}}{1+\pi(X)} p_{i,-1}, \tag{A.14}$$

where $\bar{\pi}$ is the steady state inflation.

It follows from this price adjustment mechanism that the behaviour of a firm can be described by two Bellman equations, corresponding to the two idiosyncratic states in which the firm can be. The value of a firm that is allowed to reset its price is given by $V^R(X)$ and only depends on the aggregate state. The value of a firm not allowed to reset its selling price and with last period's price $p_{\varsigma,-1}$ is denoted as $V^N(p_{\varsigma,-1}, X)$. The corresponding Bellman equations are:

$$V^R(X) = \max_{p_\varsigma} \{ \Xi + \alpha \mathbb{E}_X[M^{F\prime} V^N(p_\varsigma, X')] + (1-\alpha)\mathbb{E}_X[M^{F\prime} V^R(X')] \} \text{ and}$$

$$V^N(p_{\varsigma-1}, X) = \Xi + \alpha \mathbb{E}_X[M^{F\prime} V^N(p_\varsigma, X')] + (1-\alpha)\mathbb{E}_X[M^{F\prime} V^R(X')],$$

where $p_i(X, p_{i,-1})$ is given by (A.14), and subject to $X' = \Gamma(X, \epsilon')$.

The policy function $p^*(X)$ must satisfy the first-order condition for price re-optimizer as well as the envelope condition associated with the new price of a non-reoptimise. The first order condition is associated with the first Bellman equation is

$$\left.\frac{\partial V^R(X)}{\partial p_i}\right|_{p_i=p^*} = \Xi_p(p^*, X) + \alpha\mathbb{E}[M^F(X, X')V_p^N(p^*, X')] = 0$$

where

$$\begin{aligned}\Xi_p(p^*, X) &= y_i(X, p^*) - (p - p_m(X)) \left.\frac{\partial y_i(X, p_i)}{\partial p_i}\right|_{p_i=p^*} \\ &= [(1-\theta)(p^*)^{-\theta} + \theta p_m(X)(p^*)^{-1-\theta}]y(X).\end{aligned}$$

The envelope condition is

$$\begin{aligned}V_p^N(p_{i,-1}, X) &= \{\Xi_p(p_i, X) + \alpha\mathbb{E}[M^F(X, X')V_p^N(p_i, X')]\}\frac{\partial p_i}{\partial p_{i,-1}} \\ &= \frac{(1+\bar{\pi})^{1-\gamma}(1+\boldsymbol{\pi}_{-1})^{\gamma}}{1+\pi(X)}\{\Xi_p(p_i, X) + \alpha\mathbb{E}[M^F(X, X')V_p^N(p_i, X')]\}.\end{aligned}$$

Combining those two conditions we get the usual expressions for the determination of the optimal reset price

$$p^* = \frac{K}{F}$$

where K_p and F_p are defined recursively as follows

$$K = \mu p_m y + \alpha\mathbb{E}\left[M^{F\prime}\left(\frac{1+\pi(X')}{(1+\bar{\pi})^{1-\gamma}(1+\pi(X))^{\gamma}}\right)^{\theta} K'\right]$$

and

$$F = y + \alpha\mathbb{E}\left[M^{F\prime}\left(\frac{1+\pi(X')}{(1+\bar{\pi})^{1-\gamma}(1+\pi(X))^{\gamma}}\right)^{\theta-1} F'\right]$$

where $\mu \equiv \theta/(1-\theta)$. Later in the empirical specification of the model, we append an exogenous markup shock to the system e^{φ_p}, which appears in front of μ.

The relationship between optimal prince and inflation, $\pi(X)$, and the price dispersion, $\Lambda \equiv \int_0^1 p_i^{-\theta}\mathrm{d}i$, are given by:

$$1 = (1-\alpha)p^*(X)^{1-\theta} + \alpha\left(\frac{(1+\bar{\pi})^{1-\gamma}(1+\boldsymbol{\pi}_{-1})^{\gamma}}{1+\pi(X)}\right)^{1-\theta} \tag{A.15}$$

$$\Lambda = (1-\alpha)p^*(X)^{-\theta} + \alpha\left(\frac{(1+\bar{\pi})^{1-\gamma}(1+\boldsymbol{\pi}_{-1})^{\gamma}}{1+\pi(X)}\right)^{-\theta}\Lambda_{-1}. \tag{A.16}$$

A.4.3. *Wholesale goods firms.* A representative wholesale goods firm produce the intermediate goods using capital and labor services, with the following (static) program

$$\max_{\breve{n},\breve{k}}\{p_m(X)y_m - Q(X)\breve{n} - r_k(X)\breve{k}\} \tag{A.17}$$

subject to

$$y_m = (\breve{k})^{\phi}(\mathrm{e}^z\breve{n})^{1-\phi} \tag{A.18}$$

where $\phi \in (0,1)$ and $Q(X)$ is the price of one unit of labor services and $r_k(X)$ that of one unit of capital services both in units of the final good. $\breve{n}$ and $\breve{k}$ denote the labor and capital services, respectively (that is, $\breve{n}$ is the number of effective labor units).

The policy rules for wholesale goods firms jointly satisfy the following first-order conditions

$$r_k(X) = p_m\phi\mathrm{e}^{z(1-\phi)}\left(\frac{\breve{k}}{\breve{n}}\right)^{\phi-1} \text{ and} \tag{A.19}$$

$$Q(X) = p_m(1-\phi)\mathrm{e}^{z(1-\phi)}\left(\frac{\breve{k}}{\breve{n}}\right)^{-\phi}. \tag{A.20}$$

A.4.4. *Labor intermediaries.* Labor services are sold to intermediate goods firms by labor intermediaries, which hire raw labor in a market with search frictions. Those labor intermediaries are held by the firm owners. For every hour of work, a worker provides one unit of effective labor but an employer provides $\psi > 1$ unit of effective labor. The wage paid to a employed worker is w and that paid to an employers is ψw^F. It follows that the value to the labor intermediary of a match with a worker and an employer are given by, respectively:

$$J^W = Q - w + \mathbb{E}_X[(1-\rho')M^{F\prime}J^{W\prime}] \text{ and} J^F = \psi(Q-w) + \mathbb{E}_X[(1-\rho')M^{F\prime}J^{F\prime}]. \tag{A.21}$$

In particular, we note that $J^F(X) = \psi J^W(X)$. Assuming that the agency cannot target a particular skill type, the free entry condition implies

$$\lambda[\Omega J^W + (1-\Omega)J^F] = \kappa_v\mathrm{e}^z, \tag{A.22}$$

where $\lambda(X)$ is the vacancy-filling rate.

A.5. **Labor market flows.** Let $\tilde{\mathbf{n}}$ denote the number of employed agents (workers and firm owners) at the beginning of the period, before labor market transitions. The law of motion of $\tilde{\mathbf{n}}$ is given by the job-separation and the vacancy filling-rate, according to

$$\tilde{\mathbf{n}}' = (1-\rho(\varphi_\rho'))\tilde{\mathbf{n}} + \lambda(X)v. \tag{A.23}$$

The matching function yields thus

$$m(X) = \bar{m}(1-(1-\rho(\varphi_\rho))\tilde{\mathbf{n}})^{\chi}v(X)^{1-\chi} \tag{A.24}$$

and the relevant job-finding and vacancy-filling rates are

$$f(X) = \frac{m(X)}{1-(1-\rho(\varphi_\rho))\tilde{\mathbf{n}}} \text{ and } \lambda(X) = \frac{m(X)}{v(X)}. \tag{A.25}$$

Finally, note that under our timing workers that are separated can find a job within the period. Hence the period-to-period separation rate $s(X)$ is given by

$$s(X) = \rho(\varphi_\rho)(1-f(X)). \tag{A.26}$$

A.6. **Dynamics of the job-finding rate.** We rewrite the recursion (A.21) as follows

$$\frac{\kappa_v e^z}{\Omega+(1-\Omega)\psi}\frac{1}{\lambda(X)} = Q(X) - w(X) + \mathbb{E}\left[(1-\rho(\varphi_\rho'))M^F(X,X')\frac{\kappa_v e^{z'}}{\Omega+(1-\Omega)\psi}\frac{1}{\lambda(X')}\right]$$

A.7. **Wage.** A simple wage equation is assumed

$$w = \left(\frac{w_{-1}}{1+\pi}\right)^{\gamma_w}\left(\bar{w}e^{z+\varphi_w}\left(\frac{n}{\bar{n}}\right)^{\psi_n}\right)^{1-\gamma_w}, \tag{A.27}$$

where $\bar{n}$ is the steady-state value of n, w_{-1} is last period's wage, $\gamma_w \in [0,1]$, $\psi_n \geq 0$, and φ_w is a wage shock.

A.8. **Central Bank.** The Central Bank is assumed to set the nominal interest rate R according to the following rule:

$$\log\left(\frac{1+R}{1+\bar{R}}\right) = \rho_R\log\left(\frac{1+\mathbf{R}_{-1}}{1+\bar{R}}\right) + (1-\rho_R)\left[a_\pi\log\left(\frac{1+\pi}{1+\bar{\pi}}\right) + a_y\log\left(\frac{1+g}{1+\bar{g}}\right)\right] + \varphi_R \tag{A.28}$$

where $\bar{R}$ is the steady-state nominal interest rate, $\rho_R \in (0,1)$ an interest rate smoothing parameter, (a_π, a_y) the reaction coefficients to inflation and output growth, $g = y/\mathbf{y}_{-1} - 1$ the growth rate of final output, where $\mathbf{y}_{-1}$ is last-period final output, and φ_R a monetary policy shock.

A.9. **Market clearing and aggregation.**

Labor services. Recall from Section **??** that all households face the same labor market transition rates (f,s). Hence, in the steady state the employment rates in every family of workers and firm owners are the same. Assuming that employment is symmetric at the beginning of the date-0 labor market transition stage, by the law of large numbers they remain symmetric at every point in time, i.e.:

$$\tilde{n}^W = \tilde{n}^F = \tilde{\mathbf{n}}^W = \tilde{\mathbf{n}}^F \equiv \tilde{\mathbf{n}},\ n^W = n^F = \mathbf{n}^W = \mathbf{n}^F \equiv \mathbf{n}. \tag{A.29}$$

Because a matched firm owner provides ψ times more units of labor services than a worker, the total supply of labor services is $\Omega\mathbf{n}^W + (1-\Omega)\psi\mathbf{n}^F = (\Omega+(1-\Omega)\psi)\mathbf{n}$. Denoting by $\breve{n}$ firms' demand for labor services, market clearing requires:

$$(\Omega+(1-\Omega)\psi)\mathbf{n} = \breve{n}. \tag{A.30}$$

Asset markets. As employers are symmetric, in measure $1-\Omega$ and each of them supplies $v(X)k(X)$ units of capital services, the total supply of capital services is $(1-\Omega)v(X)k(X)$. The demand for capital services by intermediate goods firms is $\breve{k}(X)$. Clearing of the market for capital thus requires

$$(1-\Omega)v(X)k(X) = \breve{k}(X). \tag{A.31}$$

All the households participate in the market for nominal bonds, subject to the borrowing constraint. Clearing of this market thus requires

$$(1-\Omega)a^{F\prime}(X) + \Omega \sum_{\aleph} \int_a a^{W\prime}(\aleph, X)\, \mathrm{d}\boldsymbol{\mu}(a,\aleph) = 0. \tag{A.32}$$

The first term on the left hand side is the saving of the firm owners. The second term is the saving of the family. The third term is the saving of unemployed households, taking into consideration their number of consecutive periods of unemployment.

Goods markets. The aggregate demand for final goods is made of total investment (by employers), the consumption of all types of households (employers as well as employed and unemployed workers), as well as capital utilisation and vacancy costs. Clearing of this market requires that demand be equal to supply, i.e.,

$$(1-\Omega)(c^F(X) + i'(X) + \eta(v)k(X)) + \Omega \sum_{\aleph} \int_a c^W(\aleph, X)\mathrm{d}\boldsymbol{\mu}(a,\aleph) + \kappa_v \mathrm{e}^z v = y \tag{A.33}$$

The supply of wholesale goods is given by (A.18), while the intermediate sector demands one unit of wholesale goods for any unit of intermediate goods. Hence the market-clearing condition for this market is

$$\int_0^1 x_i \mathrm{d}i = y_m = (\breve{k}(X))^{\phi}(\mathrm{e}^z \breve{n}(X))^{1-\phi}. \tag{A.34}$$

The total demand for intermediate goods by the final good sector is $\int_0^1 y_i(X, p_i(X))\mathrm{d}i = y(X)\Lambda(X)$. The total supply of intermediate goods is equal to $\int_0^1 y_i \mathrm{d}i = \int_0^1 (x_i)\mathrm{d}i - \kappa_y \mathrm{e}^z$. Hence, clearing of the market for intermediate goods requires

$$\Lambda(X)y(X) = (\breve{k}(X))^{\phi}(\mathrm{e}^z \breve{n}(X))^{1-\phi} - \kappa_y \mathrm{e}^z. \tag{A.35}$$

A.10. **Equilibrium Definition.** In general the aggregate state is then given by:

$$X = \{\tilde{\boldsymbol{\mu}}(\cdot), k, a^F, i, \mathbf{c}^F, \mathbf{c}^W(\aleph)_{\aleph \in \mathbb{Z}_+}, a^e, \mathbf{R}_{-1}, \boldsymbol{\Lambda}_{-1}, \boldsymbol{\pi}_{-1}, \mathbf{y}_{-1}, \mathbf{w}_{-1}, \Phi\}, \tag{A.36}$$

where $\Phi \equiv \{z, \varphi_i, \varphi_c, \varphi_s, \varphi_R, \varphi_w, \varphi_p\}$ is the exogenous state.

Definition 1. *A symmetric recursive equilibrium is a set of value and policy functions, a set of prices, and labor market flows such that:*

(1) ***Workers**. Given $r(X)$, $w(X)$, $\tau(X)$, $b^u e^z$, $\mathbf{c}^W(\aleph)_{\aleph \in \mathbb{N}}$, $f(X)$ and $s(X)$, the value and policy functions $V^W(\mu, X)$, $g_{a^W}(a, \aleph, X)$ and $g_{c^W}(a, \aleph, X)$ solve the workers' problem;*

(2) ***Firm owners***. *Given* $r(X)$, $r_k(X)$, $w^F(X)$, $\mathbf{c}^F$, $\Upsilon(X)$, $f(X)$ *and* $s(X)$, *the value and policy functions* $V^F(n^F, k, a^F, i, X)$, $g_{a^F}(X)$, $g_{c^F}(X)$, $g_i(X)$, $g_v(X)$, *and* $g_k(X)$ *solve the firm owners' problem;*

(3) ***Final goods firms***. *Given* p_ς, $\varsigma \in [0,1]$, *the demands for wholesale goods* $y_\varsigma(p_\varsigma, X)$ *is optimal from the point of view of final goods firms;*

(4) ***Intermediate goods firms***. *Given* $p_m(X)$, $y_\varsigma(p_\varsigma, X)$, *and* $M^F(X, X')$, *the value functions* $V^R(X)$ *and* $V^N(p_{\varsigma-1}, X)$ *and the reset price* $p^*(X)$ *solve the problem of intermediate goods firms;*

(5) ***Wholesale goods firms***. *Given* $p_m(X)$, $Q(X)$ *and* $r_k(X)$, *the demand for labor and capital services* $\breve{n}(X)$ *and* $\breve{k}(X)$ *solve the problem of wholesale good firms;*

(6) ***Labour intermediaries***. *Given* $Q(X)$, $w(X)$, *and* $M^F(X, X')$, *the job values* $J^W(X)$ *and* $J^F(X)$ *are given by (A.21), the free entry condition (A.22) determines the vacancy-filling rate* $\lambda(X)$, *and* $m(X)$, $f(X)$, $v(X)$ *and* $s(X)$ *are determined according to (A.24), (A.25), and (A.26);*

(7) ***Profits***. *The profit function* $\Upsilon(X)$ *results from the optimal decision of the intermediate goods firms and the labor intermediaries.*

(8) ***Social contribution rate, real interest rate, stochastic discount factor, wages, and nominal interest rate***. *Given* $y(X)$, $\pi(X)$, *and* $b^u e^z$, *the social contribution rate* $\tau(X)$ *is so that (A.1) holds; the real return on nominal bond holdings* $r(X)$ *follows (**??**); the stochastic discount factor* $M^F(X, X')$ *is given by (A.10), firm owners' wage* $w^F(X)$ *is equal to* $\psi w(X)$, *where* $w(X)$ *is given by (A.27); the nominal interest rate* $R(X)$ *is given by (A.28); .*

(9) ***Market clearing***. *The market-clearing conditions (A.30) to (A.34) hold.*

(10) ***Laws of motion***. *Given* $p^*(X)$, *inflation* $\pi(X)$ *and price dispersion* $\Lambda(X)$ *evolve according (**??**) and (**??**), respectively. Given* $f(X)$, $s(X)$, *and* $g_{a^W}(\cdot)$ *the laws of motion from* $\tilde{\boldsymbol{\mu}}$ *to* $\boldsymbol{\mu}$, *and then from* $\boldsymbol{\mu}$ *to* $\tilde{\boldsymbol{\mu}}'$, *are given by:*

$$\tilde{\boldsymbol{\mu}} \text{ to } \boldsymbol{\mu} : \begin{cases} \boldsymbol{\mu}(a,0) = f(X)\sum_{\aleph \geq 1} \tilde{\boldsymbol{\mu}}(a,\aleph) + (1-s(X))\tilde{\boldsymbol{\mu}}(a,0) \\ \boldsymbol{\mu}(a,1) = s(X)\tilde{\boldsymbol{\mu}}(a,0) \\ \boldsymbol{\mu}(a,\aleph) = (1-f(X))\tilde{\boldsymbol{\mu}}(a,\aleph-1) \text{for } \aleph \geq 2, \end{cases}$$

$$\boldsymbol{\mu} \text{ to } \tilde{\boldsymbol{\mu}}' : \tilde{\boldsymbol{\mu}}'(\hat{a},\aleph) = \int_a \mathbf{1}_{g_{a^W}(a,\aleph,X) \leq \hat{a}} d\boldsymbol{\mu}(a,\aleph) \text{for } \aleph \geq 0.$$

(11) ***Habits***. *Given* $g_{c^F}(X)$ *and* $g_{c^W}(\cdot)$, *tomorrow's habit level of a particular household type is equal to the average consumption of this type today, i.e.,*

$$\mathbf{c}^{F\prime} = g_{c^F}(X) \text{ and } \mathbf{c}^{W\prime}(\aleph) = \int_a g_{c^W}(a,\aleph,X) d\mu(a,\aleph).$$

Definition 2. *A balanced growth path is a symmetric recursive equilibrium where:*

(1) *Innovations to the exogenous aggregate state* (ϵ) *are zero at every point in time;*

(2) *The variables* $w(X)$, $\mathbf{c}^W(\aleph)_{\aleph \in \mathbb{N}}$, $w^F(X)$, $\mathbf{c}^F$, $Q(X)$, $\Upsilon(X)$ $\breve{k}(X)$, *all grow at rate* μ_z;

(3) *The variables* $r(X)$, $r_k(X)$, $f(X)$, $s(X)$, $\lambda(X)$, $m(X)$, $\nu(X)$, $\breve{n}(X)$, $R(X)$, $p_m(X)$ *and* $\pi(X)$ *are constant.*

B. Reduction of the equilibrium

The reduction of the equilibrium to an equilibrium with a finite, countable number of relevant wealth states relies on Propositions 1 to 3 in Section 2.1 of the main text. The complete proof of Proposition 1 is provided there. A sketch of the proof of proposition 2 is also provided, and the complete proof is given in Section 2.A above. We start the present Section by restating Propostion 3 and providing its full proof.

B.1. **Proof of Proposition 3.** We first restate our key assumption about the tightness of the debt limit:

Assumption B. $\underline{a} > \underline{a}^{nat} \equiv \frac{\beta^F b^u}{\beta^F - e^{(\sigma-1)\mu_z}}$.

Proposition 3. Under Assumption **??**, in a BGP workers face a binding debt limit after a finite number of unemployment periods. Formally:

$$\exists \hat{\aleph} \in \mathbb{Z}_+, \hat{\aleph} < \infty \; : \begin{cases} \forall \aleph < \hat{\aleph}, M^{W\prime}(\aleph)(1+r') = 1, \\ \forall \aleph \geq \hat{\aleph}, M^{W\prime}(\aleph)(1+r') < 1. \end{cases}$$

Proof. The proof is by contradiction. In the economy without aggregate shock TFP grows deterministically at a rate μ_z, the real interest rate is given by $1+\bar{r}$ $= e^{\sigma\mu_z}/\beta^F$, and we denote by $\hat{c}^W(\aleph) = c^W(\aleph)e^{-z}$ and $\hat{a}^{W\prime}(\aleph) = a^{W\prime}(\aleph)e^{-z}$ the detrended consumption and assets of a type-$\aleph$ worker. If unemployed workers never faced a binding debt limit, their Euler equation (as written in the proof of proposition 2 above) would always hold with equality. Noting that in the absence of aggregate shocks we have $u_c(c^W((\aleph) - h\mathbf{c}^W(\aleph)) = (\hat{c}^W(\aleph)(e^{\mu_z} - h))^{-\sigma} e^{-\sigma z + \sigma\mu_z}$, we would thus have the following conditions :

$$\begin{aligned}
&(\hat{c}^W(\aleph)(e^{\mu_z} - h))^{-\sigma} e^{-\sigma z + \sigma\mu_z} \\
&\quad = \beta^W(1+\bar{r})[f(\hat{c}^W(0)(e^{\mu_z} - h))^{-\sigma} e^{-\sigma z + \sigma\mu_z} + (1-f)(\hat{c}^W(\aleph)(e^{\mu_z} - h))^{-\sigma} e^{-\sigma z + \sigma\mu_z}], \; \forall \aleph \geq 1
\end{aligned}$$

or, using the fact that $1+\bar{r} = e^{\sigma\mu_z}/\beta^F$ and simplifying :

$$\hat{c}^W(\aleph)^{-\sigma} = (\beta^W/\beta^F)(f\hat{c}^W(0)^{-\sigma} + (1-f)\hat{c}^W(\aleph+1)^{-\sigma}), \; \forall \aleph \geq 1$$

On the other hand, since employed workers' choice is interior we have:

$$\hat{c}^W(0)^{-\sigma} = (\beta^W/\beta^F)((1-s)\hat{c}^W(0)^{-\sigma} + s\hat{c}^W(1)^{-\sigma}), \text{ for } \aleph = 0$$

We now define $x_k \equiv \hat{c}^W(k)^{-\sigma}$ and $\tilde{\beta} \equiv \beta^W/\beta^F$. The last two conditions give rise to the following recursion:

$$x_0 = \tilde{\beta}\left((1-s)x_0 + sx_1\right)$$
$$x_k = \tilde{\beta}\left(fx_0 + (1-f)\,x_{k+1}\right)$$

that is (for $f, s, \tilde{\beta}1)$):

$$x_1 = \frac{1-\tilde{\beta}(1-s)}{\tilde{\beta}s}x_0 \text{ and}$$
$$x_{k+1} = \frac{1}{\tilde{\beta}\,(1-f)}x_k - \frac{\tilde{\beta}f}{\tilde{\beta}\,(1-f)}x_0 \text{ for } k \geq 1$$

We can rewrite the last condition as follows:

$$x_{k+1} = x^* + \frac{1}{\tilde{\beta}\,(1-f)}\,(x_k - x^*)$$

where

$$x^* = \frac{\tilde{\beta}fx_0}{1-\tilde{\beta}\,(1-f)}$$

The (x_k)-sequence diverges if $x_1 > x^*$. This is equivalent to

$$\frac{1-\tilde{\beta}(1-s)}{\tilde{\beta}s}x_0 > \frac{\tilde{\beta}f}{1-\tilde{\beta}\,(1-f)}x_0$$

or, after rearranging, $(1-\tilde{\beta})(1-\tilde{\beta}(1-s-f)) > 0$. Since this is always true, we have $\lim_{k\to+\infty} x_k = +\infty$ and hence $\hat{c}^W(\infty) = \lim_{k\to+\infty}\hat{c}^W(k) = 0$ (i.e., the consumption of a worker remaining permanently unemployed asymptotically goes to zero).

On the other hand, the budget constraint of a type-$\aleph$ worker, expressed in detrended form, is given by:

$$\hat{a}^{W\prime}(\aleph) + \hat{c}^W(\aleph) = b^u + (\mathrm{e}^{(\sigma-1)\mu_z}/\beta^F)\hat{a}^W(\aleph-1), \text{ for } \aleph = 1, 2, ...$$

If the debt limit is never binding for unemployed workers we have $a^{W\prime}(\aleph) > \underline{a}\mathrm{e}^z$, and hence $a^{W\prime}(\aleph) > \underline{a}^{nat}\mathrm{e}^z$ for all $\aleph \geq 1$ (i.e., the debt limit is strictly tighter than the natural limit, by Assumption 2). This implies (in detrended form) $\hat{a}^{W\prime}(\aleph) > \underline{a}^{nat}$ for all $\aleph \geq 1$, and hence $\lim_{k\to+\infty}\hat{a}^{W\prime}(k) \equiv \hat{a}_{\infty}^{W'} > \underline{a}^{nat}$. Taking the limit as $\aleph \to +\infty$ of the budget constraint above and using the fact that $\hat{c}^W(+\infty) = 0$ and $\underline{a}^{nat} = -\frac{\beta^F b^u}{\mathrm{e}^{(\sigma-1)\mu_z} - \beta^F}$, we get:

$$\hat{a}_{\infty}^{W'} = -\frac{\beta^F b^u}{\mathrm{e}^{(\sigma-1)\mu_z} - \beta^F} = \underline{a}^{nat},$$

a contradiction. Hence, it cannot be that the debt limit never binds for workers remaining continuously unemployed, i.e., the debt limit binds after a finite number of unemployment periods □

We describe the equilibrium conditions characterising the model's dynamics for any finite $\hat{\aleph}$ in Section 2.2 of the main paper. We now state those conditions for the case where $\hat{\aleph} = 1$, which correspond to the model being estimated thereafter.

B.2. **The case $\hat{\aleph} = 1$.**

B.2.1. *Construction.* In the case where $\hat{\aleph} = 1$ all unemployed workers face a binding borrowing limit (i.e., $a^{W\prime}(\aleph) = \underline{a}\mathrm{e}^z$ for $\aleph \geq 1$) but none of the employed workers do (i.e., $a^{W\prime}(0) > \underline{a}\mathrm{e}^z$). Employed workers' budget constraint is given by:

$$c^W(0) + a^{W\prime}(0) = (1-\tau)w + (1+r)A$$

where

$$A' = \frac{(1-s')\mathbf{n}a^{W\prime}(0) + f'(1-\mathbf{n})\underline{a}\mathrm{e}^z}{\mathbf{n}'},$$

Since the consumption level of any household depends on both beginning-of-period and end-of-period level of nominal bonds and there can be only two types of unemployed workers. First, the unemployed workers who were employed at the beginning of the labor market transitions stage receive income $b^u\mathrm{e}^z + (1+r)a$ during the production stage, so their budget constraint is:

$$c^W(1) + \underline{a}\mathrm{e}^z = b^u\mathrm{e}^z + (1+r)a(1),$$

where beginning-of-period assets a results from last period's asset accumulation, when the worker was still employed, i.e., $a'(1) = a^{W\prime}(0) > 0$. Second, the unemployed workers who were unemployed at the beginning of the labor market transitions stage receive income $b^u\mathrm{e}^z + (1+r)a$ during the production stage and thus consume:

$$c^W(2) + \underline{a}\mathrm{e}^z = b^u\mathrm{e}^z + (1+r)a(2),$$

where in this case beginning-of-period assets a result from the binding borrowing constraint, i.e., $a'(2) = a^{W\prime}(1) = \underline{a}\mathrm{e}^z$. In this equilibrium, all workers of type $\aleph \geq 2$ are symmetric so that

$$c^W(\aleph) = c^W(2) \text{ and } a^{W\prime}(\aleph) = a^{W\prime}(1) = \underline{a}\mathrm{e}^z \text{ for all } \aleph \geq 2$$

To summarize, in the conjectured equilibrium at every point in time there are three distinct types of workers: $\aleph = 0$, $\aleph = 1$ and $\aleph \geq 2$, with consumption levels $c^W(0)$, $c^W(1)$ and $c^W(2)$, respectively. These types are in numbers $\Omega\mathbf{n}^W$, $\Omega s\tilde{\mathbf{n}}^W$ and $\Omega\left(1-\mathbf{n}^W - s\tilde{\mathbf{n}}^W\right)$, respectively. Type $\aleph = 0$ workers save $a^{W\prime}(0) > \underline{a}\mathrm{e}^z$, while types $\aleph = 1$ and $\aleph \geq 2$ all save $\underline{a}\mathrm{e}^z$. Finally, because there are only three workers' types, there are only three relevant habit levels to keep track of: $\mathbf{c}^W(0)$, $\mathbf{c}^W(1)$ and $\mathbf{c}^W(2)$. Since habits levels are determined by the average consumption of the relevant group in the previous period, we have $\mathbf{c}^{W\prime}(0) = c^W(0, X)$, $\mathbf{c}^{W\prime}(1) = c^W(1, X)$ and $\mathbf{c}^{W\prime}(2) = c^W(2)$.

B.2.2. *Existence conditions.* An equilibrium with $\hat{\aleph} = 1$ prevails if and only iff employed workers' bond holding choice is intérior while all unemployed workers' bond holding choices are corner. Since all $\aleph \geq 2$ workers are identical to $\aleph = 2$, this is the case whenever the three following conditions hold :

$$\mathbb{E}_{\mu,X}\left[M^{W\prime}(0)(1+r')\right] = 1, \text{ and}$$
$$\mathbb{E}_{\mu,X}\left[M^{W\prime}(\aleph)(1+r')\right] < 1 \text{ for } \aleph = 1,2.$$

C. AGGREGATE DYNAMICS

We now summarize the set of equilibrium conditions, first formulated in recursive form and then formulated in sequential form. We then induce stationarity and compute the steady state of the model. Because the equilibrium has a particularly simple structure we can now make the following intutive change of notation. We call $a^{e\prime} = a^{W\prime}(0)$ and $a^{u\prime} = \underline{a}e^{z}$ the end-of-period assets of employed and unemployed workers, respectively, $c^{e} = c^{W}(0)$ the consumption of employed workers, and, in the same spirit $c^{eu} = c^{W}(1)$ and $c^{uu} = c^{W}(2)$ (with eu standing form "workers falling into unemployment in the current period" and uu for "unemployed workers who where so in the previous period"). Similarly, we use the notation n^{eu} to denote the share of workers currintly falling into unemployment etc. Also, we define

$$A^{e} \equiv An'.$$

C.1. **Recursive representation.** To simplify notation, we eliminate all dependence with respect to X and X'.

C.1.1. *Workers.* The relevant equations are :

$$1 + r = \frac{1 + \mathbf{R}_{-1}}{1 + \pi} e^{\varphi_c}, \tag{C.1}$$

$$M^{F\prime} = \beta^{F} \frac{u'(c^{F\prime} - h\mathbf{c}^{F\prime})}{u'(c^{F} - h\mathbf{c}^{F})}, \tag{C.2}$$

$$\mathbb{E}[M^{F\prime}(1 + r')] = 1, \tag{C.3}$$

$$1 = p_k e^{\varphi_i} \left[1 - S\left(\frac{i'}{i}\right) - S'\left(\frac{i'}{i}\right)\frac{i'}{i}\right] + \mathbb{E}\left[M^{F\prime} p_k' e^{\varphi_i'} S'\left(\frac{i''}{i'}\right)\left(\frac{i''}{i'}\right)^2\right], \tag{C.4}$$

$$\eta'(v) = r_k, \tag{C.5}$$

$$p_k = \mathbb{E}\{M^{F\prime}[r_k' v' - \eta(v') + (1 - \delta)p_k']\}, \tag{C.6}$$

$$k' = (1 - \delta)k + e^{\varphi_i}(1 - S(i'/i))i', \tag{C.7}$$

$$c^{F} + i' + a^{F\prime} = \psi w n - [r_k v - \eta(v)]k - (1 + r)a^{F} - \Upsilon, \tag{C.8}$$

$$M^{e\prime} = \beta^{W} \frac{s' u'(c^{u\prime} - h\mathbf{c}^{eu\prime}) + (1 - s')u'(c^{e\prime} - h\mathbf{c}^{e\prime})}{u'(c^{e} - h\mathbf{c}^{e})}, \tag{C.9}$$

$$\mathbb{E}[M^{e\prime}(1+r')] = 1 \text{ if } a^{e\prime} \geq \underline{a}), \tag{C.10}$$

$$a^{e\prime} + c^e = (1-\tau)w + (1+r)\frac{A^e}{n}, \tag{C.11}$$

$$A^e = (1-s)\tilde{n}a^e + f(1-\tilde{n})\underline{a}e^{z_{-1}}, \tag{C.12}$$

$$c^{eu} = b^u \mathrm{e}^z + (1+r)a^e - \underline{a}e^z, \tag{C.13}$$

$$n^{eu} = s\tilde{n}, \tag{C.14}$$

$$c^{uu} = b^u \mathrm{e}^z + (1+r)\underline{a}e^{z_{-1}} - \underline{a}e^z, \tag{C.15}$$

$$n^{uu} = 1 - n - n^{eu}, \tag{C.16}$$

$$\tilde{n}' = n. \tag{C.17}$$

C.1.2. *Firms.* The relevant equations are :

$$p^* = \frac{K}{F}, \tag{C.18}$$

$$K = \mu \mathrm{e}^{\varphi_p} p_m y + \alpha \mathbb{E}\left[M^{F\prime}\left(\frac{1+\pi'}{(1+\bar{\pi})^{1-\gamma}(1+\pi)^{\gamma}}\right)^{\theta} K'\right], \tag{C.19}$$

$$F = y + \alpha \mathbb{E}\left[M^{F\prime}\left(\frac{1+\pi'}{(1+\bar{\pi})^{1-\gamma}(1+\pi)^{\gamma}}\right)^{\theta-1} F'\right], \tag{C.20}$$

$$1 = (1-\alpha)(p^*)^{1-\theta} + \alpha\left(\frac{(1+\bar{\pi})^{1-\gamma}(1+\boldsymbol{\pi}_{-1})^{\gamma}}{1+\pi}\right)^{1-\theta}, \tag{C.21}$$

$$\Lambda = (1-\alpha)(p^*)^{-\theta} + \alpha\left(\frac{(1+\bar{\pi})^{1-\gamma}(1+\boldsymbol{\pi}_{-1})^{\gamma}}{1+\pi}\right)^{-\theta}\Lambda_{-1}, \tag{C.22}$$

$$r_k = p_m \phi \mathrm{e}^{z(1-\phi)}\left(\frac{\breve{k}}{\breve{n}}\right)^{\phi-1}, \tag{C.23}$$

$$Q = p_m(1-\phi)\mathrm{e}^{z(1-\phi)}\left(\frac{\breve{k}}{\breve{n}}\right)^{-\phi}, \tag{C.24}$$

$$\frac{\kappa_v \mathrm{e}^z}{\Omega+(1-\Omega)\psi}\frac{1}{\lambda} = Q - w + \mathbb{E}\left[(1-\rho(\varphi'_\rho))M^F \frac{\kappa_v \mathrm{e}^{z'}}{\Omega+(1-\Omega)\psi}\frac{1}{\lambda'}\right], \tag{C.25}$$

$$n = (1-\rho(\varphi_\rho)\tilde{n} + \lambda v, \tag{C.26}$$

$$m = \bar{m}(1-(1-\rho(\varphi_\rho))\tilde{n})^\chi v^{1-\chi}, \tag{C.27}$$

$$f = \frac{m}{1-(1-\rho(\varphi_\rho))\tilde{n}}, \tag{C.28}$$

$$\lambda = \frac{m}{v}, \tag{C.29}$$

$$s = \rho(\varphi_\rho)(1-f). \tag{C.30}$$

C.1.3. *Wage.* The wage equation is

$$w = \left(\frac{w_{-1}}{1+\pi}\right)^{\gamma_w}\left(\bar{w}\mathrm{e}^{z+\varphi_w}\left(\frac{n}{n_{ss}}\right)^{\psi_n}\right)^{1-\gamma_w}. \tag{C.31}$$

C.1.4. *Central Bank.* Ther Central Bank follows the rule

$$\log\left(\frac{1+R}{1+\bar{R}}\right) = \rho_R \log\left(\frac{1+\mathbf{R}_{-1}}{1+\bar{R}}\right) + (1-\rho_R)\left[a_\pi \log\left(\frac{1+\pi}{1+\bar{\pi}}\right) + a_y \log\left(\frac{1+g}{1+\bar{g}}\right)\right] + \varphi_R, \tag{C.32}$$

where

$$g = y/\mathbf{y}_{-1} - 1,$$

C.1.5. *Aggregation and market clearing.* The relevant equations are

$$[\Omega + (1-\Omega)\psi]n = \breve{n}, \tag{C.33}$$

$$(1-\Omega)vk = \breve{k}, \tag{C.34}$$

$$y\Lambda = ((1-\Omega)vk)^\phi(\mathrm{e}^{z_t}[\Omega+(1-\Omega)\psi]n)^{1-\phi} - \kappa_y \mathrm{e}^z, \tag{C.35}$$

$$(1-\Omega)(c^F + i + \eta(v)k) + \Omega(nc^e + n^{eu}c^{eu} + n^{uu}c^{uu}) + \kappa_v \mathrm{e}^z v = y, \tag{C.36}$$

$$(1-\Omega)a^{F\prime}+\Omega(A^{e\prime}+(1-n)\underline{a}\mathrm{e}^{z})=0, \tag{C.37}$$

$$\tau wn=b^{u}\mathrm{e}^{z}(1-n). \tag{C.38}$$

C.2. **Sequential representation.** It is now useful to introduce the time index before considering the stationary model. We index all variables in the current period by t, and make use of the equilibrium and symmetry conditions above to reduce the number of variables. For example, n_t is the employment rate in a particular family of workers (n^W) as well as the aggregate employment rate of workers ($\mathbf{n}^W$) ; it is also the corresponding rates for employers, since those face the same labor market transitions than the workers. Similarly, current investment (i') is denoted i_t, and so on. Finally, in as much as capital in the current period is determined by investment decisions made in the previous period, we denote it by k_{t-1}. In particular :

$$\tilde{n}_t=n_{t-1}$$

The final system is composed of the following equations:

$$1+r_t=\frac{1+R_{t-1}}{1+\pi_t}\mathrm{e}^{\varphi_{c,t}}, \tag{C.39}$$

$$M^F_{t,t+1}=\beta^F\frac{\lambda^F_{t+1}}{\lambda^F_t}, \tag{C.40}$$

$$\lambda^F_t=(c^F_t-hc^F_{t-1}), \tag{C.41}$$

$$\mathbb{E}_t\{M^F_{t,t+1}(1+r_{t+1})\}=1, \tag{C.42}$$

$$1=p_{k,t}\mathrm{e}^{\varphi_{i,t}}\left[1-S\left(\frac{i_t}{i_{t-1}}\right)-S'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right]+\mathbb{E}_t\left[M^F_{t,t+1}p_{k,t+1}\mathrm{e}^{\varphi_{i,t+1}}S'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right], \tag{C.43}$$

$$\eta'(v_t)=r_{k,t}, \tag{C.44}$$

$$p_{k,t}=\mathbb{E}_t\{M^F_{t,t+1}[r_{k,t+1}v_{t+1}-\eta(v_{t+1})+(1-\delta)p_{k,t+1}]\}, \tag{C.45}$$

$$k_t=(1-\delta)k_{t-1}+\mathrm{e}^{\varphi_{i,t}}\left(1-S\left(\frac{i_t}{i_{t-1}}\right)\right)i_t, \tag{C.46}$$

$$M^e_{t,t+1}=\beta^W\frac{(1-s_{t+1})\lambda^e_{t+1}+s_{t+1}\lambda^{eu}_{t+1}}{\lambda^e_t}, \tag{C.47}$$

$$\lambda_t^e = u'(c_t^e - hc_{t-1}^e), \tag{C.48}$$

$$\lambda_t^{eu} = u'(c_t^{eu} - hc_{t-1}^{eu}), \tag{C.49}$$

$$1 = \mathbb{E}_t[M_{t,t+1}^e(1 + r_{t+1})], \tag{C.50}$$

$$a_t^e + c_t^e = (1 - \tau_t)w_t + (1 + r_t)\frac{A_t^e}{n_t}, \tag{C.51}$$

$$A_t^e = (1 - s_t)n_{t-1}a_{t-1}^e + f_t(1 - n_{t-1})\underline{a}e^{z_{t-1}}, \tag{C.52}$$

$$c_t^{eu} = b^u e^{z_t} + (1 + r_t)a_{t-1}^e - \underline{a}e^{z_t}, \tag{C.53}$$

$$n_t^{eu} = s_t n_{t-1}, \tag{C.54}$$

$$c_t^{uu} = b^u e^{z_t} + (1 + r_t)\underline{a}e^{z_{t-1}} - \underline{a}e^{z_t} \tag{C.55}$$

$$n_t^{uu} = 1 - n_t - n_t^{eu}, \tag{C.56}$$

$$p_t^* = \frac{K_t}{F_t}, \tag{C.57}$$

$$K_t = \mu e^{\varphi_{p,t}} p_{m,t} y_t + \alpha \mathbb{E}_t\left[M_{t,t+1}^F \left(\frac{1 + \pi_{t+1}}{(1 + \pi_t^*)^{1-\gamma}(1 + \pi_t)^\gamma}\right)^\theta K_{t+1}\right], \tag{C.58}$$

$$F_t = y_t + \alpha \mathbb{E}_t\left[M_{t,t+1}^F \left(\frac{1 + \pi_{t+1}}{(1 + \pi_t^*)^{1-\gamma}(1 + \pi_t)^\gamma}\right)^{\theta-1} F_{t+1}\right], \tag{C.59}$$

$$1 = (1 - \alpha)(p_t^*)^{1-\theta} + \alpha\left(\frac{(1 + \pi_t^*)^{1-\gamma}(1 + \pi_{t-1})^\gamma}{1 + \pi_t}\right)^{1-\theta}, \tag{C.60}$$

$$\Lambda_t = (1 - \alpha)(p_t^*)^{-\theta} + \alpha\left(\frac{(1 + \pi_t^*)^{1-\gamma}(1 + \pi_{t-1})^\gamma}{1 + \pi_t}\right)^{-\theta} \Lambda_{t-1}, \tag{C.61}$$

$$r_{k,t} = p_{m,t}\phi\frac{y_{m,t}}{(1 - \Omega)v_t k_{t-1}}, \tag{C.62}$$

$$Q_t = p_{m,t}(1-\phi)\frac{y_{m,t}}{[\Omega+(1-\Omega)\psi]n_t}, \tag{C.63}$$

$$\frac{\kappa_v e^{z_t}}{\Omega+(1-\Omega)\psi}\frac{1}{\lambda_t} = Q_t - w_t + \mathbb{E}_t\left[(1-\rho_{t+1})M^F_{t,t+1}\frac{\kappa_v e^{z_{t+1}}}{\Omega+(1-\Omega)\psi}\frac{1}{\lambda_{t+1}}\right], \tag{C.64}$$

$$n_t = (1-\rho_t)n_{t-1} + \lambda_t v_t, \tag{C.65}$$

$$m_t = \bar{m}(1-(1-\rho_t)n_{t-1})^{\chi} v_t^{1-\chi}, \tag{C.66}$$

$$f_t = \frac{m_t}{1-(1-\rho_t)n_{t-1}}, \tag{C.67}$$

$$\lambda_t = \frac{m_t}{v_t}, \tag{C.68}$$

$$s_t = \rho_t(1-f_t), \tag{C.69}$$

$$w_t = \left(\frac{w_{t-1}}{1+\pi_t}\right)^{\gamma_w}\left(\bar{w}e^{z_t+\varphi_{w,t}}\left(\frac{n_t}{n_{ss}}\right)^{\psi_n}\right)^{1-\gamma_w}, \tag{C.70}$$

$$\log\left(\frac{1+R_t}{1+\bar{R}}\right) = \rho_R\log\left(\frac{1+R_{t-1}}{1+\bar{R}}\right) + (1-\rho_R)\left[a_\pi\log\left(\frac{1+\pi_t}{1+\bar{\pi}}\right) + a_y\log\left(\frac{1+g_t}{1+\bar{g}}\right)\right] + \sigma_R\epsilon_{R,t}, \tag{C.71}$$

$$1+g_t = \frac{y_t}{y_{t-1}}, \tag{C.72}$$

$$\Lambda_t y_t = y_{m,t} - \kappa_y e^{-z_t}, \tag{C.73}$$

$$(1-\Omega)(c^F + i + \eta(v)k) + \Omega(nc_t^e + (1-f_t)(1-n_{t-1})c_t^{uu} + s_t n_{t-1}c_t^{eu}) + \kappa_v e^z v_t = y_t, \tag{C.74}$$

$$(1-\Omega)a_t^F + \Omega(A_t^e + (1-n_t)\underline{a}e^{z_t}) = 0, \tag{C.75}$$

$$\tau_t w_t n_t = b^u e^{z_t}(1-n_t), \tag{C.76}$$

$$y_{m,t} = ((1-\Omega)v_t k_{t-1})^{\phi}(e^{z_t}[\Omega+(1-\Omega)\psi]n_t)^{1-\phi}. \tag{C.77}$$

C.3. **Stationary model.** Before inducing stationarity, we assume the functional forms below.

(1) The utilization cost function is of the form:

$$\eta(v) = \frac{\bar{\eta}}{\tilde{\nu}_v}[\exp(\tilde{\nu}_v(v-1)) - 1], \quad \bar{\eta} > 0, \quad \tilde{\nu}_v > 0.$$

so that in a steady state with $v = 1$, it must be the case that $\eta(1) = 0$, $\eta'(1) = \bar{\eta}$. To ensure that this condition is met for all possible MCMC draws at the estimation stage, the parameter $\bar{\eta}$ must be adjusted for each new parameter draw. As such, it is not included in the list of estimated parameters. Notice also that

$$\frac{\eta''(v)}{\eta'(v)} = \tilde{\nu}_v,$$

so that in steady state with $v = 1$, the curvature of $\eta(\cdot)$ is $\tilde{\nu}_v$. We define $\nu_v \equiv \tilde{\nu}_v/(1+\tilde{\nu}_v)$ and estimate ν_v instead of $\tilde{\nu}_v$. This allows us to eliminate numerical problems at the estimation stage.

(2) The investment adjustment cost function is of the form

$$S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\tilde{\nu}_i}{2}\left(\frac{i_t}{i_{t-1}} - \mathrm{e}^{\mu_z}\right)^2$$

where μ_z is the steady-state growth rate of technical progress, which coincides with that of investment along a balanced-growth path. Using this functional form, it must be the case that in a steady state with $i_t/i_{t-1} = \mathrm{e}^{\mu_z}$, $S(\mathrm{e}^{\mu_z}) = S'(\mathrm{e}^{\mu_z}) = 0$. We also define

$$\nu_i \equiv \tilde{\nu}_i \mathrm{e}^{2\mu_z}$$

(3) The utility function is of the form

$$u(c) = \lim_{\tilde{\sigma}\to\sigma} \frac{c^{1-\tilde{\sigma}} - 1}{1-\tilde{\sigma}}, \quad \sigma > 0$$

which encompasses the special case $\sigma = 1$ as the logarithmic utility function.

(4) The exogenous separation rate obeys

$$\rho_t = \frac{1}{1+\exp(-\bar{\rho} - \varphi_{\rho,t})},$$

so that (i) $\rho_t \longrightarrow 1$ as $\varphi_{\rho,t} \longrightarrow +\infty$, (ii) $\rho_t \longrightarrow 0$ as $\varphi_{\rho,t} \longrightarrow -\infty$, and $\bar{\rho}$ pins down the steady-state value of ρ, via

$$\bar{\rho} = \log\left(\frac{\rho}{1-\rho}\right),$$

We now define the stationary version of the originally trending variables as follows

$$\hat{y}_t = y_t \mathrm{e}^{-z_t}, \quad \hat{y}_{m,t} = y_{m,t}\mathrm{e}^{-z_t}, \quad \hat{Q}_t = Q_t \mathrm{e}^{-z_t},$$

$$\hat{\lambda}_t^F = \lambda_t^F \mathrm{e}^{\sigma z_t}, \quad \hat{\lambda}_t^e = \lambda_t^e \mathrm{e}^{\sigma z_t}, \quad \hat{\lambda}_t^{eu} = \lambda_t^{eu}\mathrm{e}^{\sigma z_t},$$

$$\hat{c}_t^F = c_t^F \mathrm{e}^{-z_t}, \quad \hat{c}_t^e = c_t^e \mathrm{e}^{-z_t}, \quad \hat{c}_t^{eu} = c_t^{eu}\mathrm{e}^{-z_t}, \quad \hat{\imath}_t = i_t \mathrm{e}^{-z_t}, \quad \hat{k}_t = k_t \mathrm{e}^{-z_t},$$

$$\hat{a}^e_t = a^e_t \mathrm{e}^{-z_t}, \quad \hat{A}^e_t = A^e_t \mathrm{e}^{-z_t}, \quad \hat{w}_t = w_t \mathrm{e}^{-z_t}, \quad \hat{F}_t = F_t \mathrm{e}^{-z_t}, \quad \hat{K}_t = K_t \mathrm{e}^{-z_t}.$$

Accordingly, the normalized system rewrites

$$1 + r_t = \frac{1 + R_{t-1}}{1 + \pi_t}, \tag{C.78}$$

$$M^F_{t,t+1} = \frac{\beta^F}{e^{\sigma \mu_z}} \mathrm{e}^{-\sigma \varphi_{z,t+1}} \frac{\lambda^F_{t+1}}{\hat{\lambda}^F_t}, \tag{C.79}$$

$$\hat{\lambda}^F_t = \left(\hat{c}^F_t - \frac{h}{\mathrm{e}^{\mu_z}} \hat{c}^F_{t-1} \mathrm{e}^{-\varphi_{z,t}} \right)^{-\sigma}, \tag{C.80}$$

$$\mathbb{E}_t [M^F_{t,t+1}(1 + r_{t+1})] = 1, \tag{C.81}$$

$$\begin{aligned} 1 = p_{k,t} \mathrm{e}^{\varphi_{i,t}} &\left[1 - \frac{\nu_i}{2} \left(\frac{\hat{\imath}_t \mathrm{e}^{\varphi_{z,t}}}{\hat{\imath}_{t-1}} - 1 \right)^2 - \nu_i \left(\frac{\hat{\imath}_t \mathrm{e}^{\varphi_{z,t}}}{\hat{\imath}_{t-1}} - 1 \right) \frac{\hat{\imath}_t \mathrm{e}^{\varphi_{z,t}}}{\hat{\imath}_{t-1}} \right] \\ &+ \mathbb{E}_t \left\{ \mathrm{e}^{\mu_z} M^F_{t,t+1} p_{k,t+1} \mathrm{e}^{\varphi_{i,t+1}} \nu_i \left(\frac{\hat{\imath}_{t+1} \mathrm{e}^{\varphi_{z,t+1}}}{\hat{\imath}_t} - 1 \right) \left(\frac{\hat{\imath}_{t+1} \mathrm{e}^{\varphi_{z,t+1}}}{\hat{\imath}_t} \right)^2 \right\}, \end{aligned} \tag{C.82}$$

$$\eta'(v_t) = r_{k,t}, \tag{C.83}$$

$$p_{k,t} = \mathbb{E}_t \left[M^F_{t,t+1} \left\{ r_{k,t+1} v_{t+1} - \eta(v_{t+1}) + (1 - \delta) p_{k,t+1} \right\} \right] \tag{C.84}$$

$$\hat{k}_t = \left(\frac{1 - \delta}{\mathrm{e}^{\mu_z}} \right) \hat{k}_{t-1} \mathrm{e}^{-\varphi_{z,t}} + \mathrm{e}^{\varphi_{i,t}} \left(1 - \frac{\nu_i}{2} \left(\frac{\hat{\imath}_t \mathrm{e}^{\varphi_{z,t}}}{\hat{\imath}_{t-1}} - 1 \right)^2 \right) \hat{\imath}_t, \tag{C.85}$$

$$M^e_{t,t+1} = \frac{\beta^W}{e^{\sigma \mu_z}} e^{-\sigma \varphi_{z,t+1}} \frac{(1 - s_{t+1}) \hat{\lambda}^e_{t+1} + s_{t+1} \hat{\lambda}^{eu}_{t+1}}{\hat{\lambda}^e_t}, \tag{C.86}$$

$$\hat{\lambda}^e_t = \left(\hat{c}^e_t - \frac{h}{e^{\mu_z}} \hat{c}^e_{t-1} \mathrm{e}^{-\varphi_{z,t-1}} \right)^{-\sigma}, \tag{C.87}$$

$$\hat{\lambda}^{eu}_t = \left(\hat{c}^{eu}_t - \frac{h}{e^{\mu_z}} \hat{c}^{eu}_{t-1} \mathrm{e}^{-\varphi_{z,t-1}} \right)^{-\sigma}, \tag{C.88}$$

$$1 = \mathbb{E}_t [M^e_{t,t+1}(1 + r_{t+1})], \tag{C.89}$$

$$\hat{a}^e_t + \hat{c}^e_t = (1 - \tau_t) \hat{w}_t + (1 + r_t) \frac{\hat{A}^e_t}{n_t}, \tag{C.90}$$

$$\mathrm{e}^{\mu_z}\hat{A}^e_t\mathrm{e}^{\varphi_{z,t}} = (1-s_t)n_{t-1}\hat{a}^e_{t-1} + f_t(1-n_{t-1})\underline{a} \tag{C.91}$$

$$\hat{c}^{eu}_t = b^u + \frac{1+r_t}{\mathrm{e}^{\mu_z}}\hat{a}^e_{t-1}\mathrm{e}^{-\varphi_{z,t}} - \underline{a}, \tag{C.92}$$

$$n^{eu}_t = s_t n_{t-1}, \tag{C.93}$$

$$\hat{c}^{uu}_t = b^u + \left(\frac{1+r_t}{\mathrm{e}^{\mu_z}} - 1\right)\underline{a} \tag{C.94}$$

$$n^{uu}_t = 1 - n_t - n^{eu}_t, \tag{C.95}$$

$$p^*_t = \frac{\hat{K}_t}{\hat{F}_t}, \tag{C.96}$$

$$\hat{K}_t = \mu\mathrm{e}^{\varphi_{p,t}}p_{m,t}\hat{y}_t + \alpha\mathbb{E}_t\left\{\mathrm{e}^{\mu_z}M^F_{t,t+1}\left(\frac{1+\pi_{t+1}}{(1+\bar{\pi})^{1-\gamma}(1+\pi_t)^{\gamma}}\right)^{\theta}\hat{K}_{t+1}\mathrm{e}^{\varphi_{z,t+1}}\right\}, \tag{C.97}$$

$$\hat{F}_t = \hat{y}_t + \alpha\mathbb{E}_t\left\{\mathrm{e}^{\mu_z}M^F_{t,t+1}\left(\frac{1+\pi_{t+1}}{(1+\bar{\pi})^{1-\gamma}(1+\pi_t)^{\gamma}}\right)^{\theta-1}\hat{F}_{+1}\mathrm{e}^{\varphi_{z,t+1}}\right\}, \tag{C.98}$$

$$1 = (1-\alpha)(p^*_t)^{1-\theta} + \alpha\left(\frac{(1+\bar{\pi})^{1-\gamma}(1+\pi_{t-1})^{\gamma}}{1+\pi_t}\right)^{1-\theta}, \tag{C.99}$$

$$\Lambda_t = (1-\alpha)(p^*_t)^{-\theta} + \alpha\left(\frac{(1+\bar{\pi})^{1-\gamma}(1+\pi_{t-1})^{\gamma}}{1+\pi_t}\right)^{-\theta}\Lambda_{t-1}, \tag{C.100}$$

$$n_t = (1-\rho)n_{t-1} + \lambda_t v_t, \tag{C.101}$$

$$m_t = \bar{m}\mathrm{e}^{\varphi_{m,t}}(1-(1-\rho_t)n_{t-1})^{\chi}v_t^{1-\chi}, \tag{C.102}$$

$$s_t = \rho_t(1-f_t), \tag{C.103}$$

$$f_t = \frac{m_t}{1-(1-\rho_t)n_{t-1}}, \tag{C.104}$$

$$\lambda_t = \frac{m_t}{v_t}, \tag{C.105}$$

$$\frac{\kappa_v}{\Omega+(1-\Omega)\psi}\frac{1}{\lambda_t}=\hat{Q}_t-\hat{w}_t+\mathbb{E}_t\left[(1-\rho_{t+1})M^F_{t,t+1}\frac{\kappa_v \mathrm{e}^{\mu_z+\varphi_{z,t+1}}}{\Omega+(1-\Omega)\psi}\frac{1}{\lambda_{t+1}}\right], \tag{C.106}$$

$$\hat{y}_{m,t}=\left((1-\Omega)v_t\frac{\hat{k}_{t-1}}{\mathrm{e}^{\mu_z}}\mathrm{e}^{-\varphi_{z,t}}\right)^{\phi}\left(\left[\Omega+(1-\Omega)\psi\right]n_t\right)^{1-\phi}, \tag{C.107}$$

$$\Lambda_t\hat{y}_t=\hat{y}_{m,t}-\kappa_y, \tag{C.108}$$

$$(1-\Omega)\left(\hat{c}^F_t+\hat{\imath}_t+\eta(v_t)\frac{\hat{k}_{t-1}}{\mathrm{e}^{\mu_z}}\mathrm{e}^{-\varphi_{z,t}}\right)+\Omega(n_t\hat{c}^e_t+n^{uu}_t\hat{c}^{uu}_t+n^{eu}_t\hat{c}^{eu}_t)+\kappa_v v_t=\hat{y}_t \tag{C.109}$$

$$(1-\Omega)\hat{a}^F_t+\Omega(\hat{A}^e_t+(1-n_t)\underline{a})=0, \tag{C.110}$$

$$\tau_t\hat{w}_t n_t=b^u(1-n_t), \tag{C.111}$$

$$r_{k,t}=p_{m,t}\phi\mathrm{e}^{\mu_z}\frac{\hat{y}_{m,t}}{(1-\Omega)v_t\hat{k}_{t-1}}\mathrm{e}^{\varphi_{z,t}}, \tag{C.112}$$

$$\hat{Q}_t=p_{m,t}(1-\phi)\frac{\hat{y}_{m,t}}{[\Omega+(1-\Omega)\psi]n_t} \tag{C.113}$$

$$\hat{w}_t=\left(\frac{\hat{w}_{t-1}\mathrm{e}^{-\mu_z-\varphi_{z,t}}}{1+\pi_t}\right)^{\gamma_w}\left(\bar{w}\mathrm{e}^{\varphi_{w,t}}\left(\frac{n_t}{n_{ss}}\right)^{\psi_n}\right)^{1-\gamma_w}, \tag{C.114}$$

$$\log\left(\frac{1+R_t}{1+\bar{R}}\right)=\rho_R\log\left(\frac{1+R_{t-1}}{1+\bar{R}}\right)+(1-\rho_R)\left[a_\pi\log\left(\frac{1+\pi_t}{1+\bar{\pi}}\right)+a_y\log\left(\frac{\hat{y}_t\mathrm{e}^{\varphi_{z,t}}}{\hat{y}_{t-1}}\right)\right]+\sigma_R\epsilon_{R,t}, \tag{C.115}$$

$$\rho_t=\frac{1}{1+\exp(-\bar{\rho}-\varphi_{\rho,t})}. \tag{C.116}$$

C.4. **Steady state.** Several variables have trivial steady-state values: $\Lambda = 1$, $p_k = 1$, $p^* = 1$. Once we get rid of these, the reduced steady state is solution to the system

$$1 + r = \frac{1+R}{1+\bar{\pi}}, \tag{C.117}$$

$$M^F = \frac{\beta^F}{e^{\sigma \mu_z}}, \tag{C.118}$$

$$\hat{\lambda}^F = \left(\left(1 - \frac{h}{e^{\mu_z}}\right)\hat{c}^F\right)^{-\sigma}, \tag{C.119}$$

$$M^F(1+r) = 1, \tag{C.120}$$

$$\eta'(v) = r_k, \tag{C.121}$$

$$1 = M^F(r_k + 1 - \delta) \tag{C.122}$$

$$\left[1 - \left(\frac{1-\delta}{e^{\mu_z}}\right)\right]\hat{k} = \hat{\imath}, \tag{C.123}$$

$$M^e = \frac{\beta^W}{e^{\sigma \mu_z}} \frac{(1-s)\hat{\lambda}^e + s\hat{\lambda}^{eu}}{\hat{\lambda}^e}, \tag{C.124}$$

$$\hat{\lambda}^e = \left(\left(1 - \frac{h}{e^{\mu_z}}\right)\hat{c}^e\right)^{-\sigma}, \tag{C.125}$$

$$\hat{\lambda}^{eu} = \left(\left(1 - \frac{h}{e^{\mu_z}}\right)\hat{c}^{eu}\right)^{-\sigma}, \tag{C.126}$$

$$1 = M^e(1+r), \tag{C.127}$$

$$\hat{a}^e + \hat{c}^e = (1-\tau)\hat{w} + (1+r)\frac{\hat{A}^e}{n}, \tag{C.128}$$

$$e^{\mu_z}\hat{A}^e = (1-s)n\hat{a}^e \tag{C.129}$$

$$\hat{c}^{eu} = b^u + \frac{1+r}{e^{\mu_z}}\hat{a}^e, \tag{C.130}$$

$$n^{eu} = sn, \tag{C.131}$$

$$\hat{c}^{uu} = b^u \tag{C.132}$$

$$n^{uu} = 1 - n - n^{eu}, \tag{C.133}$$

$$1 = \frac{\hat{K}}{\hat{F}}, \tag{C.134}$$

$$\hat{K} = \mu p_m \hat{y} + \alpha \mathrm{e}^{\mu_z} M^F \hat{K}, \tag{C.135}$$

$$\hat{F} = \hat{y} + \alpha \mathrm{e}^{\mu_z} M^F \hat{F}, \tag{C.136}$$

$$\rho n = \lambda v, \tag{C.137}$$

$$m = \bar{m}(1 - (1-\rho)n)^{\chi} v^{1-\chi}, \tag{C.138}$$

$$s = \rho(1 - f), \tag{C.139}$$

$$f = \frac{m}{1 - (1-\rho)n}, \tag{C.140}$$

$$\lambda = \frac{m}{v}, \tag{C.141}$$

$$\frac{\kappa_v}{\Omega + (1-\Omega)\psi} \frac{1}{\lambda} = \hat{Q} - \hat{w} + \left[(1-\rho) M^F \frac{\kappa_v \mathrm{e}^{\mu_z}}{\Omega + (1-\Omega)\psi} \frac{1}{\lambda} \right] \tag{C.142}$$

$$\hat{y}_m = \left((1-\Omega) \frac{\hat{k}}{\mathrm{e}^{\mu_z}} \right)^{\phi} ([\Omega + (1-\Omega)\psi] n)^{1-\phi}, \tag{C.143}$$

$$\hat{y} = \hat{y}_m - \kappa_y, \tag{C.144}$$

$$(1-\Omega)(\hat{c}^F + \hat{\imath}) + \Omega(n\hat{c}^e + (1-f)(1-n)\hat{c}^{uu} + sn\hat{c}^{eu}) + \kappa_v v = \hat{y} \tag{C.145}$$

$$(1-\Omega)\hat{a}^F + \Omega \hat{A}^e = 0, \tag{C.146}$$

$$\tau \hat{w} n = b^u (1 - n), \tag{C.147}$$

$$r_k = p_m \phi \mathrm{e}^{\mu_z} \frac{\hat{y}_m}{(1 - \Omega) v \hat{k}}, \tag{C.148}$$

$$\hat{Q} = p_m (1 - \phi) \frac{\hat{y}_m}{[\Omega + (1 - \Omega)\psi] n}, \tag{C.149}$$

$$\hat{w} = \left(\frac{\mathrm{e}^{-\mu_z}}{1 + \pi} \right)^{\frac{\gamma w}{1 - \gamma w}} \bar{w}. \tag{C.150}$$

There are 34 equations in the system for the 34 unknown variables: r, R, M^F, M^e, $\hat{\lambda}^F$, $\hat{\lambda}^e$, $\hat{\lambda}^{eu}$, $\hat{c}^F$, $\hat{c}^e$, $\hat{c}^{eu}$, $\hat{c}^{uu}$, $\hat{a}^F$, $\hat{a}^e$, $\hat{A}^e$, λ, m, v, s, f, n, n^{eu}, n^{uu}, $\hat{k}$, $\hat{\imath}$, v, $\hat{y}$, $\hat{y}_m$, r_k, $\hat{w}$, $\hat{Q}$, τ, p_m, $\hat{K}_p$, $\hat{F}_p$.

D. Estimation and Empirical Results

Let $\hat{X}$ denote the vector collecting the deviation from steady state of the normalized state variables and let ϵ denote the vector collecting the innovations to the aggregate shocks. The law of motion of $\hat{X}$ is of the form:

$$\hat{X}' = \mathbf{F}(\boldsymbol{\vartheta})\hat{X} + \mathbf{G}(\boldsymbol{\vartheta})\epsilon', \tag{D.1}$$

where

$$\boldsymbol{\vartheta} = (\Omega, \sigma, h, \beta^F, \beta^W, \delta, \theta, \phi, \kappa_v, \kappa_y, \bar{\rho}, \chi, \bar{m}, \nu_i, \nu_v, \bar{\eta}, \alpha, \gamma_p, \psi, \bar{w}, \psi_n, \gamma_w,$$
$$b_u, \underline{a}, \bar{\pi}, \rho, a_\pi, a_y, \mu_z, \rho_x \text{for} x \in \{c, i, w, s, p, R\}, \sigma_x \text{ for } x \in \{c, i, w, s, p, R, z\})$$

is the vector of model's structural parameters. The matrices $\mathbf{F}(\boldsymbol{\vartheta})$ and $\mathbf{G}(\boldsymbol{\vartheta})$ are functions of the model's structural parameters.

As mentioned in the paper, the vector of structural parameters $\boldsymbol{\vartheta}$ is split into two subsets $\boldsymbol{\vartheta}_1$ and $\boldsymbol{\vartheta}_2$. The first one, that we call the calibrated parameters,

$$\boldsymbol{\vartheta}_1 = (\delta, \theta, \chi, \bar{m}, \Omega, \underline{a}, \bar{\pi}, \mu_z, \beta^F, \beta^W, b^u, \phi, \kappa_v, \bar{\rho}, \kappa_y, \psi, \bar{w}),$$

contains structural parameters that are not estimated. The calibrated parameters are either outright calibrated or tied to some restriction implied by the fact that we force the steady-state to match some unconditional moments. The remaining structural parameters, contained in $\boldsymbol{\vartheta}_2$, are estimated.

D.1. **Calibrated Parameters.** The calibrated parameters are set to match certain calibration restrictions, either imposed directly at the specification stage or stemming from calibration targets (i.e. a discipline imposed by the data). Below, we list these two sorts of restrictions.

$$\boldsymbol{\vartheta}_1 = (\delta, \chi, \bar{m}),$$

(1) Restrictions at the specification stage
- The steady-state utilization rate $v = 1$, so that $\bar{\eta}$ is not strictly speaking a free parameter;
- Fixed costs are such that aggregate profits are zero in the steady state;

(2) Outright and data-based restrictions
- We set $\delta = 0.015$;
- Relative share of workers $\Omega = 0.6$;
- The parameter $\bar{m}$ is normalized to 1.
- The elasticity of the matching function with respect to vacancies is set to 0.5, so that $\chi = 0.5$;
- The borrowing limit $\underline{a} = 0$;

- The price markup is set to 20%, thus restricting $\theta = 6$;
- Vacancy posting costs are such that $\kappa_v v/\hat{y}$ are equal to a 1%, which restricts κ_v. Below, we define $vac \equiv \kappa_v v/\hat{y}$;
- The labor share, which we will denote lsh, and is equal to $\hat{w}[\Omega + (1-\Omega)\psi]n/y$, is set to 64%;
- Wage premium ψ is set so as to match the share of consumption of the 60% poorest in aggregate consumption, denoted s_{60};
- The nominal interest rate R is set to the observed average Fed Funds rate $\bar{R}$, which restricts β^F;
- The inflation target $\bar{\pi}$ is set to the observed average inflation;
- The growth rate μ_z is set to the observed average output growth;
- The steady-state values of s and f are set to their observed average (at the estimation stage), which restricts $\bar{\rho}$ and $\bar{\varpi}$;
- The consumption loss $\hat{c}^{eu}/\hat{c}^e = 1 - (1-0.79)/0.6$ so that on average (across the whole population of employees), the consumption drop when falling into unemployment is close to 20%, which restricts β^W. Below, we define $loss \equiv \hat{c}^{eu}/\hat{c}^e$;
- We also impose a replacement ratio (50%), so that $b^u = 0.5\hat{w}$. In the following we define the parameter governing the replacement ratio $reprat = 0.5$.

It is useful to define the above calibration restrictions in terms of a new vector $\boldsymbol{\vartheta}_{\text{target}}$ which simply lists them. In practice, we treat $\boldsymbol{\vartheta}_{\text{target}}$ as the parameters and $\boldsymbol{\vartheta}_1$ as the variables solving the a system of the form

$$Z(\boldsymbol{\vartheta}_{\text{target}}, \boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2) = 0.$$

Now, given $\boldsymbol{\vartheta}_2$, our task is to find a one-to-one mapping between the calibration restrictions $\boldsymbol{\vartheta}_{\text{target}}$ and the parameters we want to pin down $\boldsymbol{\vartheta}_1$. To that end, we develop the following alogrithm.

(1) From equations (C.134), (C.135), and (C.136), we obtain

$$1 = \mu p_m, \tag{D.2}$$

(2) The steady-state profits of firm i are given by

$$\hat{y}(i) - p_m(\hat{y}(i) + \kappa_y).$$

Since $\mu p_m = 1$, we obtain

$$\left(1 - \frac{1}{\mu}\right)\hat{y} = \frac{1}{\mu}\kappa_y \Longrightarrow (\mu - 1)\,\hat{y} = \kappa_y.$$

Since

$$\hat{y} = \hat{y}_m - \kappa_y$$

and since profits are zero in steady state, we obtain

$$\hat{y}_m = \mu \hat{y}.$$

We will use this restriction repeatedly in the remainder.

(3) From (C.118), (C.120), and (C.127)

$$M^e = M^F = \frac{\beta^F}{e^{\sigma \mu_z}}, \tag{D.3}$$

$$r = \frac{e^{\sigma \mu_z}}{\beta^F} - 1, \tag{D.4}$$

and from (C.117)

$$R = \frac{e^{\sigma \mu_z}}{\beta^F}(1+\bar{\pi}) - 1. \tag{D.5}$$

This implies the restriction on β^F

$$\beta^F = e^{\sigma \mu_z} \frac{1+\bar{\pi}}{1+\bar{R}}.$$

(4) The arbitrage condition between capital and bonds (equations C.120 and C.122, using C.148) yields

$$\begin{aligned} r_k - \delta = p_m \phi e^{\mu_z} \frac{\hat{y}_m}{(1-\Omega)\hat{k}} - \delta &= r \\ p_m \phi e^{\mu_z} \frac{\hat{y}_m}{(1-\Omega)\hat{k}} &= \frac{e^{\sigma \mu_z} - \beta^F (1-\delta)}{\beta^F} \\ p_m \phi \frac{\hat{y}_m}{(1-\Omega)\hat{k}} &= \frac{e^{(\sigma-1)\mu_z} - \beta^F \frac{1-\delta}{e^{\mu_z}}}{\beta^F} \end{aligned} \tag{D.6}$$

so that

$$\frac{\phi}{\mu} \frac{\hat{y}_m}{(1-\Omega)\hat{k}} = \frac{e^{(\sigma-1)\mu_z} - \beta^F \frac{1-\delta}{e^{\mu_z}}}{\beta^F}.$$

which using $\hat{y}_m = \mu \hat{y}$ simplifies to

$$\frac{\hat{y}}{(1-\Omega)\hat{k}} = \frac{e^{(\sigma-1)\mu_z} - \beta^F \frac{1-\delta}{e^{\mu_z}}}{\phi \beta^F}.$$

It follows that

$$r_k = p_m \phi e^{\mu_z} \frac{\hat{y}_m}{(1-\Omega)\hat{k}} = \phi e^{\mu_z} \frac{\hat{y}}{(1-\Omega)\hat{k}} = \frac{e^{\sigma \mu_z} - \beta^F (1-\delta)}{\beta^F},$$

where we imposed $v = 1$. Thus, through equation (C.121), this imposes the constraint

$$\bar{\eta} = \frac{e^{\sigma \mu_z} - \beta^F (1-\delta)}{\beta^F}. \tag{D.7}$$

(5) We now work out a restriction on ϕ.Using equations (C.118), (C.141), (C.142), and (C.149), and the constraint imposed on $lsh \equiv \hat{w}[\Omega + (1-\Omega)\psi]n/y$, we obtain

$$\frac{\kappa_v}{\Omega + (1-\Omega)\psi}\frac{1}{\lambda} = \frac{\hat{Q} - \hat{w}}{1-(1-\rho)e^{\mu_z}M^F} \tag{D.8}$$

$$\frac{\kappa_v v}{m} = \frac{(1-\phi)\frac{\hat{y}}{n} - \hat{w}[\Omega + (1-\Omega)\psi]}{1-(1-\rho)e^{\mu_z}M^F} \tag{D.9}$$

$$\frac{\kappa_v v}{m} = \frac{\hat{y}}{n}\frac{(1-\phi) - lsh}{1-(1-\rho)e^{\mu_z}M^F} \tag{D.10}$$

$$vac = \frac{m}{n}\frac{(1-\phi) - lsh}{1-(1-\rho)e^{(1-\sigma)\mu_z}\beta^F} \tag{D.11}$$

Also, from (C.137), we obtain

$$\frac{m}{n} = \frac{\lambda v}{n} = \frac{\rho n}{n} = \rho.$$

We thus obtain

$$vac = \rho\frac{(1-\phi) - lsh}{1-(1-\rho)e^{(1-\sigma)\mu_z}\beta^F} \tag{D.12}$$

$$\Leftrightarrow \tag{D.13}$$

$$\phi = 1 - \frac{1-(1-\rho)e^{(1-\sigma)\mu_z}\beta^F}{\rho}vac - lsh \tag{D.14}$$

which is an equation restricting the admissible value of ϕ, i.e. the value consistent with the constraints imposed on $vac \equiv \kappa_v v/\hat{y}$ and on the labor share.

(6) From eq. (C.123), we obtain

$$\frac{\hat{\imath}}{\hat{k}} = \left[1 - \left(\frac{1-\delta}{e^{\mu_z}}\right)\right],$$

so that

$$(1-\Omega)\frac{\hat{\imath}}{\hat{y}} = (1-\Omega)\frac{\hat{k}}{\hat{y}}\frac{\hat{\imath}}{\hat{k}}.$$

Recall that

$$(1-\Omega)\frac{\hat{k}}{\hat{y}} = \frac{\phi\beta^F}{e^{(\sigma-1)\mu_z} - \beta^F\frac{1-\delta}{e^{\mu_z}}}.$$

Hence

$$(1-\Omega)\frac{\hat{\imath}}{\hat{y}} = \frac{\phi\beta^F\left(1 - \left(\frac{1-\delta}{e^{\mu_z}}\right)\right)}{e^{(\sigma-1)\mu_z} - \beta^F\frac{1-\delta}{e^{\mu_z}}}.$$

(7) Let us define aggregate consumption as

$$\hat{c} = (1-\Omega)\hat{c}^F + \Omega(n\hat{c}^e + (1-f)(1-n)\hat{c}^{uu} + sn\hat{c}^{eu})$$

Notice that, from (C.145), we have

$$\frac{\hat{c}}{\hat{y}} = 1 - vac - (1-\Omega)\frac{\hat{\imath}}{\hat{y}}. \tag{D.15}$$

(8) We can also back out ρ from the average values of s and f obtained at the estimation stage, through equation (C.139)

$$\rho = \frac{s}{1-f}. \tag{D.16}$$

(9) Now, notice that from equations (C.137), (C.139), (C.140), and (C.141) , we get

$$n = \frac{f}{s+f}, \tag{D.17}$$

so that n is known when s and f are known. It follows that we can deduce the value of m through equations (C.137) and (C.141)

$$\rho n = \lambda v = m, \tag{D.18}$$

yielding

$$\rho\frac{f}{s+f} = m. \tag{D.19}$$

(10) We now want to find an analytical formula for $s_{60} \equiv \hat{c}_{60}/\hat{c}$, where

$$\hat{c}_{60} \equiv \Omega(n\hat{c}^e + (1-f)(1-n)\hat{c}^{uu} + sn\hat{c}^{eu}).$$

For simplicity, we also define

$$\hat{c}_{40} \equiv (1-\Omega)\,\hat{c}^F.$$

Now, we have from eqations (C.128), (C.129), and (C.130)

$$n\hat{c}^e = (1-\tau)\hat{w}n + \left[\frac{1+r}{e^{\mu_z}}(1-s)n - n\right]\hat{a}^e, \tag{D.20}$$

$$sn\hat{c}^{eu} = snb^u + \frac{1+r}{e^{\mu_z}}sn\hat{a}^e, \tag{D.21}$$

so that

$$n\hat{c}^e + sn\hat{c}^{eu} = (1-\tau)\hat{w}n + snb^u + n\left[\frac{1+r}{e^{\mu_z}} - 1\right]\hat{a}^e$$

and

$$n\hat{c}^e + sn\hat{c}^{eu} + (1-n-sn)\hat{c}^{uu} = (1-n)b^u + (1-\tau)\hat{w}n + n\left[\frac{1+r}{e^{\mu_z}} - 1\right]\hat{a}^e.$$

Also, we get from (C.147)

$$\tau\hat{w}n = b^u(1-n)$$

so that

$$n\hat{c}^{e} + sn\hat{c}^{eu} + (1-n-sn)\hat{c}^{uu} = \hat{w}n + n\left[\frac{1+r}{\mathrm{e}^{\mu_z}} - 1\right]\hat{a}^{e}.$$

Recall that

$$1-n-sn = (1-f)(1-n)$$

Thus

$$\hat{c}_{60} = \Omega\left[\hat{w}n + n\left(\frac{1+r}{\mathrm{e}^{\mu_z}} - 1\right)\hat{a}^{e}\right].$$

Also, from the restriction imposed on s_{60}, we get

$$\frac{\hat{c}_{60}}{\hat{y}} = s_{60}\frac{\hat{c}}{\hat{y}}.$$

Using this, we obtain

$$\psi = \frac{\Omega}{1-\Omega}\left[\left(1 + \left(\frac{1+r}{\mathrm{e}^{\mu_z}} - 1\right)\frac{\hat{a}^{e}}{\hat{w}}\right)\frac{lsh}{s_{60}\frac{\hat{c}}{\hat{y}}} - 1\right].$$

Now

$$\frac{\hat{c}^{e}}{\hat{w}} = \left(\frac{1+r}{\mathrm{e}^{\mu_z}}(1-s) - 1\right)\frac{\hat{a}^{e}}{\hat{w}} + 1 - reprat\frac{1-n}{n}, \tag{D.22}$$

$$\left(\frac{\hat{c}^{eu}}{\hat{c}^{e}}\right)\frac{\hat{c}^{e}}{\hat{w}} = reprat + \frac{1+r}{\mathrm{e}^{\mu_z}}\frac{\hat{a}^{e}}{\hat{w}}, \tag{D.23}$$

so that

$$\frac{\hat{a}^{e}}{\hat{w}} = \frac{loss\left(1 - reprat\frac{1-n}{n}\right) - reprat}{\left(\frac{1+r}{\mathrm{e}^{\mu_z}}\left(1 - loss(1-s)\right) + loss\right)}. \tag{D.24}$$

Plugging this back above yields ψ.

(11) Now that we solved for n and ψ, we obtain from equations (C.131) and (C.133)

$$n^{eu} = sn$$

$$n^{uu} = 1 - n - n^{eu}$$

and from equation (C.143)

$$\hat{k} = \mathrm{e}^{\mu_z}\left(\mu\mathrm{e}^{\mu_z}\frac{\hat{y}}{(1-\Omega)\hat{k}}\right)^{\frac{1}{\phi-1}}\frac{[\Omega + (1-\Omega)\psi]n}{(1-\Omega)} \tag{D.25}$$

$$= \mathrm{e}^{\mu_z}\left(\mu\frac{\mathrm{e}^{\sigma\mu_z} - \beta^{F}(1-\delta)}{\phi\beta^{F}}\right)^{\frac{1}{\phi-1}}\frac{[\Omega + (1-\Omega)\psi]n}{(1-\Omega)} \tag{D.26}$$

from which we can back out $\hat{y}_m$, $\hat{y}$, and thus from (C.144)

$$\kappa_y = (\mu - 1)\hat{y}. \tag{D.27}$$

We also obtain from equation (C.123)

$$\hat{\imath} = \left[1 - \left(\frac{1-\delta}{\mathrm{e}^{\mu_z}}\right)\right]\hat{k}.$$

Finally, we can back out $\bar{w}$, $\hat{w}^s$, $\hat{Q}^s$ from the restrictions and equations (C.149) and (C.150). In particular

$$\hat{y}_m = \left(\left(\mu\frac{e^{\sigma\mu_z} - \beta^F(1-\delta)}{\phi\beta^F}\right)^{\frac{1}{\phi-1}}\right)^{\phi}([\Omega + (1-\Omega)\psi]n)$$

hence

$$\hat{y} = \frac{1}{\mu}\left(\left(\mu\frac{e^{\sigma\mu_z} - \beta^F(1-\delta)}{\phi\beta^F}\right)^{\frac{1}{\phi-1}}\right)^{\phi}([\Omega + (1-\Omega)\psi]n).$$

Now recall that

$$\hat{w} = lsh\frac{\hat{y}}{[\Omega + (1-\Omega)\psi]n}$$

so that

$$\hat{w} = \frac{1}{\mu}lsh\left(\mu\frac{e^{\sigma\mu_z} - \beta^F(1-\delta)}{\phi\beta^F}\right)^{\frac{\phi}{\phi-1}}$$

$$\bar{w} = \hat{w}\left(\frac{e^{-\mu_z}}{1+\pi}\right)^{\frac{\gamma_w}{\gamma_w-1}}.$$

Also, from equations (C.135) and (C.136), we can back out $\hat{K}$ and $\hat{F}$ via

$$\hat{K} = \hat{F} = \frac{\hat{y}}{1 - \alpha e^{\mu_z}M^F}, \tag{D.28}$$

(12) Then, imposing $\bar{m} = 1$, we can back out v from (C.138), yielding

$$v = \frac{m^{\frac{1}{1-\chi}}}{(1-(1-\rho)n)^{\frac{\chi}{1-\chi}}}, \tag{D.29}$$

from which we deduce κ_v.

(13) Finally equation (C.124), (C.125), and (C.126) imply

$$\frac{\hat{\lambda}^{eu}}{\hat{\lambda}^e} = loss^{-\sigma} = \frac{1}{s}\left(\left(\frac{\beta^F}{\beta^W}\right) - (1-s)\right). \tag{D.30}$$

From the restriction on the ratio $\hat{c}^{eu}/\hat{c}^e$, we can back out β^W using

$$\frac{\beta^F}{\beta^W} = s(loss^{-\sigma}) + (1-s).$$

Equation (C.147) together with the constraint on the replacement ratio allows us to back out

$$\tau = reprat\frac{(1-n)}{n}, \tag{D.31}$$

and from equation (C.132) we get

$$\hat{c}^{uu} = b^u. \tag{D.32}$$

(14) Then, using equations (C.128), (C.129), (C.130) we can obtain

$$\left(\frac{1}{loss}\right)\hat{c}^{eu} = (1-\tau)\hat{w} + \left[\frac{1+r}{e^{\mu_z}}(1-s) - 1\right]\frac{e^{\mu_z}}{1+r}(\hat{c}^{eu} - b^u), \tag{D.33}$$

so that we can solve for $\hat{c}^{eu}$

$$\hat{c}^{eu} = \frac{(1-\tau)\hat{w} - \left[1 - s - \frac{e^{\mu_z}}{1+r}\right] b^u}{\left(\frac{1}{loss}\right) - \left(1 - s - \frac{e^{\mu_z}}{1+r}\right)}. \tag{D.34}$$

We then back out $\hat{c}^e$.

(15) From equations (C.129), (C.130), and (C.146) we can back out

$$\hat{a}^e = \frac{e^{\mu_z}}{1+r}(\hat{c}^{eu} - b^u), \tag{D.35}$$

$$\hat{A}^e = e^{-\mu_z}(1-s)n\hat{a}^e, \tag{D.36}$$

$$\hat{a}^F = -\frac{\Omega}{1-\Omega}\hat{A}^e, \tag{D.37}$$

and finally to back out $\hat{c}^F$ from equation (C.145)

$$\hat{c}^F = \frac{1}{1-\Omega}[\hat{y} - \Omega(n\hat{c}^e + (1-f)(1-n)\hat{c}^{uu} + sn\hat{c}^{eu}) - \kappa_v v - (1-\Omega)\hat{\imath}]. \tag{D.38}$$

To complete the calculations, we also back out the Lagrange multipliers from equations (C.119), (C.125), and (C.126)

$$\hat{\lambda}^F = \left(\left(1 - \frac{h}{e^{\mu_z}}\right)\hat{c}^F\right)^{-\sigma}, \tag{D.39}$$

$$\hat{\lambda}^e = \left(\left(1 - \frac{h}{e^{\mu_z}}\right)\hat{c}^e\right)^{-\sigma}, \tag{D.40}$$

$$\hat{\lambda}^{eu} = \left(\left(1 - \frac{h}{e^{\mu_z}}\right)\hat{c}^{eu}\right)^{-\sigma}. \tag{D.41}$$

The procedure just described first imposes constraints on the parameters ψ, Ω, and $\underline{a}$. These restrictions are a priori independent from the MCMC draws. Then, a number of steady-state shares are also restricted. These restrictions must hold for each possible MCMC draw. They thus yield parameter restrictions on β^F, ρ, ϕ, $\bar{\eta}$, κ_y, κ_v, ϖ, β^W that need to readjust at each MCMC draw. The remaining parameters can be freely estimated.

D.2. **Data and Choice of Priors.** The data equivalent variables used for estimation come from the Bureau of Economic Analysis (BEA), the Federal Reserve Bank of St. Louis' FRED II database, and the Bureau of Labor Statistics (BLS). All the series are seasonally adjusted except for population. Our sample runs from 1982Q1 to 2007Q4.

Consumption is defined as the sum of personal consumption expenditures on nondurable goods and services (PCNDS) and government consumption expenditures and gross investment (GCE). The resulting series is deflated by the implicit GDP deflator (GDPDEF). Investment is defined as the sum of gross private domestic investment (GPDI) and personal consumption expenditures on durable goods (PCDG). The resulting series is also deflated by the implicit GDP deflator. These two series are converted to per-capita terms by dividing them by the civilian population, age 16 and over (CNP16OV). Inflation is calculated using the GDP deflator and the nominal interest rate is defined as the Effective Federal Funds Rate (FEDFUNDS). Finally, we measure nominal wages as the average weekly earnings of production and nonsupervisory employees, fro the Current Employment Statistics survey (CES0500000030).

For the labor-market transition probabilities, we proceed as follows. First, we compute monthly job-finding probabilities using Current Population Survey (CPS) data on unemployment and short-run unemployment, using the approach of Shimer (2005, 2012). We then compute monthly separation probabilities as residuals from a monthly flow equation similar to equation (1). Using these two series, we construct transition matrices across employment statuses for every month in the sample, and then multiply those matrices over the three consecutive months of each quarter to obtain quarterly transition probabilities.

To construct the consumption share of the poorest 60 percent, we first aggregate nondurable items in the Consumer Expenditure Survey (CEX) to compute individual nondurables consumption (using the same categories as Heathcote, Perri, and Violante, 2010). More precisely, nondurable goods are defined as the sum of the vehicle services and other vehicle expenses (insurance, maintenance, etc.), the housing services, the rent paid, other lodging expenses, household equipment and entertainment. These items are deflated using the CPI. This measure corresponds to the variable $ndpnd0$ in Heathcote et al. (2010). Heathcote et al. (2010) construct this variable from 1980Q1 to 2007Q1. We use the same methodology to extend their time series from 2007Q1 to 2012Q4. Second, in each quarter we sort households by income to aggregate the consumption of the bottom 60%, to obtain $c^{\star}_{60,t}$.

For the labor-market transition probabilities, we proceed as follows. First, we compute monthly job-finding probabilities using CPS data on unemployment and short-run unemployment, using the two-state approach of Shimer (2005, 2012). First, using deseasonalized monthly data on employment (BLS series LNS12000000), unemployment (LNS13000000) and short-run unemployment (LNS13008396), we construct monthly series for the unemployment and short-run unemployment rates, and from then for the monthly job-finding and job-loss rate (following Shimer, 2012, the short-run unemployment series is made homogenous over the entire sample by multiplying the raw series by 1.1 from 1994M1 onwards). We then construct transition matrices across employment statuses for every month in the sample, and then multiply those matrices over the three consecutive months of each quarter to obtain quarterly transition probabilities.

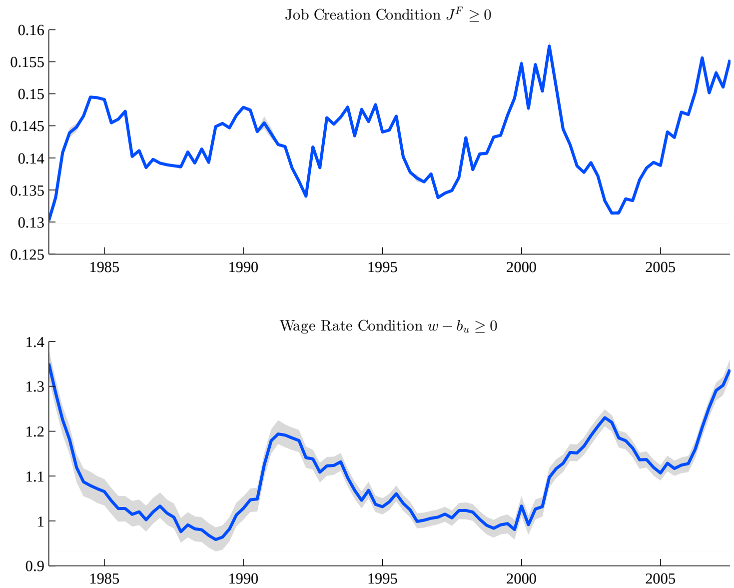
Our choice of priors is as follows.

- We impose Beta distributions for all the parameters which theoretical support is the compact $[0, 1]$;
- We use Gamma distributions for positive parameters;
- Finally, we use Inverse Gamma distributions for the standard errors of structural shocks;
- All the parameters governing the serial correlation of structural shocks have priors centred on 0.5, with standard deviation 0.1, with only one exception concerning ρ_z which is centered on 0.2, reflecting our prior that shocks to productivity growth have a lower degree of serial correlation than other shocks;
- All the standard errors of shocks have priors centred on 1, with standard deviation set to 0.2;
- The transformed curvature of the utilisation cost ν_u has a prior mean of 0.5, with standard deviation equal to 0.1
- The curvature of investment adjustment costs ν_i has a prior mean equal to 2, with standard deviation set to 0.2;
- The risk-aversion parameter σ has prior mean 1.5, with standard deviation set to 0.2;
- The degree of habit formation h, the degree of price stickiness α, the degree of price indexation γ_p, and the degree of wage indexation γ_w all have prior distributions centred on 0.5, with standard deviation equal to 0.1;
- The sensitivity of the real wage to employment ψ_n has a prior centered on 1 with standard error equal to 0.2;
- Finally, when it comes to the Taylor rule coefficient, a_π has prior mean 1.5, with standard error 0.15, a_y has prior mean 0.13, with standard error 0.10, and ρ has prior mean 0.75, with standard error 0.1.

D.3. **Verification that the wage rate is in the bargaining set.** We have to check that given the history of aggregate shocks and the uncertainty about the estimated parameters, workers, firm owners and labor intermediaries all extract a positive surplus from the match in every period in the sample. To make sure that the wage rate is indeed in the bargaining set, it must be the case that

$$w_t - b^u \mathrm{e}^{z_t} > 0$$

FIGURE 2. Wage Rate in the Bargaining Set



Note: The thick red line is the posterior mean path, the grey area is the 90 percent HPD interval

and

$$J_t^F > 0.$$

These are the conditions that make sure that, given the postulated wage rule, the labor contract lies within the bounds of the bargaining set: It is profitable for a labor intermediary to post vacancies and, at the same time, the wage rate is higher than unemployment benefits, so that an unemployed worker will not automatically turn down a job offer.

As in the preceding section, figure 2 reports the in-sample counterpart of each of these (each appropriately normalized), respectively, over the estimation sample, as implied by the smoothed values of the state variables. In each panel, we also report the associated 90 percent HPD interval (the grey area delineated by the thin, black dashed lines). Figure 2 also makes clear the posterior probability that these conditions is indeed satisfied is close to one.

D.4. **Variance decomposition.** Finally, Table 1 reports the variance decomposition for output, consumption, and investment growth, and for the employment rate. For each of these variables, the table reports the portion of the variance explained by each of the seven structural shocks, ϵ_h, $h \in \{R, c, i, w, p, , z, s\}$, at quarters 0, 4, 8, and 12.

Table 1 shows that when it comes to output, consumption, and investment, demand shocks, i.e. the monetary policy shock ϵ_R, the risk-premium shock ϵ_c, and the investment shock ϵ_i (according to the taxonomy proposed by Smets and Wouters (2007)), are significant drivers at business cycle frequencies. Indeed, these shocks explain about or more than 60 percent of the variance on impact

TABLE 1. Variance Decomposition

Quarter	ϵ_R	ϵ_c	ϵ_i	ϵ_w	ϵ_p	ϵ_z	ϵ_s
				Output Growth			
0	34.13	14.82	9.07	5.06	22.56	13.75	0.61
4	23.84	10.00	6.12	6.45	20.99	32.01	0.60
8	23.92	9.79	5.86	6.95	22.22	30.70	0.57
12	23.65	9.68	5.82	7.39	22.51	30.38	0.57
				Consumption Growth			
Quarter	ϵ_R	ϵ_c	ϵ_i	ϵ_w	ϵ_p	ϵ_z	ϵ_s
0	30.95	14.11	20.09	3.15	17.12	14.53	0.04
4	23.72	10.34	19.56	3.81	17.77	24.75	0.05
8	23.10	9.84	21.84	4.08	18.04	23.05	0.05
12	22.31	9.50	23.96	4.09	17.77	22.31	0.05
				Investment Growth			
Quarter	ϵ_R	ϵ_c	ϵ_i	ϵ_w	ϵ_p	ϵ_z	ϵ_s
0	16.33	6.75	53.87	3.30	12.68	5.58	1.49
4	12.04	4.85	43.80	5.00	12.31	20.67	1.33
8	12.07	4.74	43.65	5.31	13.15	19.82	1.27
12	11.69	4.59	44.40	5.69	13.11	19.29	1.23
				Employment			
Quarter	ϵ_R	ϵ_c	ϵ_i	ϵ_w	ϵ_p	ϵ_z	ϵ_s
0	33.82	14.40	9.32	8.85	19.00	14.27	0.33
4	18.67	5.76	10.86	24.33	37.12	3.21	0.05
8	13.87	4.24	13.41	28.49	37.50	2.45	0.04
12	12.75	3.90	15.12	28.33	36.92	2.94	0.04

Note: The variance decomposition is obtained from the $MA(\infty)$ form of the model solution. ϵ_R, ϵ_c, ϵ_i, ϵ_w, ϵ_p, ϵ_z, ϵ_s stand for monetary policy shocks, risk-premium shocks, investment shocks, wage shocks, markup shocks, technology shocks, and separation shocks (respectively).

and more than 40 percent at the fourth quarter. Similar to results reported in Justiniano, Primiceri, and Tambalotti (2010), the investment shock alone is the main driver of the business cycle.

This is not to say that supply shocks, such as the markup shock ϵ_p or the technology shock ϵ_z do not play any role. As a matter of fact, these two shocks alone explain about or more than a quarter of the variance of output and consumption at business cycle frequencies. Their contribution to investment fluctuations is less marked, however.

Finally, the contribution of labor market shocks, i.e. the wage shock ϵ_w and the job-separation shock ϵ_s, is much smaller. For example, the job-separation shock never explains more than 2 percent of the variance of any of the variables considered. The wage shock has a larger contribution for employment. However, it is dwarfed by that demand or supply shocks.

D.5. **Investigating the incentive compatibility of the family structure.** In this sectrion, we consider the following thought experiment. The economy is in its BGP. At a given period, a member of the family (whtehr employed or unemployed) is offered the possibility to leave the family and live on its own. This offer is under a veil of ignorance, that is, the offer is made before the agent knows her current status in the family and initial wealth. The agent being atomistic, she takes as given and does not affect either the labor-market transition probabilities or any of the aggregate state variable. In addition, we assume that the agent faces the same habit stocks as members who stayed in the family, for any employment history. Hence, there are, jst as for the family members, only three levels of habit stocks: $\hat{\mathbf{c}}^W(0)$ for employed agents, $\hat{\mathbf{c}}^W(1)$ for agents who just fell into unemployed at the beginning of the period, and $\hat{\mathbf{c}}^W(2)$ for agents who have been unemployed for one period or more. As a consequence, there are three different value functions to be considered, one for employed agents and two for unemployed agents. These value functions are expressed in terms of normalized variables:

$$\begin{aligned}
\hat{a} &= a\mathrm{e}^{-z} \\
\hat{c} &= c\mathrm{e}^{-z} \\
\hat{\mathbf{c}}(0) &= \mathbf{c}(0)\mathrm{e}^{-z} \\
\hat{\mathbf{c}}(1) &= \mathbf{c}(1)\mathrm{e}^{-z} \\
\hat{\mathbf{c}}(2) &= \mathbf{c}(2)\mathrm{e}^{-z}.
\end{aligned}$$

Formally, the value function for an employed agent outside the family is

$$\begin{aligned}
V(\hat{a},0) &= \max\left\{u(\hat{c}-\eta\hat{\mathbf{c}}^W(0)) + \beta\mathrm{e}^{(1-\sigma)\mu_z}[(1-s)V(\hat{a}',0)+sV(\hat{a}',1)]\right\} \\
\hat{c}+\mathrm{e}^{\mu_z}\hat{a}' &= (1-\tau)\hat{w}+(1+r)\hat{a} \\
\hat{a}' &\geq 0.
\end{aligned}$$

Similarly, the value function of an agent outside the family, who has just fell into unemployed, is

$$\begin{aligned}
V(\hat{a},1) &= \max\left\{u\left(\hat{c}-\eta\hat{\mathbf{c}}^W(1)\right) + \beta\mathrm{e}^{(1-\sigma)\mu_z}[fV(\hat{a}',0)+(1-f)V(\hat{a}',2)]\right\} \\
\hat{c}+\mathrm{e}^{\mu_z}\hat{a}' &= b^u+(1+r)\hat{a} \\
\hat{a}' &\geq 0.
\end{aligned}$$

Finally, the value function of an agent outside the family, who has been unemployed for one period or more, is

$$\begin{aligned}
V(\hat{a},2) &= \max\left\{u(\hat{c}-\eta\hat{\mathbf{c}}^W(2)) + \beta\mathrm{e}^{(1-\sigma)\mu_z}[fV(\hat{a}',0)+(1-f)V(\hat{a}',2)]\right\} \\
\hat{c}+\mathrm{e}^{\mu_z}\hat{a}' &= b^u+(1+r)\hat{a} \\
a' &\geq 0.
\end{aligned}$$

Note that in the above value functions, the aggregate state vector is not explicitly an argument since it is a constant.

The ex ante welfare of an agent offered the possibility of leaving the family is

$$V^{\text{outside}} = n^W V(\hat{A}, 0) + sn^W V(\hat{A}, 1) + (1 - n^W - sn^W) V(0, 2)$$

where $\hat{A} = A\mathrm{e}^{-z}$. In words, the welfare of an employed agent who leaves the family is $V(\hat{A}, 0)$. This is so because such an agent would leave the family with her fair share of assets $\hat{A}$. There is a proportion n^W of such agents in the family. Similarly, the welfare of an agent who has just fell into unemployment and who leaves the family is $V(\hat{A}, 1)$. Here again, this agent would leave the family with his fair share of assets $\hat{A}$. There is a proportion sn^W of such agents in the family. Finally, the welfare of an agent who has been unemployed for one period or more and who leaves the family is $V(0, 2)$. Here, this agent leaves the family with zero assets given the distribution of wealth within the family. There is a proportion $1 - n^W - sn^W$ of such agents.

By way of constrast, the ex ante welfare of an agent inside the family is the steady-state, normalized value of $V^W(\mu, X)$, which we denote V^{inside}.

$$V^{\text{inside}} = \begin{pmatrix} n^W & sn^W & 1 - n^W - sn^W \end{pmatrix} \left[I_3 - \beta^W \mathrm{e}^{(1-\sigma)\mu_z} \begin{pmatrix} 1-s & s & 0 \\ f & 0 & 1-f \\ f & 0 & 1-f \end{pmatrix} \right]^{-1} \times \begin{pmatrix} u((1-\eta)\hat{\mathbf{c}}^W(0)) \\ u((1-\eta)\hat{\mathbf{c}}^W(1)) \\ u((1-\eta)\hat{\mathbf{c}}^W(2)) \end{pmatrix}. \tag{D.42}$$

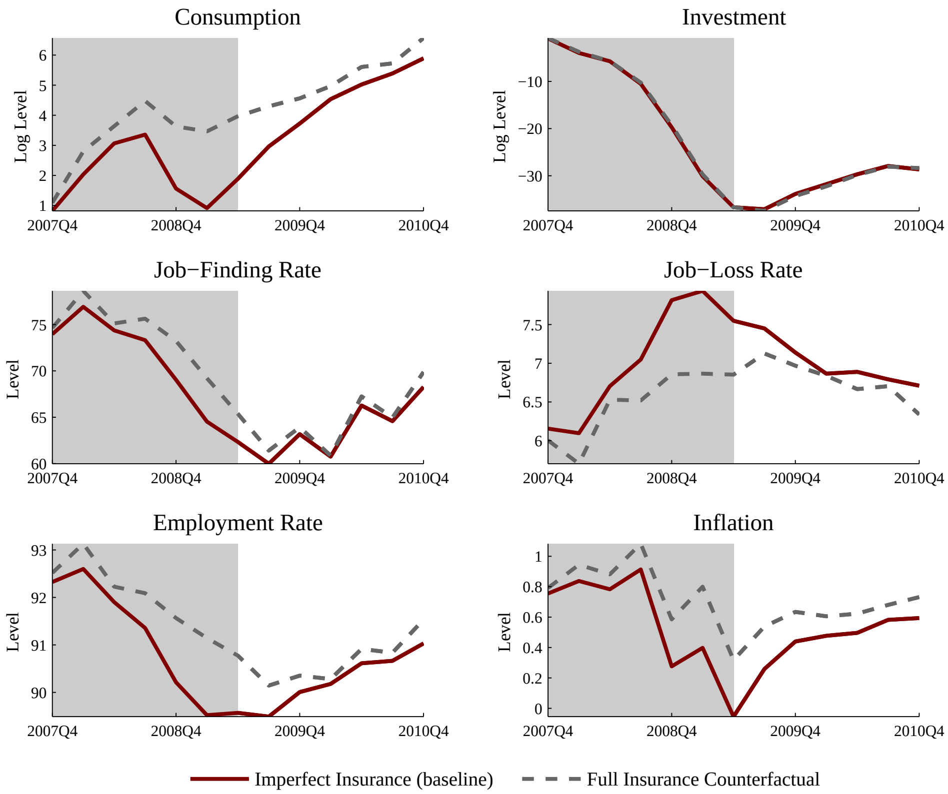
At the posterior mean of $\boldsymbol{\vartheta}_2$ and using the calibrated values for $\boldsymbol{\vartheta}_1$, we obtain:

$$V^{\text{inside}} = 141.945 > V^{\text{outside}} = 141.936.$$

To solve the above programs numerically, we resort to discrete value function iteration, with a grid with 1000 exponentially spaced points and linear interpolation to obtain values outside these grid points.

D.6. **Alternative Counterfactual Simulations.** In the benchmark counterfactual analysis, we compared the paths of key macroeconomic variables in the perfect- and in the imperfect-insurance models for the three recessions present in the sample. In doing so, we first ran the Kalman smoother at the posterior mean of the parameter distribution of the imperfect-insurance model, allowing us to back out the structural shocks. We then fed these very same shocks in the perfect-insurance model, making sure that this was done under the same parameters as those used to back out the shocks in the first place. In this procedure, it is clear that the dynamics in the perfect- and imperfect-insurance models can differ first because these two economies react in different ways to the same shocks, and, second, because these two economies may start their recessions episodes with different state variables (and possibly react in different ways to these state variables).

FIGURE 3. The Great Recession – Alternative Counterfactual Analysis



Note: The red lines correspond to the paths of consumption, investment, the job-finding rate, the job-loss rate, and the employment rate obtained under the alternative simulation strategy. The dashed, grey lines correspond to the counterfactual sample paths. Consumption and investment are reported in proportional deviation from their level at the beginning of the recession. All the other variables are expressed in level deviation from their values at the beginning of the recession. The grey area indicates the recession dates.

In this section, we complement our results by running the same exercise except that this time we force the state variables in each model version to zero at the start of each of the recessions considered. The results are reported in figures (3) and (4)

FIGURE 4. The 1990Q3 and 2001Q1 Recessions – Alternative Counterfactual Analysis

(a) 1990Q3 Recession

(b) 2001Q1 Recession

Note: The red lines correspond to the paths of consumption, investment, the job-finding rate, the job-loss rate, and the employment rate obtained under the alternative simulation strategy. The dashed, grey lines correspond to the counterfactual sample paths. Consumption and investment are reported in proportional deviation from their level at the beginning of the recession. All the other variables are expressed in level deviation from their values at the beginning of the recession. The grey area indicates the recession dates.

E. THE PERFECT-INSURANCE MODEL

The perfect-insurance model is one where in each family of workers, the head of family is allowed to transfer resources between employed and unemployed members. Private insurance within the family acts as a complement to the public UI scheme, ensuring full risk sharing between family members. However, workers, whether employed or unemployed, are still borrowing-constrained. Because they are more impatient than firm-owners, their borrowing constraint always binds. Imposing $\underline{a} = 0$ as before, this implies that total revenues at the family scale are

$$(1 - \tau_t)\hat{w}_t n_t + b^u(1 - n_t),$$

where we directly state the relevant equations in normalized terms (i.e. in deviation form the stochastic trend e^{z_t}). As before, we assume that the UI scheme is balanced in each period, so that

$$\tau_t w_t n_t = b_t^u(1 - n_t).$$

Combining these two relations, total revenues at the family scale are

$$(1 - \tau_t)\hat{w}_t n_t + \tau_t\hat{w}_t n_t = \hat{w}_t n_t.$$

Since all family members are ex ante alike, they all receive an equal share of the above revenues.

E.1. The Normalized Dynamic System.

Accordingly, the normalized system rewrites

$$1 + r_t = \frac{1 + R_{t-1}}{1 + \pi_t}, \tag{E.1}$$

$$M^F_{t,t+1} = \frac{\beta^F}{e^{\sigma\mu_z}} \mathrm{e}^{-\sigma\varphi_{z,t+1}} \frac{\lambda^F_{t+1}}{\hat{\lambda}^F_t}, \tag{E.2}$$

$$\hat{\lambda}^F_t = \left(\hat{c}^F_t - \frac{h}{\mathrm{e}^{\mu_z}}\hat{c}^F_{t-1}\mathrm{e}^{-\varphi_{z,t}}\right)^{-\sigma}, \tag{E.3}$$

$$\mathbb{E}_t[M^F_{t,t+1}(1 + r_{t+1})] = 1, \tag{E.4}$$

$$1 = p_{k,t}\mathrm{e}^{\varphi_{i,t}}\left[1 - \frac{\nu_i}{2}\left(\frac{\hat{\imath}_t\mathrm{e}^{\varphi_{z,t}}}{\hat{\imath}_{t-1}} - 1\right)^2 - \nu_i\left(\frac{\hat{\imath}_t\mathrm{e}^{\varphi_{z,t}}}{\hat{\imath}_{t-1}} - 1\right)\frac{\hat{\imath}_t\mathrm{e}^{\varphi_{z,t}}}{\hat{\imath}_{t-1}}\right] + \mathbb{E}_t\left\{\mathrm{e}^{\mu_z}M^F_{t,t+1}p_{k,t+1}\mathrm{e}^{\varphi_{i,t+1}}\nu_i\left(\frac{\hat{\imath}_{t+1}\mathrm{e}^{\varphi_{z,t+1}}}{\hat{\imath}_t} - 1\right)\left(\frac{\hat{\imath}_{t+1}\mathrm{e}^{\varphi_{z,t+1}}}{\hat{\imath}_t}\right)^2\right\}, \tag{E.5}$$

$$\eta'(v_t) = r_{k,t}, \tag{E.6}$$

$$p_{k,t} = \mathbb{E}_t[M^F_{t,t+1}\{r_{k,t+1}v_{t+1} - \eta(v_{t+1}) + (1-\delta)p_{k,t+1}\}] \tag{E.7}$$

$$\hat{k}_t = \left(\frac{1-\delta}{\mathrm{e}^{\mu_z}}\right)\hat{k}_{t-1}\mathrm{e}^{-\varphi_{z,t}} + \mathrm{e}^{\varphi_{i,t}}\left(1 - \frac{\nu_i}{2}\left(\frac{\hat{\imath}_t \mathrm{e}^{\varphi_{z,t}}}{\hat{\imath}_{t-1}} - 1\right)^2\right)\hat{\imath}_t, \tag{E.8}$$

$$M^e_{t,t+1} = \frac{\beta^W}{e^{\sigma\mu_z}}e^{-\sigma\varphi_{z,t+1}}\frac{(1-s_{t+1})\hat{\lambda}^e_{t+1} + s_{t+1}\hat{\lambda}^{eu}_{t+1}}{\hat{\lambda}^e_t}, \tag{E.9}$$

$$\hat{\lambda}^e_t = \left(\hat{c}^e_t - \frac{h}{e^{\mu_z}}\hat{c}^e_{t-1}\mathrm{e}^{-\varphi_{z,t-1}}\right)^{-\sigma}, \tag{E.10}$$

$$\hat{\lambda}^{eu}_t = \left(\hat{c}^{eu}_t - \frac{h}{e^{\mu_z}}\hat{c}^{eu}_{t-1}\mathrm{e}^{-\varphi_{z,t-1}}\right)^{-\sigma}, \tag{E.11}$$

$$1 = \mathbb{E}_t[M^e_{t,t+1}(1 + r_{t+1})], \tag{E.12}$$

$$\hat{c}^e_t = \hat{w}_t n_t, \tag{E.13}$$

$$\hat{A}^e_t = 0, \tag{E.14}$$

$$\hat{c}^{eu}_t = \hat{w}_t n_t, \tag{E.15}$$

$$n^{eu}_t = s_t n_{t-1}, \tag{E.16}$$

$$\hat{c}^{uu}_t = \hat{w}_t n_t \tag{E.17}$$

$$n^{uu}_t = 1 - n_t - n^{eu}_t, \tag{E.18}$$

$$p^*_t = \frac{\hat{K}_t}{\hat{F}_t}, \tag{E.19}$$

$$\hat{K}_t = \mu\mathrm{e}^{\varphi_{p,t}}p_{m,t}\hat{y}_t + \alpha\mathbb{E}_t\left\{\mathrm{e}^{\mu_z}M^F_{t,t+1}\left(\frac{1+\pi_{t+1}}{(1+\bar{\pi})^{1-\gamma}(1+\pi_t)^\gamma}\right)^\theta \hat{K}_{t+1}\mathrm{e}^{\varphi_{z,t+1}}\right\}, \tag{E.20}$$

$$\hat{F}_t = \hat{y}_t + \alpha\mathbb{E}_t\left\{\mathrm{e}^{\mu_z}M^F_{t,t+1}\left(\frac{1+\pi_{t+1}}{(1+\bar{\pi})^{1-\gamma}(1+\pi_t)^\gamma}\right)^{\theta-1} \hat{F}_{+1}\mathrm{e}^{\varphi_{z,t+1}}\right\}, \tag{E.21}$$

$$1 = (1-\alpha)(p_t^*)^{1-\theta} + \alpha\left(\frac{(1+\bar{\pi})^{1-\gamma}(1+\pi_{t-1})^{\gamma}}{1+\pi_t}\right)^{1-\theta}, \tag{E.22}$$

$$\Lambda_t = (1-\alpha)(p_t^*)^{-\theta} + \alpha\left(\frac{(1+\bar{\pi})^{1-\gamma}(1+\pi_{t-1})^{\gamma}}{1+\pi_t}\right)^{-\theta}\Lambda_{t-1}, \tag{E.23}$$

$$n_t = (1-\rho)n_{t-1} + \lambda_t v_t, \tag{E.24}$$

$$m_t = \bar{m}\mathrm{e}^{\varphi_{m,t}}(1-(1-\rho_t)n_{t-1})^{\chi} v_t^{1-\chi}, \tag{E.25}$$

$$s_t = \rho_t(1-f_t), \tag{E.26}$$

$$f_t = \frac{m_t}{1-(1-\rho_t)n_{t-1}}, \tag{E.27}$$

$$\lambda_t = \frac{m_t}{v_t}, \tag{E.28}$$

$$\frac{\kappa_v}{\Omega+(1-\Omega)\psi}\frac{1}{\lambda_t} = \hat{Q}_t - \hat{w}_t + \mathbb{E}_t\left[(1-\rho_{t+1})M_{t,t+1}^F \frac{\kappa_v \mathrm{e}^{\mu_z+\varphi_{z,t+1}}}{\Omega+(1-\Omega)\psi}\frac{1}{\lambda_{t+1}}\right], \tag{E.29}$$

$$\hat{y}_{m,t} = \left((1-\Omega)v_t \frac{\hat{k}_{t-1}}{\mathrm{e}^{\mu_z}}\mathrm{e}^{-\varphi_{z,t}}\right)^{\phi}\left(\left[\Omega+(1-\Omega)\psi\right]n_t\right)^{1-\phi}, \tag{E.30}$$

$$\Lambda_t \hat{y}_t = \hat{y}_{m,t} - \kappa_y, \tag{E.31}$$

$$(1-\Omega)\left(\hat{c}_t^F + \hat{\imath}_t + \eta(v_t)\frac{\hat{k}_{t-1}}{\mathrm{e}^{\mu_z}}\mathrm{e}^{-\varphi_{z,t}}\right) + \Omega\hat{w}_t n_t + \kappa_v v_t = \hat{y}_t \tag{E.32}$$

$$\hat{a}_t^F = 0, \tag{E.33}$$

$$\tau_t \hat{w}_t n_t = b^u(1-n_t), \tag{E.34}$$

$$r_{k,t} = p_{m,t}\phi\mathrm{e}^{\mu_z}\frac{\hat{y}_{m,t}}{(1-\Omega)v_t\hat{k}_{t-1}}\mathrm{e}^{\varphi_{z,t}}, \tag{E.35}$$

$$\hat{Q}_t = p_{m,t}(1-\phi)\frac{\hat{y}_{m,t}}{[\Omega+(1-\Omega)\psi]n_t} \tag{E.36}$$

$$\hat{w}_t = \left(\frac{\hat{w}_{t-1}\mathrm{e}^{-\mu_z-\varphi_{z,t}}}{1+\pi_t}\right)^{\gamma_w}\left(\bar{w}\mathrm{e}^{\varphi_{w,t}}\left(\frac{n_t}{n_{ss}}\right)^{\psi_n}\right)^{1-\gamma_w}, \tag{E.37}$$

$$\log\left(\frac{1+R_t}{1+\bar{R}}\right) = \rho_R \log\left(\frac{1+R_{t-1}}{1+\bar{R}}\right) + (1-\rho_R)\left[a_\pi \log\left(\frac{1+\pi_t}{1+\bar{\pi}}\right) + a_y \log\left(\frac{\hat{y}_t \mathrm{e}^{\varphi_{z,t}}}{\hat{y}_{t-1}}\right)\right] + \sigma_R \epsilon_{R,t}, \tag{E.38}$$

$$\rho_t = \frac{1}{1+\exp(-\bar{\rho}-\varphi_{\rho,t})}. \tag{E.39}$$

E.2. **Associated Steady State.** As before, several variables have trivial steady-state values: $\Lambda = 1$, $p_k = 1$, $p^* = 1$. Once we get rid of these, the reduced steady state is solution to the system

$$1+r = \frac{1+R}{1+\bar{\pi}}, \tag{E.40}$$

$$M^F = \frac{\beta^F}{\mathrm{e}^{\sigma\mu_z}}, \tag{E.41}$$

$$\hat{\lambda}^F = \left(\left(1-\frac{h}{\mathrm{e}^{\mu_z}}\right)\hat{c}^F\right)^{-\sigma}, \tag{E.42}$$

$$M^F(1+r) = 1, \tag{E.43}$$

$$\eta'(v) = r_k, \tag{E.44}$$

$$1 = M^F(r_k + 1 - \delta) \tag{E.45}$$

$$\left[1-\left(\frac{1-\delta}{\mathrm{e}^{\mu_z}}\right)\right]\hat{k} = \hat{\imath}, \tag{E.46}$$

$$M^e = \frac{\beta^W}{\mathrm{e}^{\sigma\mu_z}}, \tag{E.47}$$

$$\hat{\lambda}^e = \left(\left(1-\frac{h}{\mathrm{e}^{\mu_z}}\right)\hat{c}^e\right)^{-\sigma}, \tag{E.48}$$

$$\hat{\lambda}^{eu} = \left(\left(1-\frac{h}{\mathrm{e}^{\mu_z}}\right)\hat{c}^{eu}\right)^{-\sigma}, \tag{E.49}$$

$$\hat{a}^e = 0, \tag{E.50}$$

$$\hat{c}^e = \hat{w}n, \tag{E.51}$$

$$\hat{A}^e = 0 \tag{E.52}$$

$$\hat{c}^{eu} = \hat{w}n, \tag{E.53}$$

$$n^{eu} = sn, \tag{E.54}$$

$$\hat{c}^{uu} = \hat{w}n \tag{E.55}$$

$$n^{uu} = 1 - n - n^{eu}, \tag{E.56}$$

$$1 = \frac{\hat{K}}{\hat{F}}, \tag{E.57}$$

$$\hat{K} = \mu p_m \hat{y} + \alpha \mathrm{e}^{\mu_z} M^F \hat{K}, \tag{E.58}$$

$$\hat{F} = \hat{y} + \alpha \mathrm{e}^{\mu_z} M^F \hat{F}, \tag{E.59}$$

$$\rho n = \lambda v, \tag{E.60}$$

$$m = \bar{m}(1 - (1-\rho)n)^{\chi} v^{1-\chi}, \tag{E.61}$$

$$s = \rho(1 - f), \tag{E.62}$$

$$f = \frac{m}{1 - (1-\rho)n}, \tag{E.63}$$

$$\lambda = \frac{m}{v}, \tag{E.64}$$

$$\frac{\kappa_v}{\Omega + (1-\Omega)\psi}\frac{1}{\lambda} = \hat{Q} - \hat{w} + \left[(1-\rho)M^F \frac{\kappa_v e^{\mu_z}}{\Omega + (1-\Omega)\psi}\frac{1}{\lambda}\right] \tag{E.65}$$

$$\hat{y}_m = \left((1-\Omega)\frac{\hat{k}}{e^{\mu_z}}\right)^{\phi} ([\Omega + (1-\Omega)\psi]n)^{1-\phi}, \tag{E.66}$$

$$\hat{y} = \hat{y}_m - \kappa_y, \tag{E.67}$$

$$(1-\Omega)(\hat{c}^F + \hat{\imath}) + \Omega(n\hat{c}^e + (1-f)(1-n)\hat{c}^{uu} + sn\hat{c}^{eu}) + \kappa_v v = \hat{y} \tag{E.68}$$

$$(1-\Omega)\hat{a}^F + \Omega\hat{A}^e = 0, \tag{E.69}$$

$$\tau\hat{w}n = b^u(1-n), \tag{E.70}$$

$$r_k = p_m \phi e^{\mu_z}\frac{\hat{y}_m}{(1-\Omega)u\hat{k}}, \tag{E.71}$$

$$\hat{Q} = p_m(1-\phi)\frac{\hat{y}_m}{[\Omega + (1-\Omega)\psi]n} \tag{E.72}$$

$$\hat{w} = \left(\frac{e^{-\mu_z}}{1+\pi}\right)^{\frac{\gamma w}{1-\gamma w}} \bar{w}. \tag{E.73}$$

E.3. **Restrictions in the Perfect-Insurance Model.** When comparing the imperfect-insurance model with its perfect-insurance counterpart at the estimation stage, we have to make sure that the latter matches the same calibration constraints as the imperfect-insurance model. In this section, we provide details as to how this is done in practice.

We use the same set of restrictions as before. More precisely, we treat ”calibration targets” as parameters and let ”parameters” adjust endogenously to match the latter. In the contexte of the perfect-insurance model, this requires that we adapt the algorithm to solve for the steady state accordingly. In particular, we can no longer target a replacement ratio or an average consumption drop in this perfect-insurance setup.

(1) As before, we obtain

$$1 = \mu p_m, \tag{E.74}$$

(2) Recall that κ_y is selected so that aggregate monopolistic profits are zero in the steady state. The steady-state profits of firm i are given by

$$\hat{y}(i) - p_m(\hat{y}(i) + \kappa_y).$$

Since $\mu p_m = 1$, we obtain

$$\left(1 - \frac{1}{\mu}\right)\hat{y} = \frac{1}{\mu}\kappa_y \Longrightarrow (\mu - 1)\,\hat{y} = \kappa_y.$$

Since

$$\hat{y} = \hat{y}_m - \kappa_y$$

we obtain

$$\hat{y}_m = \mu\hat{y}.$$

We will use this restriction repeatedly in the remainder.

(3) Following the same line of reasoning as before

$$M^F = \frac{\beta^F}{e^{\sigma\mu_z}}, \tag{E.75}$$

$$r = \frac{e^{\sigma\mu_z}}{\beta^F} - 1, \tag{E.76}$$

and

$$R = \frac{e^{\sigma\mu_z}}{\beta^F}(1 + \bar{\pi}) - 1. \tag{E.77}$$

In fact, since output growth, R_t and π_t will be observable variables, we will directly estimate μ_z, R and π, so that β^F will be deduced from the above equation. Thus, in the preamble of the model block, we will impose

$$\beta^F = e^{\sigma\mu_z}\frac{1 + \bar{\pi}}{1 + R}.$$

(4) The arbitrage condition between capital and bonds yields

$$\begin{aligned}
r_k - \delta = p_m\phi e^{\mu_z}\frac{\hat{y}_m}{(1-\Omega)\hat{k}} - \delta &= r \\
p_m\phi e^{\mu_z}\frac{\hat{y}_m}{(1-\Omega)\hat{k}} &= \frac{e^{\sigma\mu_z} - \beta^F(1-\delta)}{\beta^F} \\
p_m\phi\frac{\hat{y}_m}{(1-\Omega)\hat{k}} &= \frac{e^{(\sigma-1)\mu_z} - \beta^F\frac{1-\delta}{e^{\mu_z}}}{\beta^F}
\end{aligned} \tag{E.78}$$

so that

$$\frac{\phi}{\mu}\frac{\hat{y}_m}{(1-\Omega)\hat{k}} = \frac{e^{(\sigma-1)\mu_z} - \beta^F\frac{1-\delta}{e^{\mu_z}}}{\beta^F}.$$

which using $\hat{y}_m = \mu\hat{y}$ simplifies to

$$\frac{\hat{y}}{(1-\Omega)\hat{k}} = \frac{e^{(\sigma-1)\mu_z} - \beta^F\frac{1-\delta}{e^{\mu_z}}}{\phi\beta^F}.$$

It follows that

$$r_k = p_m \phi e^{\mu_z} \frac{\hat{y}_m}{(1-\Omega)\hat{k}} = \phi e^{\mu_z} \frac{\hat{y}}{(1-\Omega)\hat{k}} = \frac{e^{\sigma\mu_z} - \beta^F(1-\delta)}{\beta^F},$$

where we imposed $u = 1$. Thus, this imposes the constraint

$$\bar{\eta} = \frac{e^{\sigma\mu_z} - \beta^F(1-\delta)}{\beta^F}. \tag{E.79}$$

(5) Also we obtain

$$\frac{\hat{\imath}}{\hat{k}} = \left[1 - \left(\frac{1-\delta}{e^{\mu_z}}\right)\right],$$

so that

$$(1-\Omega)\frac{\hat{\imath}}{\hat{y}} = (1-\Omega)\frac{\hat{k}}{\hat{y}}\frac{\hat{\imath}}{\hat{k}}.$$

Recall that

$$(1-\Omega)\frac{\hat{k}}{\hat{y}} = \frac{\phi\beta^F}{e^{(\sigma-1)\mu_z} - \beta^F \frac{1-\delta}{e^{\mu_z}}}.$$

Hence

$$(1-\Omega)\frac{\hat{\imath}}{\hat{y}} = \frac{\phi\beta^F\left(1 - \left(\frac{1-\delta}{e^{\mu_z}}\right)\right)}{e^{(\sigma-1)\mu_z} - \beta^F \frac{1-\delta}{e^{\mu_z}}}.$$

(6) Let us define aggregate consumption as

$$\hat{c} = (1-\Omega)\hat{c}^F + \Omega(n\hat{c}^e + (1-f)(1-n)\hat{c}^{uu} + sn\hat{c}^{eu})$$

Notice that we have

$$\frac{\hat{c}}{\hat{y}} = 1 - \frac{\kappa_v v}{\hat{y}} - (1-\Omega)\frac{\hat{\imath}}{\hat{y}}. \tag{E.80}$$

(7) We can also back out ρ from the average values of s and f obtained at the estimation stage, through

$$\rho = \frac{s}{1-f}. \tag{E.81}$$

(8) We now work out a restriction on ϕ. We obtain

$$\frac{\kappa_v}{\Omega + (1-\Omega)\psi}\frac{1}{\lambda}, = \frac{\hat{Q} - \hat{w}}{1 - (1-\rho)e^{\mu_z} M^F} \tag{E.82}$$

$$\frac{\kappa_v v}{m} = \frac{(1-\phi)\frac{\hat{y}}{n} - \hat{w}[\Omega + (1-\Omega)\psi]}{1 - (1-\rho)e^{\mu_z} M^F} \tag{E.83}$$

$$\frac{\kappa_v v}{m} = \frac{\hat{y}}{n}\frac{(1-\phi) - \left(\frac{\hat{w}[\Omega+(1-\Omega)\psi]n}{\hat{y}}\right)}{1 - (1-\rho)e^{\mu_z} M^F} \tag{E.84}$$

$$\left(\frac{\kappa_v v}{\hat{y}}\right) = \frac{m}{n}\frac{(1-\phi) - \left(\frac{\hat{w}[\Omega+(1-\Omega)\psi]n}{\hat{y}}\right)}{1 - (1-\rho)e^{(1-\sigma)\mu_z}\beta^F} \tag{E.85}$$

Also we obtain

$$\frac{m}{n} = \frac{\lambda v}{n} = \frac{\rho n}{n} = \rho.$$

We thus obtain

$$\left(\frac{\kappa_v v}{\hat{y}}\right) = \rho \frac{(1-\phi) - \left(\frac{\hat{w}[\Omega+(1-\Omega)\psi]n}{\hat{y}}\right)}{1-(1-\rho)\mathrm{e}^{(1-\sigma)\mu_z}\beta^F} \tag{E.86}$$

$$\Leftrightarrow \tag{E.87}$$

$$\phi = 1 - \frac{1-(1-\rho)\mathrm{e}^{(1-\sigma)\mu_z}\beta^F}{\rho}\left(\frac{\kappa_v v}{\hat{y}}\right) - \left(\frac{\hat{w}[\Omega+(1-\Omega)\psi]n}{\hat{y}}\right) \tag{E.88}$$

which is an equation restricting the admissible value of ϕ, i.e. the value consistent with the constraints imposed on $\kappa_v v/\hat{y}$ and on the labor share.

(9) Now, we get

$$n = \frac{f}{s+f}, \tag{E.89}$$

so that n is known when s and f are known. It follows that we can deduce the value of m through

$$\rho n = \lambda v = m, \tag{E.90}$$

yielding

$$\rho \frac{f}{s+f} = m. \tag{E.91}$$

(10) We now want to find an analytical formula for $s_{60} = \hat{c}_{60}/\hat{c}$, where

$$\hat{c}_{60} \equiv \Omega(n\hat{c}^e + (1-f)(1-n)\hat{c}^{uu} + sn\hat{c}^{eu}).$$

Thus

$$\hat{c}_{60} = \Omega\hat{w}n.$$

Also, from the restriction imposed on s_{60}, we get

$$\frac{\hat{c}_{60}}{\hat{y}} = s_{60}\frac{\hat{c}}{\hat{y}}.$$

Using this, we obtain

$$\psi = \frac{\Omega}{1-\Omega}\left[\frac{\mathrm{lsh}}{s_{60}\frac{\hat{c}}{\hat{y}}} - 1\right].$$

(11) Now that we solved for n and ψ, we obtain

$$n^{eu} = sn$$

$$n^{uu} = 1 - n - n^{eu}$$

and

$$\hat{k} = e^{\mu_z} \left(\mu e^{\mu_z} \frac{\hat{y}}{(1-\Omega)\hat{k}} \right)^{\frac{1}{\phi-1}} \frac{[\Omega + (1-\Omega)\psi]n}{(1-\Omega)} \tag{E.92}$$

$$= e^{\mu_z} \left(\mu \frac{e^{\sigma\mu_z} - \beta^F(1-\delta)}{\phi\beta^F} \right)^{\frac{1}{\phi-1}} \frac{[\Omega + (1-\Omega)\psi]n}{(1-\Omega)} \tag{E.93}$$

from which we can back out $\hat{y}_m$, $\hat{y}$, and thus

$$\kappa_y = (\mu - 1)\hat{y}. \tag{E.94}$$

We also obtain

$$\hat{\imath} = \left[1 - \left(\frac{1-\delta}{e^{\mu_z}} \right) \right] \hat{k}.$$

Finally, we can back out $\bar{w}$, $\hat{w}^s$, $\hat{Q}^s$. In particular

$$\hat{y}_m = \left(\left(\mu \frac{e^{\sigma\mu_z} - \beta^F(1-\delta)}{\phi\beta^F} \right)^{\frac{1}{\phi-1}} \right)^{\phi} ([\Omega + (1-\Omega)\psi]n)$$

hence

$$\hat{y} = \frac{1}{\mu} \left(\left(\mu \frac{e^{\sigma\mu_z} - \beta^F(1-\delta)}{\phi\beta^F} \right)^{\frac{1}{\phi-1}} \right)^{\phi} ([\Omega + (1-\Omega)\psi]n).$$

Now recall that

$$\hat{w} = \left(\frac{\hat{w}[\Omega + (1-\Omega)\psi]n}{\hat{y}} \right) \frac{\hat{y}}{[\Omega + (1-\Omega)\psi]n}$$

so that

$$\hat{w} = \frac{1}{\mu} \left(\frac{\hat{w}[\Omega + (1-\Omega)\psi]n}{\hat{y}} \right) \left(\mu \frac{e^{\sigma\mu_z} - \beta^F(1-\delta)}{\phi\beta^F} \right)^{\frac{\phi}{\phi-1}}$$

$$\bar{w} = \hat{w} \left(\frac{e^{-\mu_z}}{1+\pi} \right)^{\frac{\gamma_w}{\gamma_w - 1}}.$$

Also, we can back out $\hat{K}_p$ and $\hat{F}_p$ via

$$\hat{K} = \hat{F} = \frac{\hat{y}}{1 - \alpha e^{\mu_z} M^F}, \tag{E.95}$$

(12) Then, imposing $\bar{m} = 1$, we can back out v, yielding

$$v = \frac{m^{\frac{1}{1-\chi}}}{(1-(1-\rho)n)^{\frac{\chi}{1-\chi}}}, \tag{E.96}$$

from which we deduce κ_v.

(13) Finally, we impose the same β^F and the same β^W as in the imperfect-insurance model.

(14) We can back out $\hat{c}^F$

$$\hat{c}^F = \frac{1}{1-\Omega}[\hat{y} - \Omega\hat{w}n - \kappa_v v - (1-\Omega)\hat{\imath}]. \tag{E.97}$$

To complete the calculations, we also back out the Lagrange multipliers

$$\hat{\lambda}^F = \left(\left(1 - \frac{h}{e^{\mu_z}}\right)\hat{c}^F\right)^{-\sigma}, \tag{E.98}$$

$$\hat{\lambda}^e = \left(\left(1 - \frac{h}{e^{\mu_z}}\right)\hat{c}^e\right)^{-\sigma}, \tag{E.99}$$

$$\hat{\lambda}^{eu} = \left(\left(1 - \frac{h}{e^{\mu_z}}\right)\hat{c}^{eu}\right)^{-\sigma}. \tag{E.100}$$

The procedure just described first imposes constraints on the parameters ψ, Ω, and $\underline{a}$. These restrictions are a priori independent from the MCMC draws. Then, a number of steady-state shares are also restricted. These restrictions must hold for each possible MCMC draw. They thus yield parameter restrictions on β^F, ρ, ϕ, $\bar{\eta}$, κ_y, κ_v, $\bar{w}$, β^W that need to readjust at each MCMC draw. The remaining parameters can be freely estimated.

E.4. **Estimation Results in the Perfect-Insurance Model.** In the main paper, we compared the marginal likelihoods of the baseline model and its perfect-insurance counterpart. Recall that in this exercise, we impose (i) the exact same calibration restriction in both model versions and (ii) use the exact same priors as in the imperfect-insurance model.

Table 2 reports the estimation outcome under the perfect-insurance model. Overall, the posterior mean of the parameters obtained in the perfect-insurance model do not differ much from their imperfect-insurance counterparts. Two notable exceptions are the curvature of the utility function σ and the degree of external habits h. In particular, σ is higher in the perfect-insurance model than in the imperfect-insurance setup, while the converse holds for h.

E.5. **Analytical steady state in perfect-insurance model.** In section 5, when we simulate the perfect-insurance model, we use it to assess what would have happened in a counterfactual setup with perfect insurance. To make this exercise well defined, it is important to make sure that the parameters in the perfect-insurance model have the exact same values as their counterparts in the imperfect-insurance model. In this case, we no longer impose the calibration restrictions detailed before.

Doing so requires manipulating an analytical steady state for the perfect-insurance model. In this section, we provide details on how we proceeded. After straigthforward manipulations, we arrive at the following set of equations.

$$M^F = \frac{\beta^F}{e^{\sigma\mu_z}}, \tag{E.101}$$

TABLE 2. Estimation Results - HM Model, Full Set of Observable Variables

Parameter	Prior shape	Prior Mean	Prior S.D.	Post. Mean	Post. S.D.	Low	High
σ	Gamma	1.50	0.20	0.83	0.10	0.66	0.99
h	Beta	0.50	0.10	0.53	0.04	0.46	0.60
ν_i	Gamma	2.00	0.20	1.88	0.19	1.57	2.20
ν_u	Beta	0.50	0.10	0.69	0.08	0.56	0.82
α	Beta	0.50	0.10	0.75	0.03	0.70	0.81
γ_p	Beta	0.50	0.10	0.28	0.09	0.13	0.42
γ_w	Beta	0.50	0.10	0.86	0.03	0.82	0.91
ψ_n	Gamma	1.00	0.20	1.59	0.28	1.13	2.04
ρ	Beta	0.75	0.10	0.49	0.06	0.40	0.58
a_π	Gamma	1.50	0.10	1.96	0.10	1.80	2.14
a_y	Gamma	0.13	0.10	0.50	0.17	0.22	0.77
ρ_z	Beta	0.20	0.10	0.51	0.07	0.40	0.61
ρ_c	Beta	0.50	0.10	0.65	0.04	0.59	0.71
ρ_w	Beta	0.50	0.10	0.72	0.08	0.59	0.84
ρ_i	Beta	0.50	0.10	0.87	0.03	0.81	0.92
ρ_p	Beta	0.50	0.10	0.89	0.04	0.84	0.95
ρ_s	Beta	0.50	0.10	0.67	0.05	0.58	0.76
ρ_R	Beta	0.50	0.10	0.60	0.08	0.46	0.73
ρ_u	Beta	0.50	0.10	0.82	0.04	0.76	0.88
σ_c	Inverted Gamma	1.00	0.20	0.63	0.08	0.51	0.76
σ_w	Inverted Gamma	1.00	0.20	0.49	0.04	0.43	0.55
σ_i	Inverted Gamma	1.00	0.20	2.48	0.31	1.98	2.99
σ_p	Inverted Gamma	1.00	0.20	1.53	0.26	1.13	1.91
σ_z	Inverted Gamma	1.00	0.20	1.19	0.08	1.06	1.32
σ_R	Inverted Gamma	1.00	0.20	0.42	0.03	0.36	0.47
σ_s	Inverted Gamma	1.00	0.20	6.94	0.46	6.17	7.69
σ_u	Inverted Gamma	1.00	0.20	0.79	0.05	0.70	0.88

Note: Low and High stand for the lower and upper boundaries of the 90 percent HPD interval, respectively.

$$M^e = \frac{\beta^W}{e^{\sigma\mu_z}}, \tag{E.102}$$

$$r = 1/M^F - 1, \tag{E.103}$$

$$R = (1+r)(1+\bar{\pi}) - 1, \tag{E.104}$$

$$\eta'(u) = r_k \Rightarrow u = 1, \tag{E.105}$$

$$r_k = 1/M^F + \delta - 1 \tag{E.106}$$

$$\frac{\hat{\imath}}{\hat{k}} = 1 - \left(\frac{1-\delta}{e^{\mu_z}}\right), \tag{E.107}$$

$$p_m = \frac{1}{\mu} \tag{E.108}$$

$$\hat{y} = \mu^{-1}\hat{y}_m \tag{E.109}$$

$$\frac{(1-\Omega)r_k}{\phi e^{\mu_z}} = \frac{\hat{y}}{\hat{k}}, \tag{E.110}$$

$$\hat{w} = \left(\frac{e^{-\mu_z}}{1+\pi}\right)^{\frac{\gamma w}{1-\gamma w}} \bar{w}. \tag{E.111}$$

$$\frac{\hat{y}}{(1-\Omega)\frac{\hat{k}}{e^{\mu_z}}} = \mu\left(\frac{(1-\Omega)\frac{\hat{k}}{e^{\mu_z}}}{[\Omega+(1-\Omega)\psi]n}\right)^{\phi-1}, \tag{E.112}$$

$$\frac{\hat{y}}{[\Omega+(1-\Omega)\psi]n} = \mu\left(\frac{(1-\Omega)\frac{\hat{k}}{e^{\mu_z}}}{[\Omega+(1-\Omega)\psi]n}\right)^{\phi}, \tag{E.113}$$

$$\hat{Q} = (1-\phi)\frac{\hat{y}}{[\Omega+(1-\Omega)\psi]n} \tag{E.114}$$

$$\frac{\kappa_v}{\lambda} = \frac{(\Omega+(1-\Omega)\psi)\,(\hat{Q}-\hat{w})}{1-(1-\rho)M^F e^{\mu_z}} \tag{E.115}$$

$$\lambda = \bar{m}\left(\frac{1-(1-\rho)n}{v}\right)^{\chi}, \tag{E.116}$$

$$m = \bar{m}(1-(1-\rho)n)^{\chi}v^{1-\chi}, \tag{E.117}$$

$$f = \bar{m}\left(\frac{1-(1-\rho)n}{v}\right)^{\chi-1}, \tag{E.118}$$

$$s = \rho(1-f), \tag{E.119}$$

$$\lambda = \frac{m}{v}, \tag{E.120}$$

$$\rho n = \lambda v, \tag{E.121}$$

$$\hat{c}^{uu} = \hat{c}^{eu} = \hat{c}^{e} = \hat{w} n, \tag{E.122}$$

$$n^{eu} = sn, \tag{E.123}$$

$$n^{uu} = 1 - n - n^{eu}, \tag{E.124}$$

$$(1-\Omega)(\hat{c}^F + \hat{\imath}) + \Omega \hat{w} + \kappa_v v = \hat{y} \tag{E.125}$$

$$\hat{\lambda}^F = \left(\left(1 - \frac{h}{e^{\mu_z}}\right)\hat{c}^F\right)^{-\sigma}, \tag{E.126}$$

$$\hat{\lambda}^e = \left(\left(1 - \frac{h}{e^{\mu_z}}\right)\hat{c}^e\right)^{-\sigma}, \tag{E.127}$$

$$\hat{\lambda}^{eu} = \left(\left(1 - \frac{h}{e^{\mu_z}}\right)\hat{c}^{eu}\right)^{-\sigma}, \tag{E.128}$$

$$1 = \frac{\hat{K}}{\hat{F}}, \tag{E.129}$$

$$\hat{F} = \hat{K} = \hat{y} + \alpha e^{\mu_z} M^F \hat{K}, \tag{E.130}$$

$$\hat{y} = \hat{y}_m - \kappa_y, \tag{E.131}$$

$$\hat{y}_m = \left((1-\Omega)\frac{\hat{k}}{e^{\mu_z}}\right)^{\phi} \left([\Omega + (1-\Omega)\psi] n\right)^{1-\phi}, \tag{E.132}$$

$$\tau \hat{w} n = b^u (1-n), \tag{E.133}$$

This system is entirely recursive.

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